Robust Control

Spring, 2019

Instructor: Prof. Masayuki Fujita (S5-303B)

3rd class

Tue., 23rd April, 2019, 10:45～12:15,

S423 Lecture Room

3. Robustness and Uncertainty

3.1 Why Robustness? [SP05, Sec. 4.1.1, 7.1, 9.2]

3.2 Representing Uncertainty [SP05, Sec. 7.2, 7.3, 7.4]

3.3 Uncertain Systems [SP05, Sec. 8.1, 8.2, 8.3]

3.4 Systems with Structured Uncertainty [SP05, Sec. 8.2]

Reference:

[SP05] S. Skogestad and I. Postlethwaite, ***Multivariable Feedback Control; Analysis and Design,*** Second Edition, Wiley, 2005.

Why Robustness?

Birth of Modern Control Theory

- Modern Control Theory by State Space Method

1960 1st IFAC World Congress @Moscow

State Space

R.E.Kalman   R. Bellman   L.S.Pontryagin

On the General Theory of Control Systems

R.E.Kalman, 1st IFAC World Congress, 1960

[AM08, Fig. 2.5(b), p. 36]

Glory of LQG Control

- LQG (Linear Quadratic Gaussian) Control

Special Issue on Linear-Quadratic-Gaussian Problem

IEEE TAC Special Issue,16 - 6, 1971 (About 340 pages)

M.Athans

Linear System

$$\begin{cases} \dot{x} = Ax + Bu + \xi \\ y = Cx + \eta \end{cases}$$

Cost Function

$$J = E\left[\lim_{T\to\infty} \frac{1}{T}\int_0^T y^T y + u^T u dt\right]$$

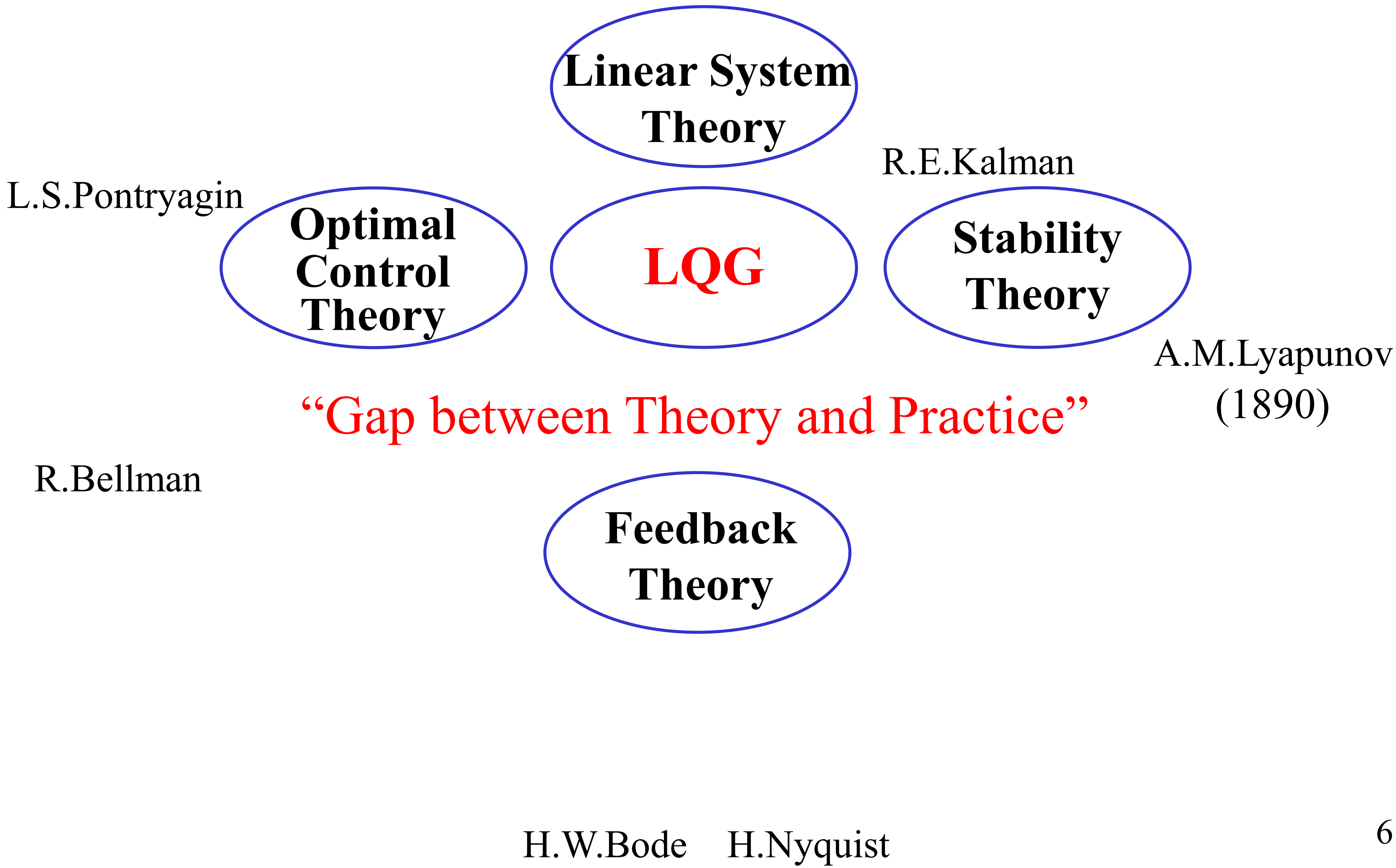
$$\begin{cases} u = K\hat{x} \\ \dot{\hat{x}} = A\hat{x} + Bu - L(y - C\hat{x}) \end{cases} \quad \begin{matrix} K = -B^T P \\ L = -SC^T \end{matrix}$$

$$P \geq 0; \quad PA + A^T P - PBB^T P + C^T C = 0$$

$$S \geq 0; \quad AS + SA^T - SC^T CS + V = 0$$

Trends in the 1970s

Glory and Drawback of LQG Control

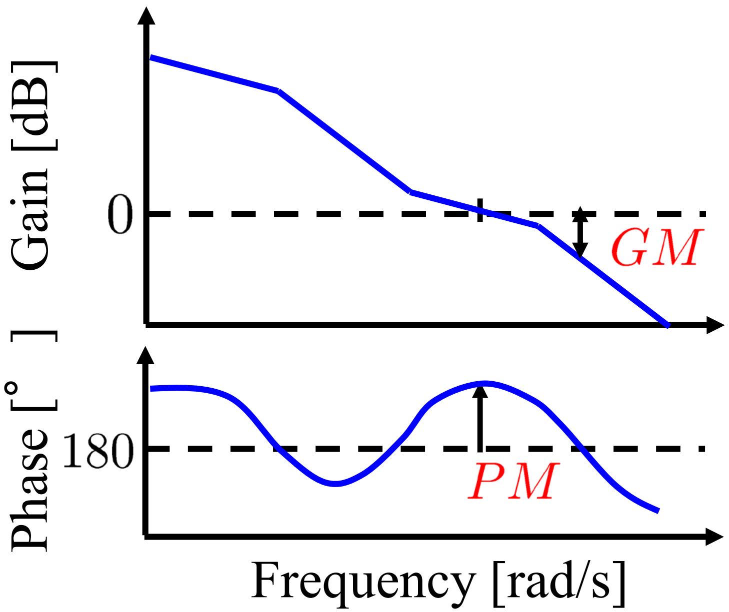


Drawback of LQG Control

- Stability Margin in Multivariable Systems from frequency domain

Good, Bad, or Optimal?

H.H.Rosenbrock (UMIST), IEEE TAC Special Issue, 16 - 6, 1971



essential requirement … that changes of loop gains … in all combinations, should leave the system with an adequate stability margin.

Catastrophe of LQG

Applications of LQG Control

A.E.Bryson. Jr., IEEE TAC, 22 - 5, 1977

- F-8C Crusader Aircraft
- Trident Submarine (1975)

Stability Margin in
Multivariable Systems

Discussions

… very limited success …
… not very practical …

Blind Spot of LQG Control

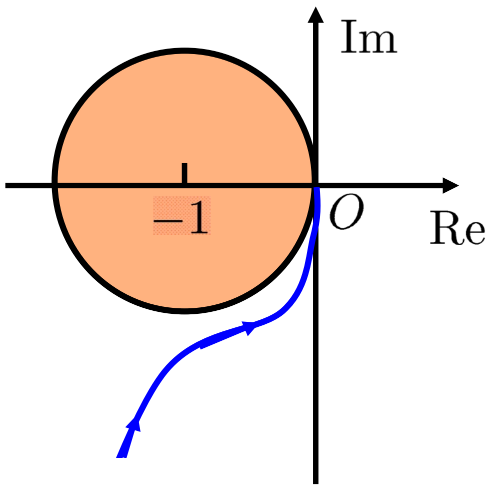
Stability Margin of LQ Control

1964 Circle Criterion Inverse Problem

In the frequency domain, the vector locus of the open loop transfer function $-K\Phi(j\omega)$ never enters the circle centered at -1 with radius 1

(i) Gain Margin : ∞
(ii) Phase Margin : More than or equal 60°
(iii) Allowable Range of Gain Decrease : Until 50% (1/2)

When is a Linear Control System Optimal?



Multivariable LQ

R.E. Kalman, ASME, 86 - D, 1964

Nyquist Plot of $-K\Phi(j\omega)$

M.Safonov

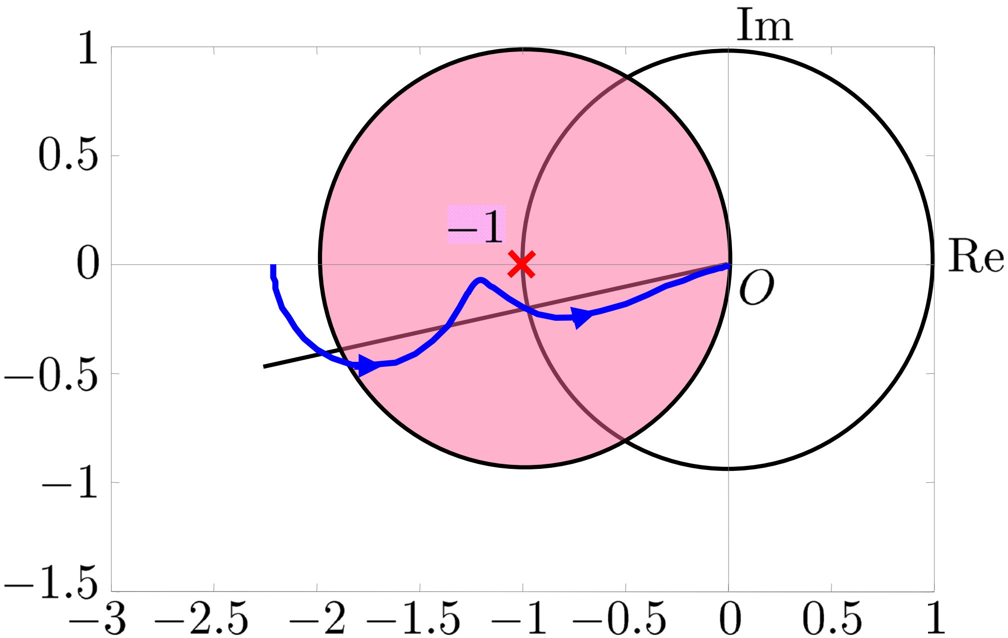
Blind Spot of LQG Control

Stability Margin of LQG Control (Fragile)

J.Doyle, G.Stein, IEEE TAC, 24 - 4, 1979

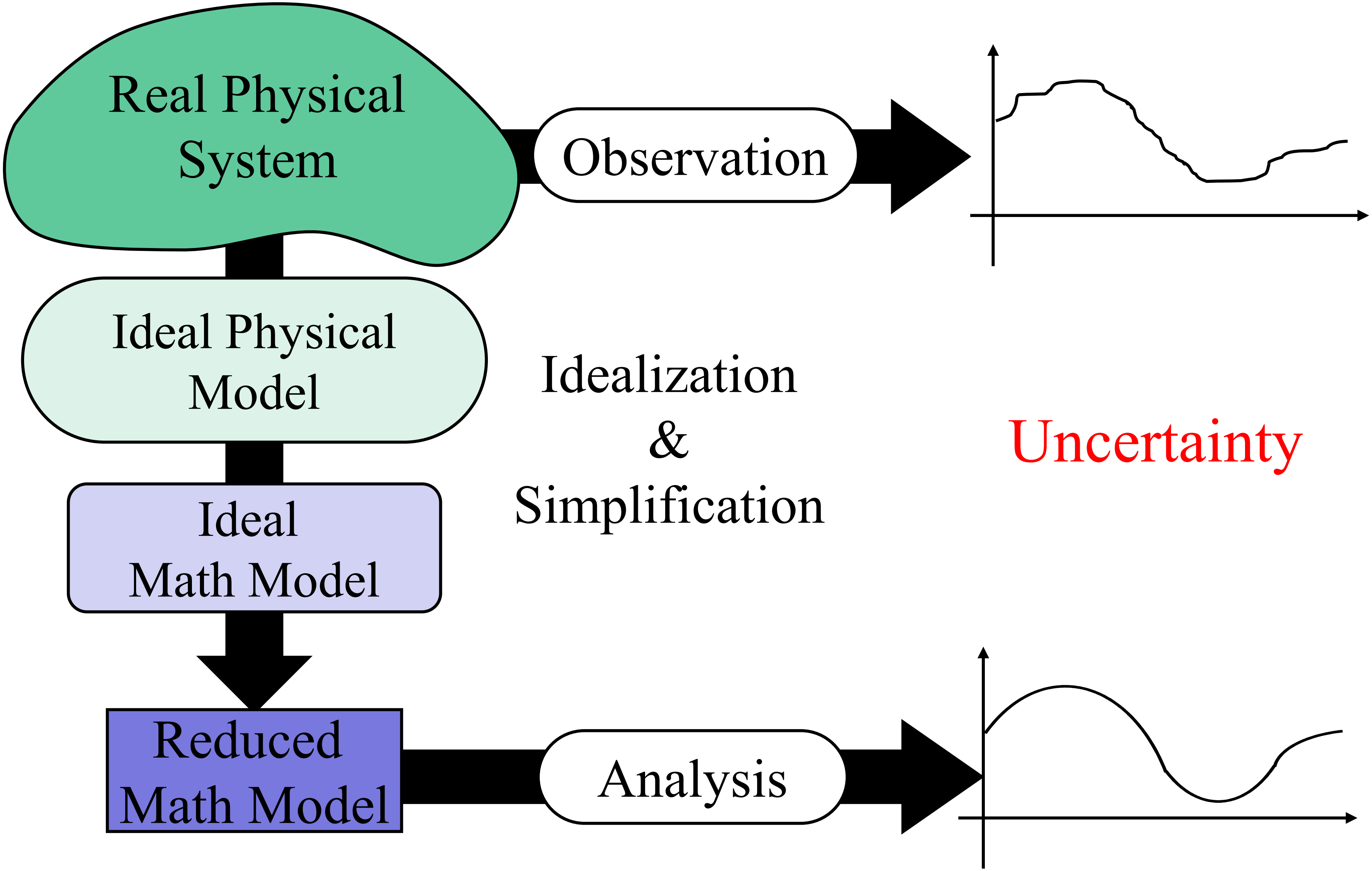
LQG Regulator

Phase Margin: 15° Oops…



Nyquist plot for the resulting observer-based controller is shown in Fig. 2. Oops… less than 15° phase margin.

System and Model



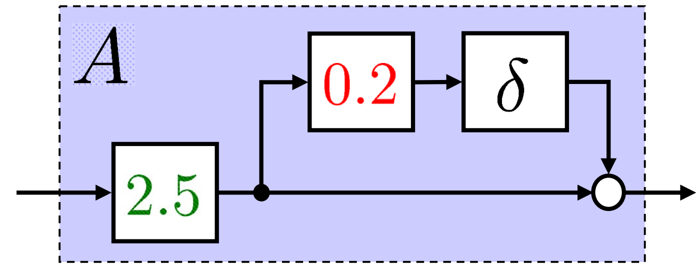
Representing Uncertainty in SISO Systems

Uncertainty Regions [SP05, p. 265]

[SP05, Ex., p. 265] First Order Plant Model $P(s) = \dfrac{A}{Ts+1}$

Case 1: Uncertain Gain $2 \le A \le 3$

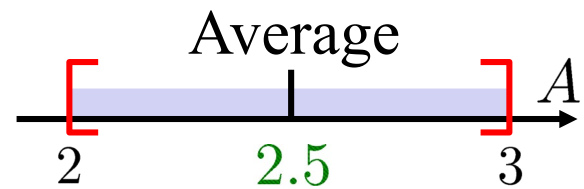
$$A = 2.5(1 + \delta \cdot 0.2), \ \ |\delta| \le 1$$



$$\left\{ \begin{array}{ll} \text{Nominal Value} & 2.5 \\ \text{Uncertainty } \pm 20\% & \text{any } |\delta| \le 1 \end{array} \right\}$$

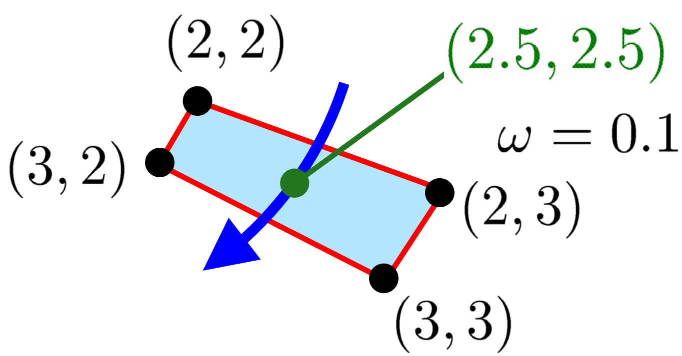
[Ex.] $A = 3$ for $\delta = 1$

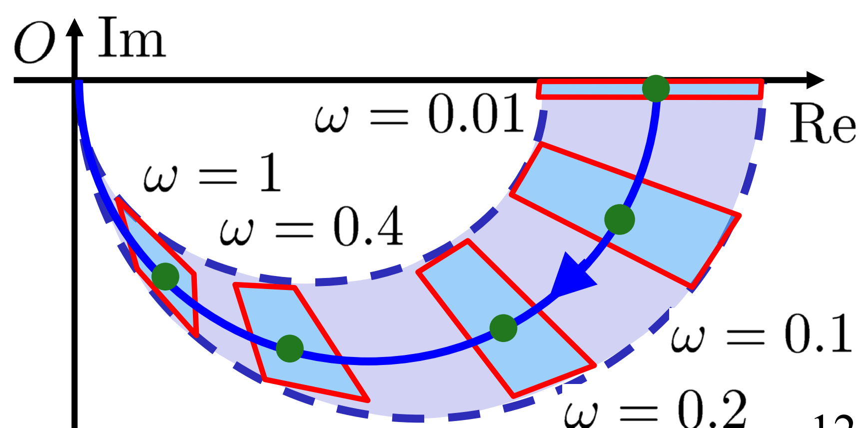
$A = 2$ for $\delta = -1$



Case 2: Uncertain Gain/Time Constant

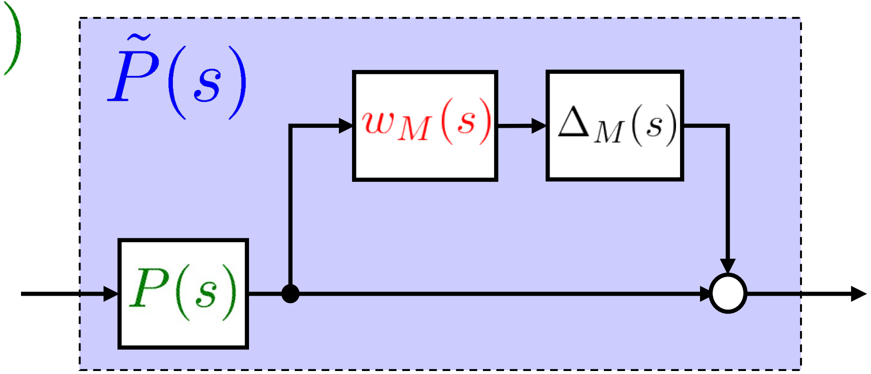
$$2 \le A \le 3, \ \ 2 \le T \le 3$$





Multiplicative Uncertainty in SISO Systems [SP05, p. 267]

$$\tilde{P}(s) = (1 + \Delta_M(s) w_M(s)) P(s)$$



$\tilde{P}(s)$: Perturbed Plant Model

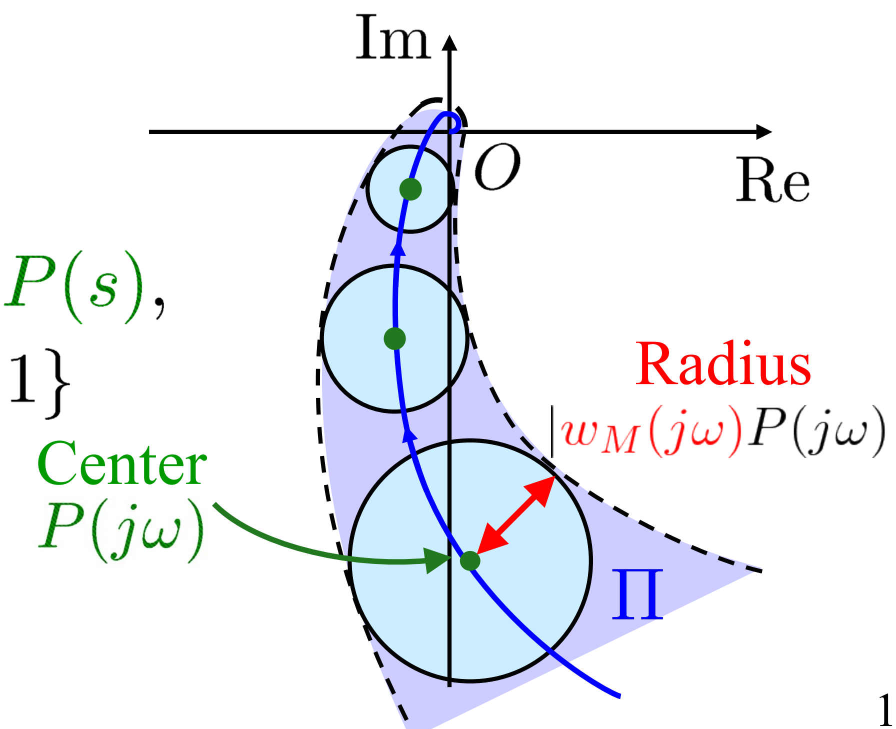
$P(s)$: *Nominal* Plant Model

$w_M(s)$: Uncertainty Weight

any $\|\Delta_M\|_\infty \le 1$

A Set of Plant Models

$$\Pi = \{\tilde{P}(s) | \tilde{P}(s) = (1 + \Delta_M(s) w_M(s)) P(s), \|\Delta_M\|_\infty \le 1\}$$

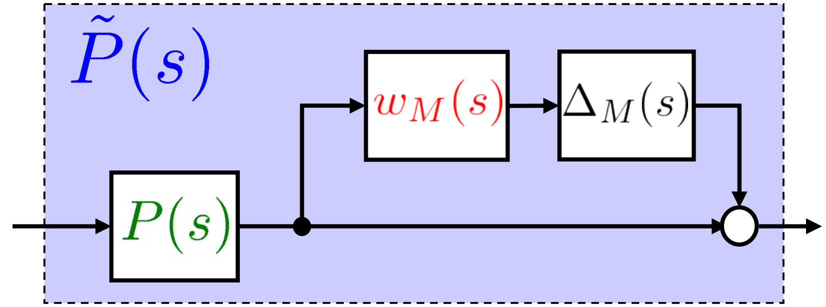


Disc Uncertainty

Center: $P(j\omega)$
Radius: $|w_M(j\omega)P(j\omega)|$

Obtaining Uncertainty Weight $w_M(s)$ [SP05, p. 268]

$$\tilde{P}(s) = (1 + \Delta_M(s) w_M(s)) P(s), \quad \|\Delta_M\|_\infty \le 1$$



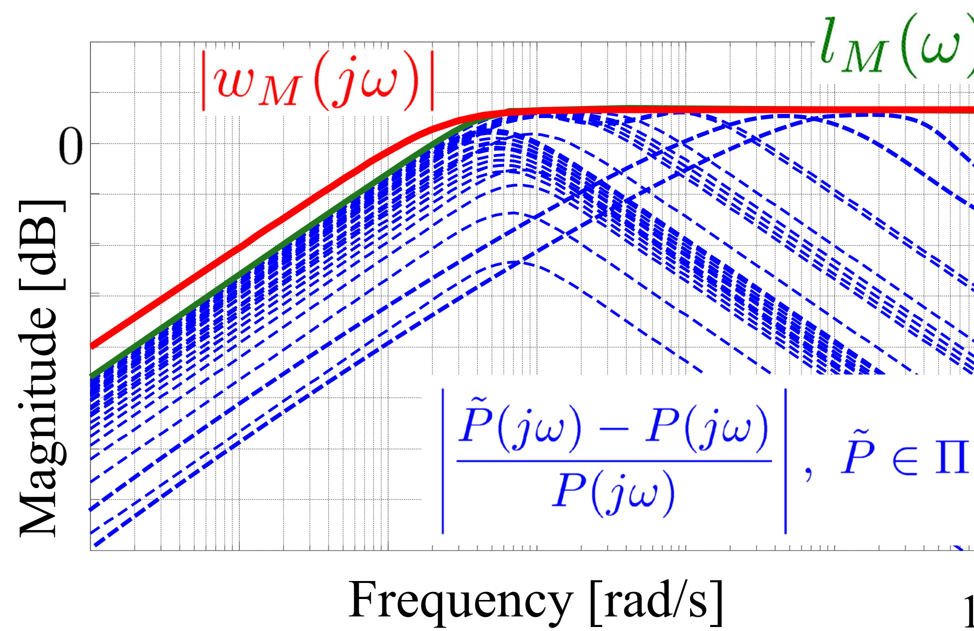
Step 1. Select a nominal model $P(s)$

Step 2. At each frequency, find the smallest radius $l_M(\omega)$ which includes the possible plants $\tilde{P} \in \Pi$:

$$l_M(\omega) = \max_{\tilde{P} \in \Pi} \left| \frac{\tilde{P}(j\omega) - P(j\omega)}{P(j\omega)} \right|$$

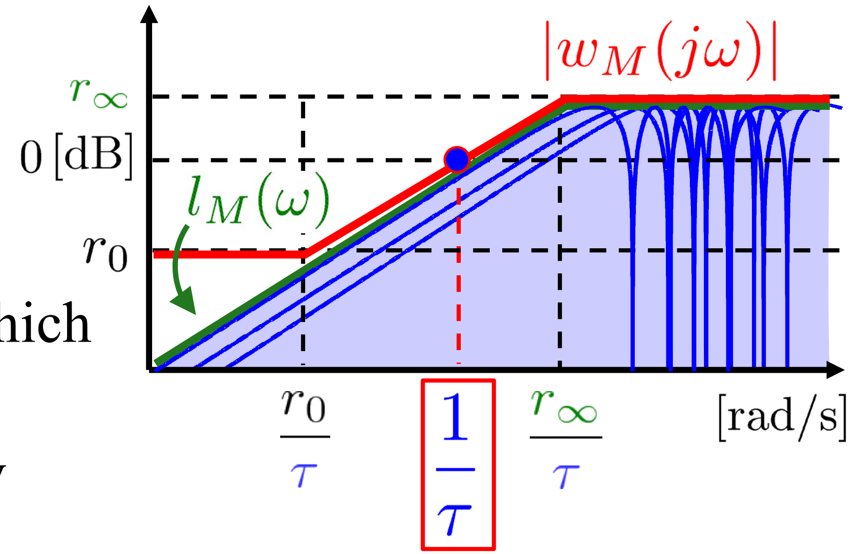
Step 3. Choose a (reduced order) weight $w_M(s)$ to cover the set:

$$|w_M(j\omega)| \ge l_M(\omega), \forall \omega$$



Uncertainty Weight $w_M(s)$ [SP05, p. 273]

$$w_M(s) = \frac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1}$$



$1/\tau$: (Approximately) the frequency at which the relative uncertainty reaches 100%.

r_∞ : Magnitude of w_M at high frequency

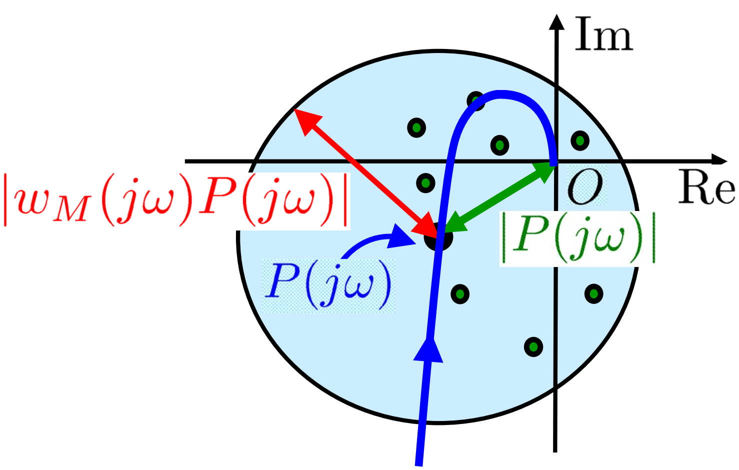
r_0 : Relative uncertainty at steady-state

$$|w_M(j\omega)| \geq 1 \ \ (\omega \geq 1/\tau)$$

$$|w_M(j\omega)P(j\omega)| \geq |P(j\omega)|$$

Frequency at which the relative uncertainty exceeds 100%

Phase Information: Lost



[SP05, Ex. 7.6] Time-delay Variations (p. 269)

$$\tilde{P}(s) = \frac{1}{s+1}e^{-\theta s}, \quad 0 \le \theta \le 1$$

$$\Pi = \{\tilde{P}(s) | \tilde{P}(s) = (1 + \Delta_M(s) w_M(s))P(s), \ \|\Delta_M\| \le 1\}$$

Step 1. Nominal Model: $\theta = 0$ ➡ $P(s) = \frac{1}{s+1}$

Step 2. $l_M(\omega) = \max_{\tilde{P}\in\Pi} \left| \frac{\tilde{P}(j\omega) - P(j\omega)}{P(j\omega)} \right| = \max_{\tilde{P}\in\Pi} |e^{-j\omega\theta} - 1| \le 2$

Step 3. $w_M(s) = \frac{\tau s}{\frac{\tau}{r_\infty}s + 1}$

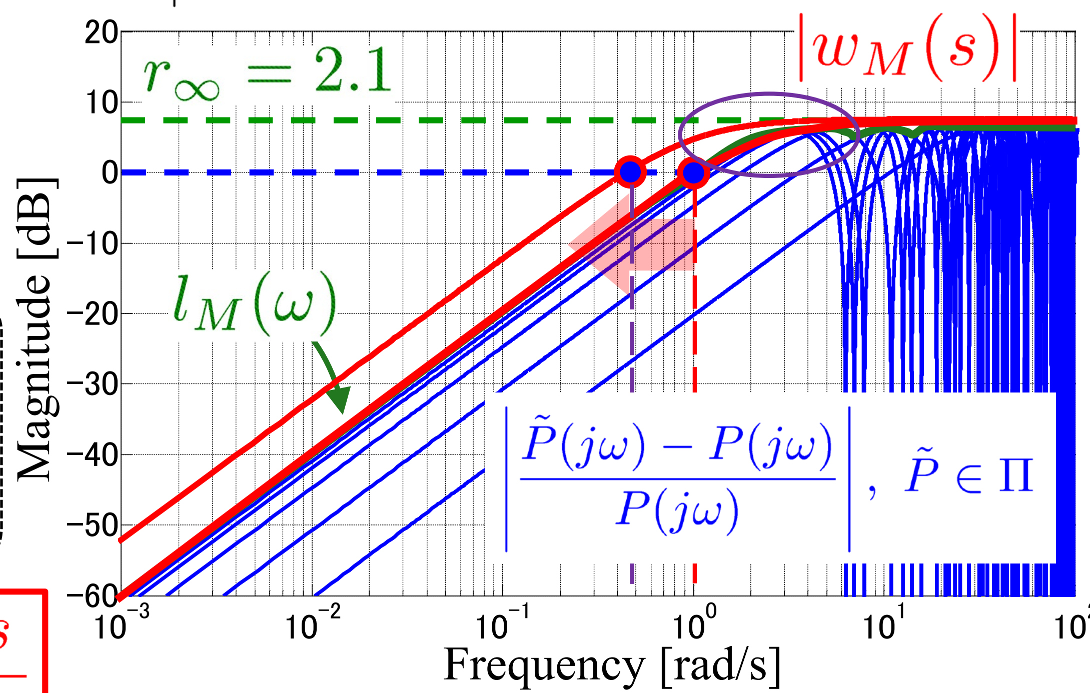
$l_M(\omega) \le 2$ ➡ $r_\infty = 2.1$

$\frac{1}{\tau} = 1$ **?** $(\tau = 1)$ $\left(\omega_c \le \frac{1}{\theta} = 1\right)$

➡ $|w_M(j\omega)| \ge l_M(\omega), \ ^\forall\omega$ (✗)

$\frac{1}{\tau} = 0.48$ $(\tau = 2.1)$

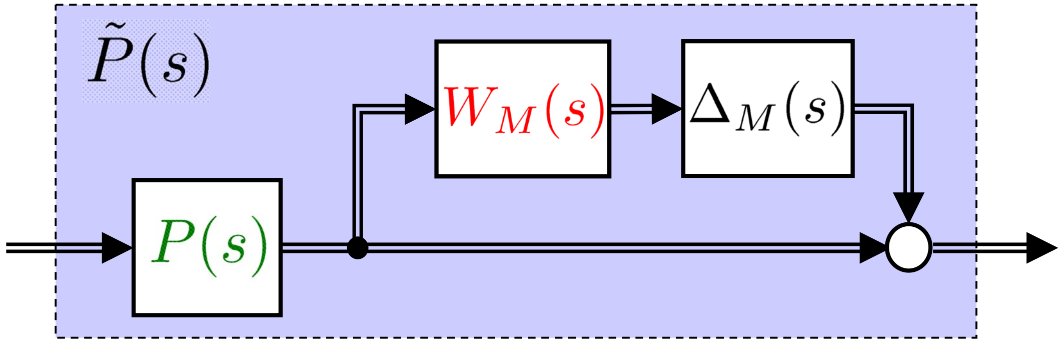
$$w_M(s) = \frac{2.1s}{s+1}$$



Representing Uncertainty in MIMO Systems

Multiplicative (Output) Uncertainty

$$\Pi_0 = \{\tilde{P}(s) \mid \tilde{P}(s) = (I + \Delta_M(s) W_M(s)) P(s),\ \|\Delta_M\|_\infty \le 1\}$$



Uncertainty Weight $W_M(s)$

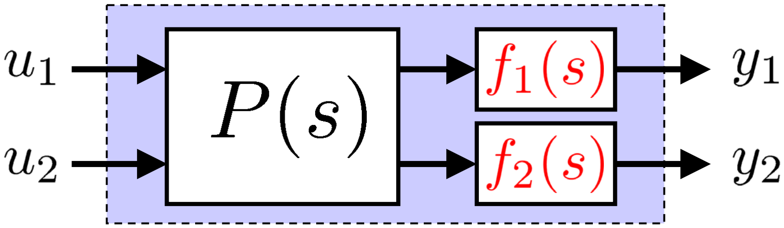
$$\begin{bmatrix} w_{M11}(s) & \cdots & w_{M1n}(s) \\ \vdots & \ddots & \vdots \\ w_{Mn1}(s) & \cdots & w_{Mnn}(s) \end{bmatrix}, \quad \begin{bmatrix} w_{M1}(s) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & w_{Mn}(s) \end{bmatrix}$$

$$W_M(s) = w_M(s) I = \begin{bmatrix} w_M(s) & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & w_M(s) \end{bmatrix}, \quad w_M(s) = \frac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1}$$

[Ex.] Spinning Satellite: Uncertainty Weight [SP05, p. 295]

Uncertain Plant Model (Real System)

$$\tilde{P}(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix} P(s)$$



$$P(s) = \begin{bmatrix} \frac{s-100}{s^2+100} & \frac{10s+10}{s^2+100} \\ \frac{-10s-10}{s^2+100} & \frac{s-100}{s^2+100} \end{bmatrix}$$

$$f_i(s) = k_i \frac{-\frac{\theta_i}{2}s+1}{\frac{\theta_i}{2}s+1}, \quad i = 1, 2$$

Gain Margin: $0.8 \leq k_i \leq 1.2$

($\pm$20%, GM = 2dB)

Delay Margin: $0 \leq \theta_i \leq 0.02$

sampling time of controller: 20ms

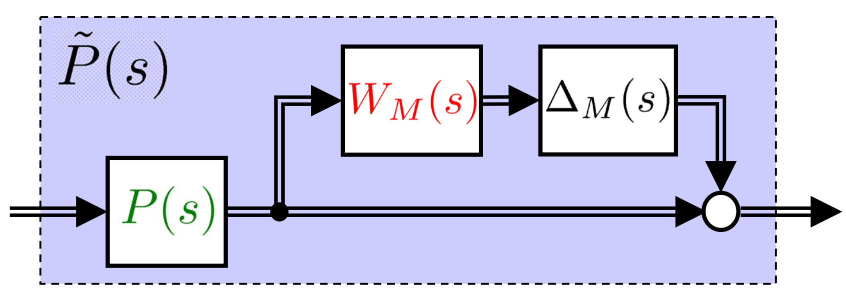
Multiplicative (Output) Uncertainty

$$\Pi_0 = \{\tilde{P}(s) |\ \tilde{P}(s) = (I + \Delta_M(s) W_M(s)) P(s),\ \|\Delta_M\|_\infty \leq 1\}$$

Step 1. Nominal Model:

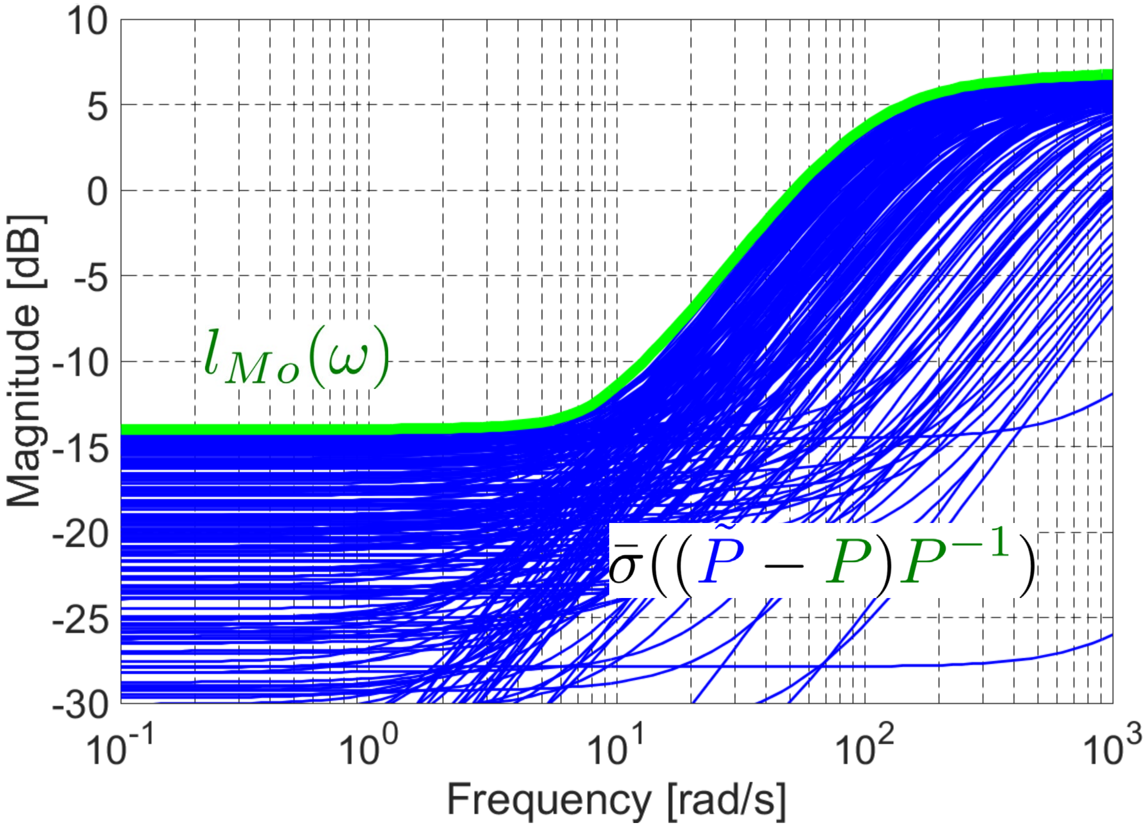
$$k_i = 1,\ \theta_i = 0\ ,\quad i = 1, 2$$

$\Rightarrow$ $P(s) = P(s)$



[Ex.] Spinning Satellite: Uncertainty Weight [SP05, p. 295]

Step 2. $l_{Mo}(\omega) = \max_{\tilde{P}\in\Pi_0} \bar{\sigma}((\tilde{P}(j\omega) - P(j\omega))P^{-1}(j\omega))$



MATLAB Command

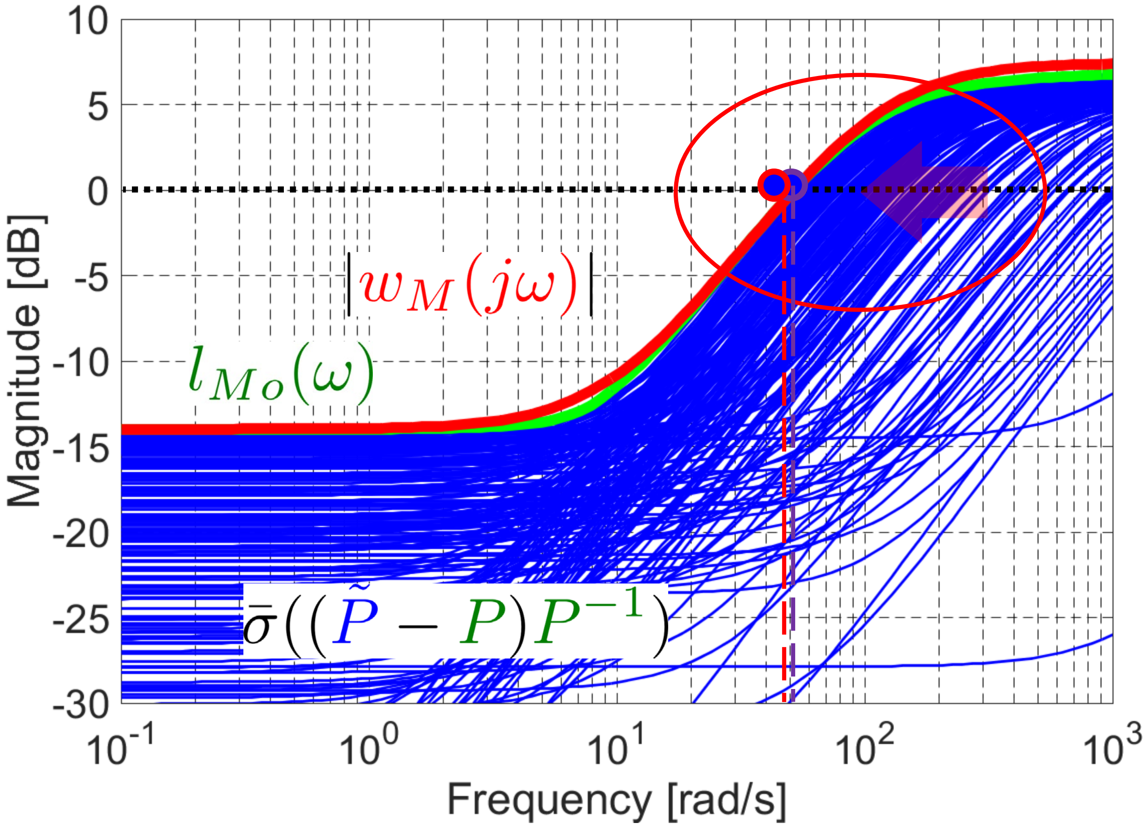
```
k1 = ureal('k1',1,'Per',[-20 20]);
k2 = ureal('k2',1,'Per',[-20 20]);
L1 = ureal('L1',0.01,'Range',[0 0.02]);
L2 = ureal('L2',0.01,'Range',[0 0.02]);
f1 = k1*tf([-L1/2 1],[L1/2 1]);
f2 = k2*tf([-L2/2 1],[L2/2 1]);
f = [f1 0;0 f2];
farray = usample(f,100);
```

100 randomly generated parameters

```
Parray=farray*Pnom;
Pfarray=frd(Parray,logspace(-1,3,100));
Eo=(Pfarray-Pnom)*inv(Pnom);
figure
sigma(Eo,'b-');
hold on; grid on;
```

[Ex.] Spinning Satellite: Uncertainty Weight [SP05, p. 295]

Step 3. $W_M(s) = w_M(s) I_2$, $w_M(s) = \dfrac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1}$ $|w_M(j\omega)| \geq l_{Mo}(\omega), {}^\forall \omega$



$r_0 = 0.2$, $r_\infty = 2.3$

$\frac{1}{\tau} = 50$ $(\tau = 0.02)$?

$\left(\omega_c \leq \frac{1}{\theta} = 50\right)$

$\frac{1}{\tau} = 48$ $(\tau = 0.021)$

$$w_M(s) = \frac{0.021s + 0.2}{0.0091s + 1}$$

Manual Fitting

MATLAB Command

```
r0 = 0.2; rinf = 2.3; tau = 0.021;
wM = tf([tau r0], [tau/rinf 1]);
WM = eye(2)*wM;
sigma(WM,'r');
```

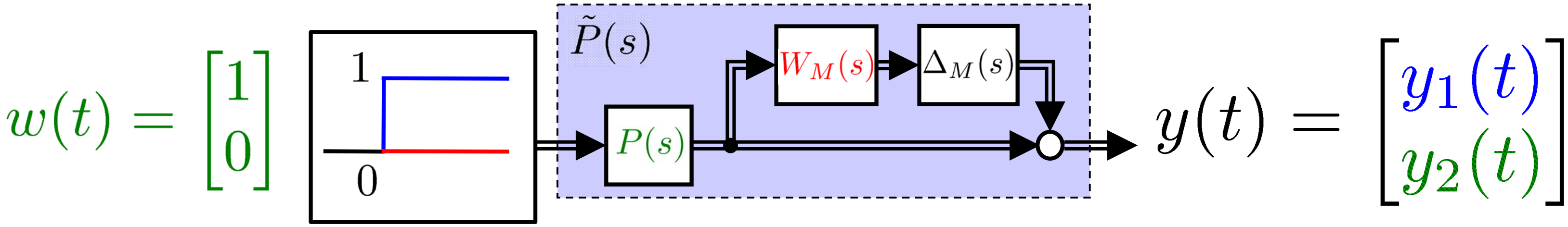
Automatic Fitting

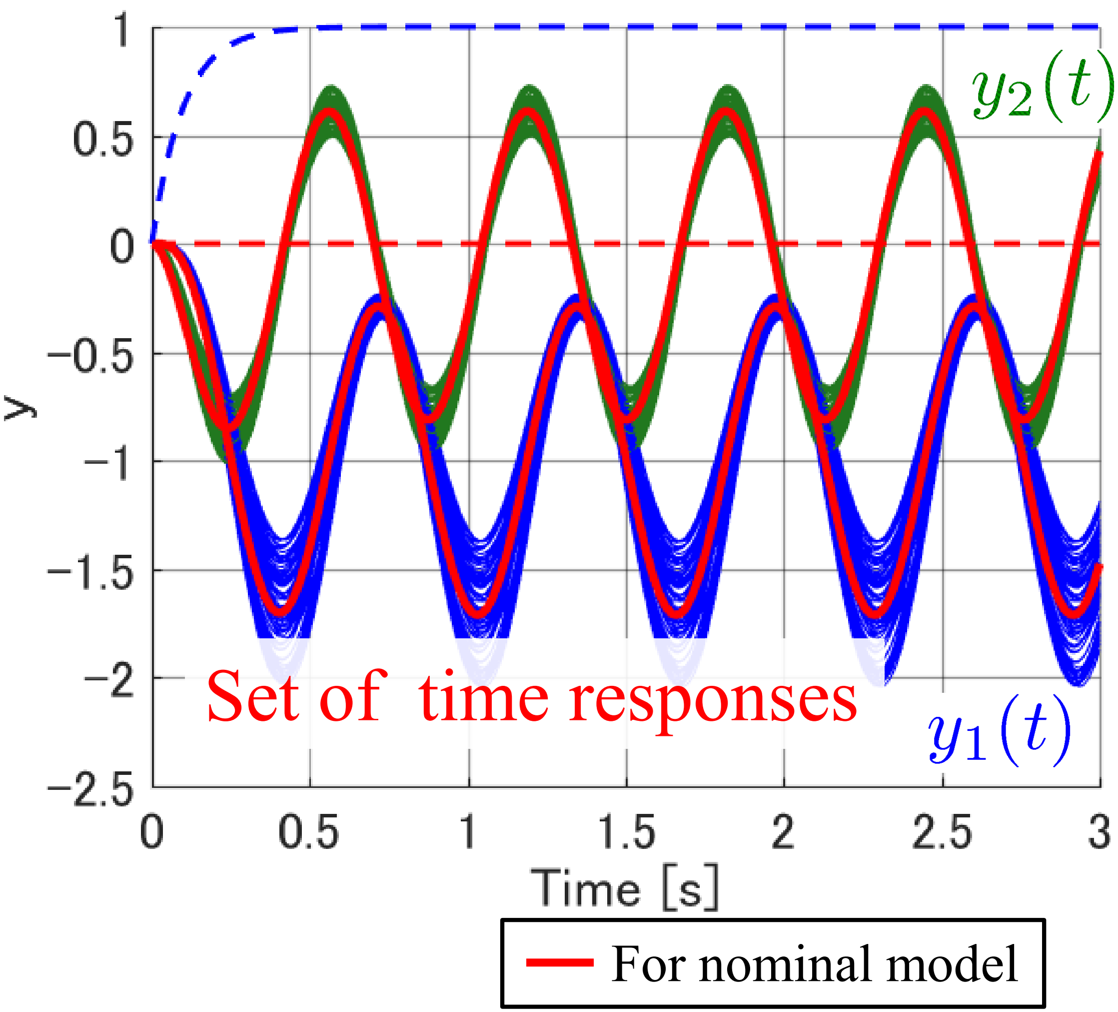
MATLAB Command

Order of w_M

```
[Usys,uInfo] = ucover(Parray,Pnom,1,'OutputMult');
sigma(uInfo.W1opt,'g-');
wM = uInfo.W1; WM = eye(2)*wM;
sigma(WM,'r');
```

[Ex.] Spinning Satellite: Time Responses for Uncertain Plant

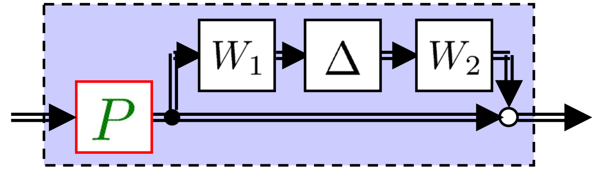
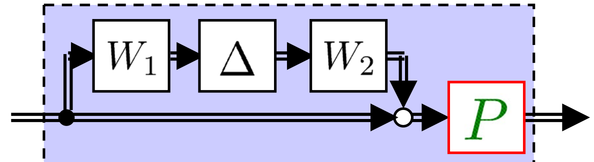
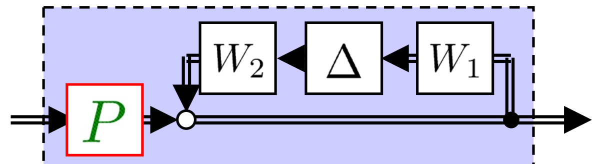
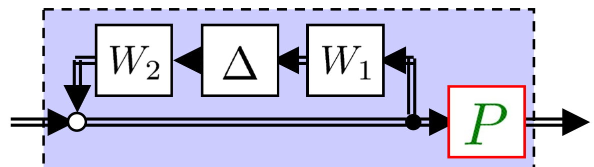
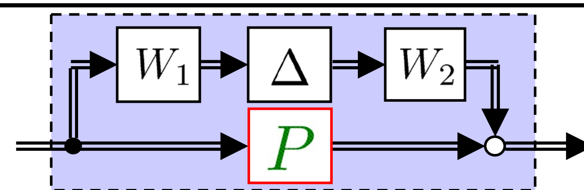
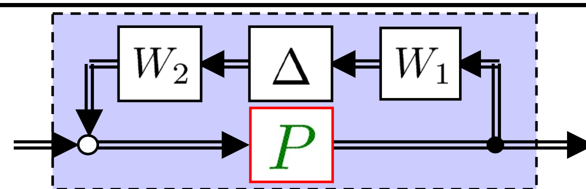




MATLAB Command

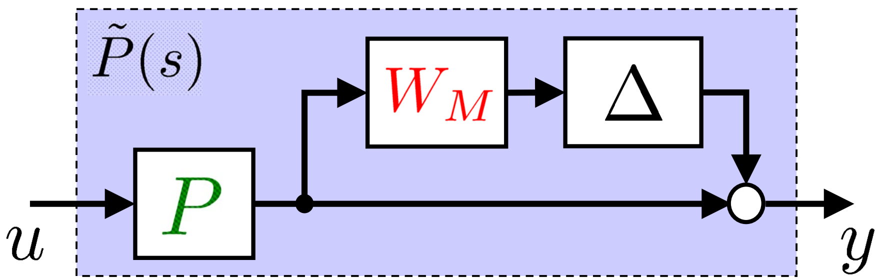
```
time = 0:0.01:3;
step_ref = ones(1,length(time));
Filter = tf(1,[0.1 1]);
step_ref_filt = lsim(Filter,step_ref,time);
ref = [step_ref_filt'; zeros(1,length(time))];

figure
hold on; grid on;
Parray=farray*Pnom;
for i = 1 : 100
    [yhi,t] = lsim(Parray(:,:,i),ref,time);
    plot(t,yhi(:,1),'b-');
    plot(t,yhi(:,2),'g-');
end

[yhi1,t] = lsim(Pnom,ref,time);
plot(t,yhi1,'r-');
plot(time,ref,'g-.');
```

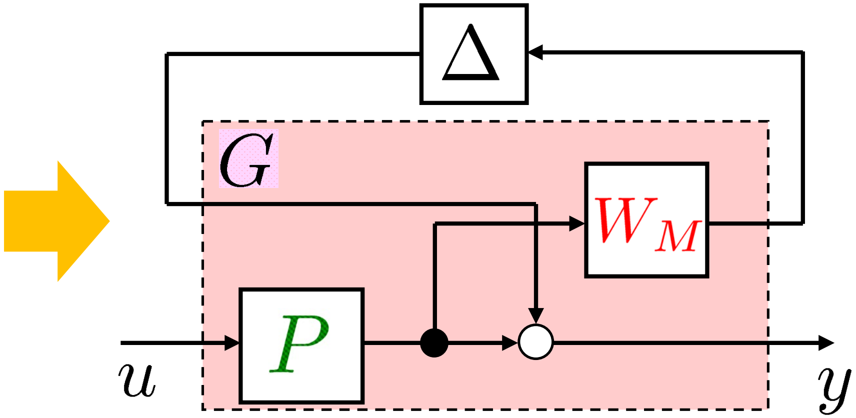
Unstructured Uncertainty [SP05, p. 293]

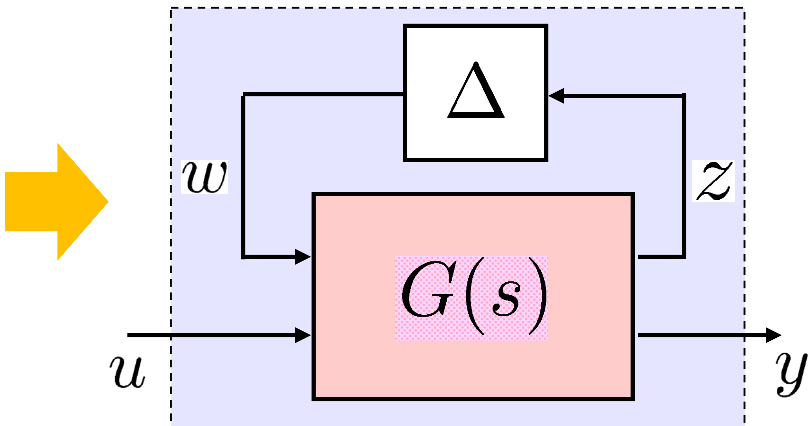
Unstructured Uncertainty	Perturbed Model Set Π		
Multiplicative (Output)	Π_1		$(I + W_2 \Delta W_1) P$
Multiplicative (Input)	Π_2		$P(I + W_2 \Delta W_1)$
Inverse Multiplicative (Output)	Π_3		$(I - W_2 \Delta W_1)^{-1} P$
Inverse Multiplicative (Input)	Π_4		$P(I - W_2 \Delta W_1)^{-1}$
Additive	Π_5		$P + W_2 \Delta W_1$
Inverse Additive	Π_6		$P(I - W_2 \Delta W_1 P)^{-1}$

$W_2 \Delta W_1 \neq W_1 \Delta W_2$ $\quad \|\Delta\|_\infty \leq 1$

Uncertain Systems [SP05, pp. 113, 543]







$$w = \Delta z \;, \quad \|\Delta\|_\infty \leq 1$$

$$\begin{bmatrix} z \\ y \end{bmatrix} = G(s) \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

Upper Linear Fractional Transformation (LFT):

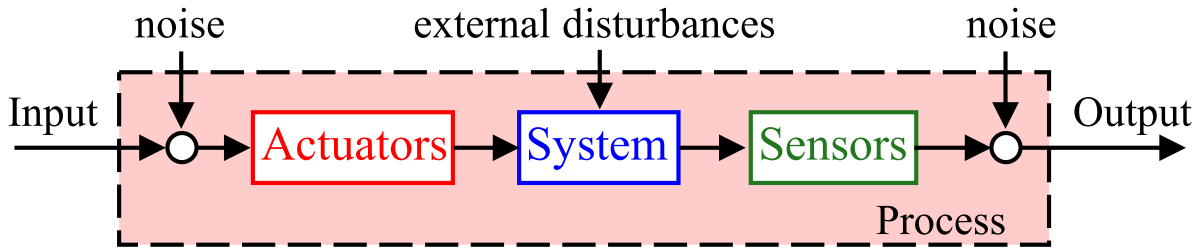
$$y = F_u(G, \Delta) u$$

$$F_u(G, \Delta) = G_{22} + G_{21} \Delta (I - G_{11} \Delta)^{-1} G_{12}$$

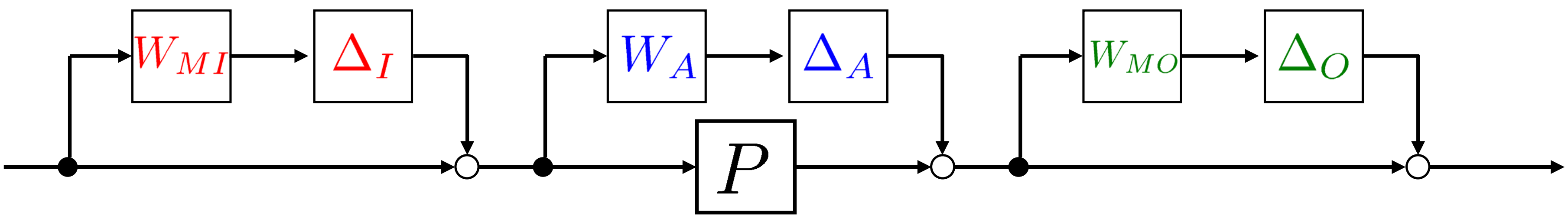
Systems with Structured Uncertainty [SP05, p. 296]

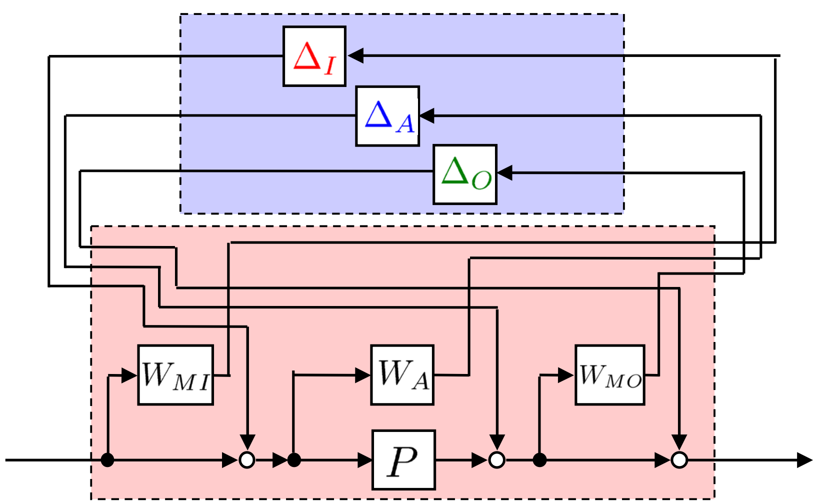
Additive, Input and Output Multiplicative Uncertainty

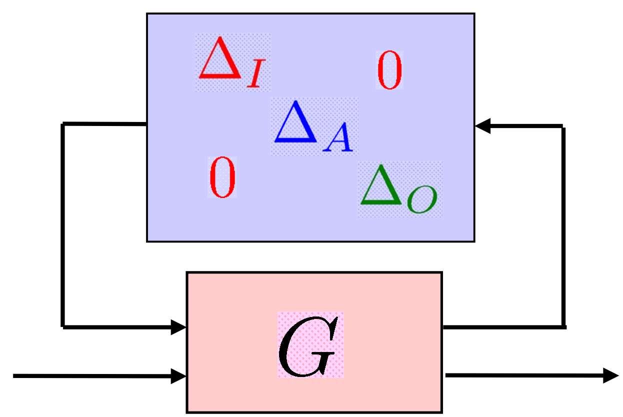
[Ex.]



X-29 Aircraft





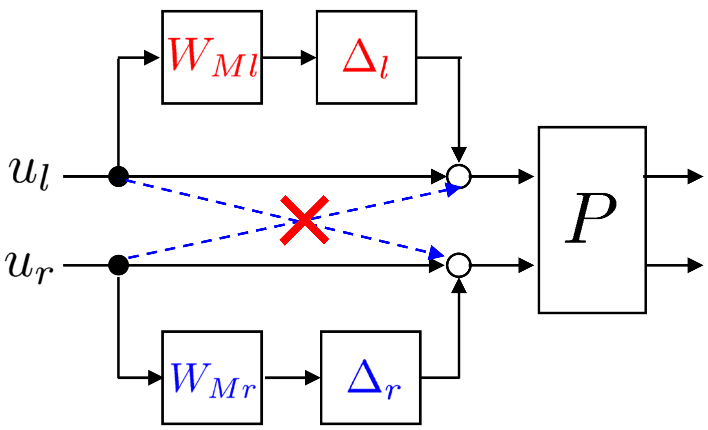


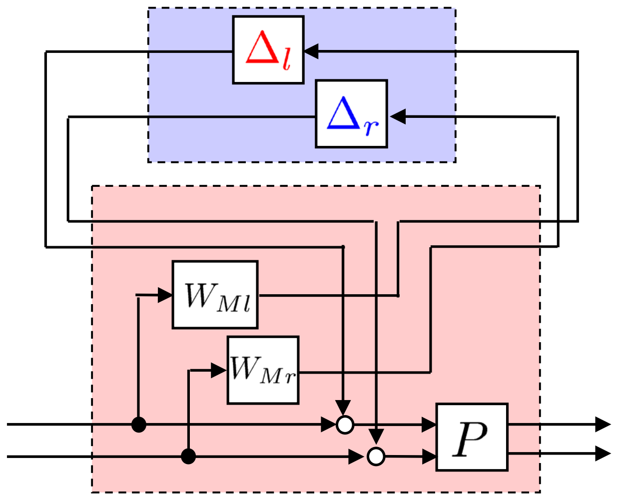
Block Diagonal

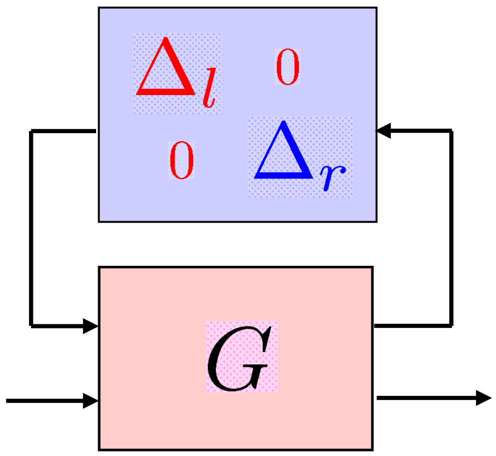
Input Multiplicative/Diagonal Uncertainty

[Ex.]

NASA HIMAT





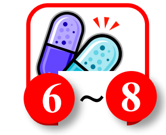


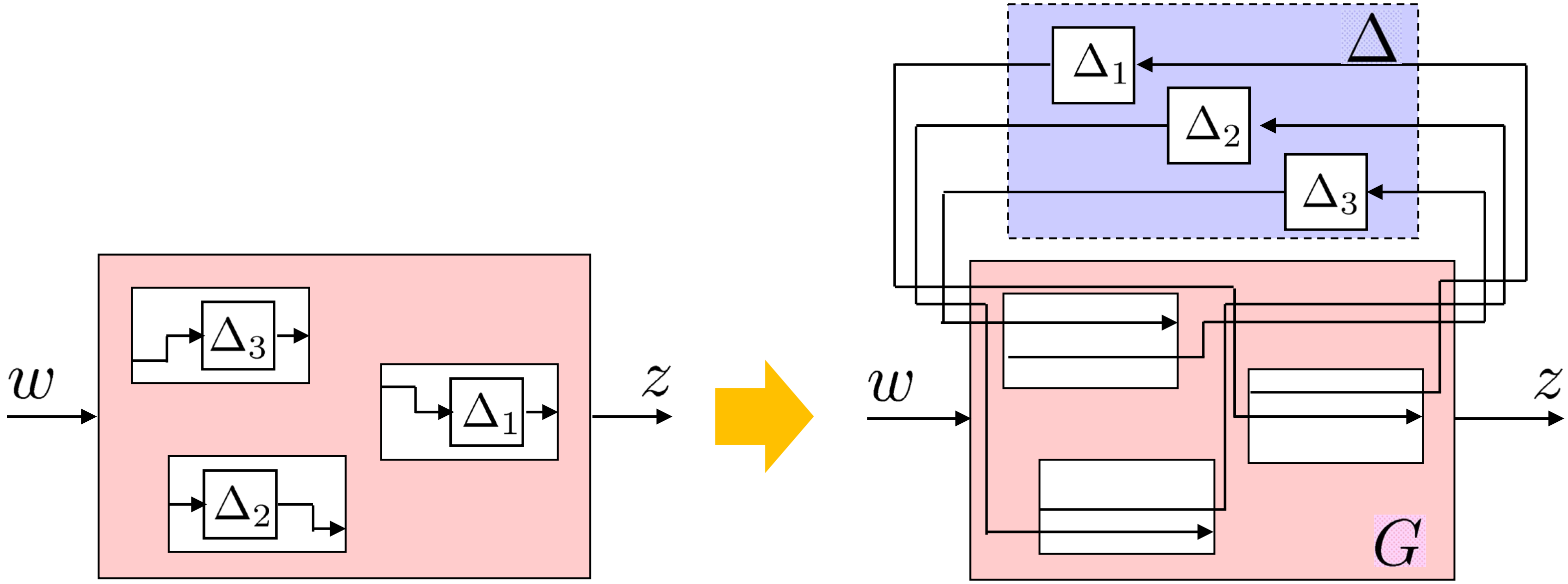
Block Diagonal

Stability Margin in Multivariable Systems

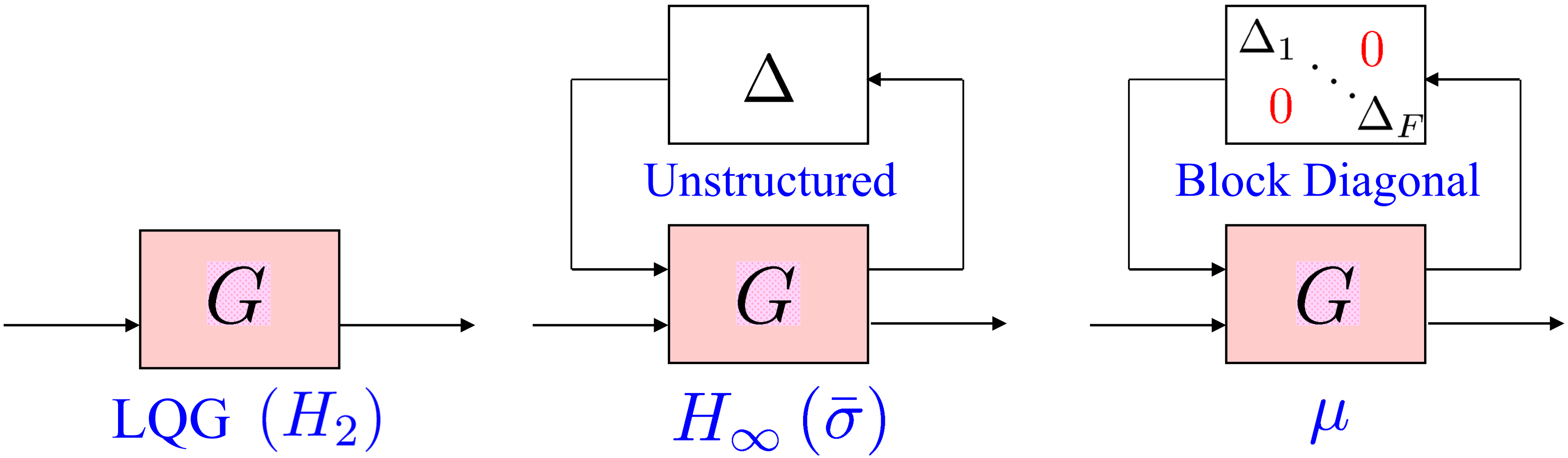
A.E.Bryson. Jr., IEEE TAC, 22 - 5, 1977

Structured Uncertainty [SP05, p. 296]

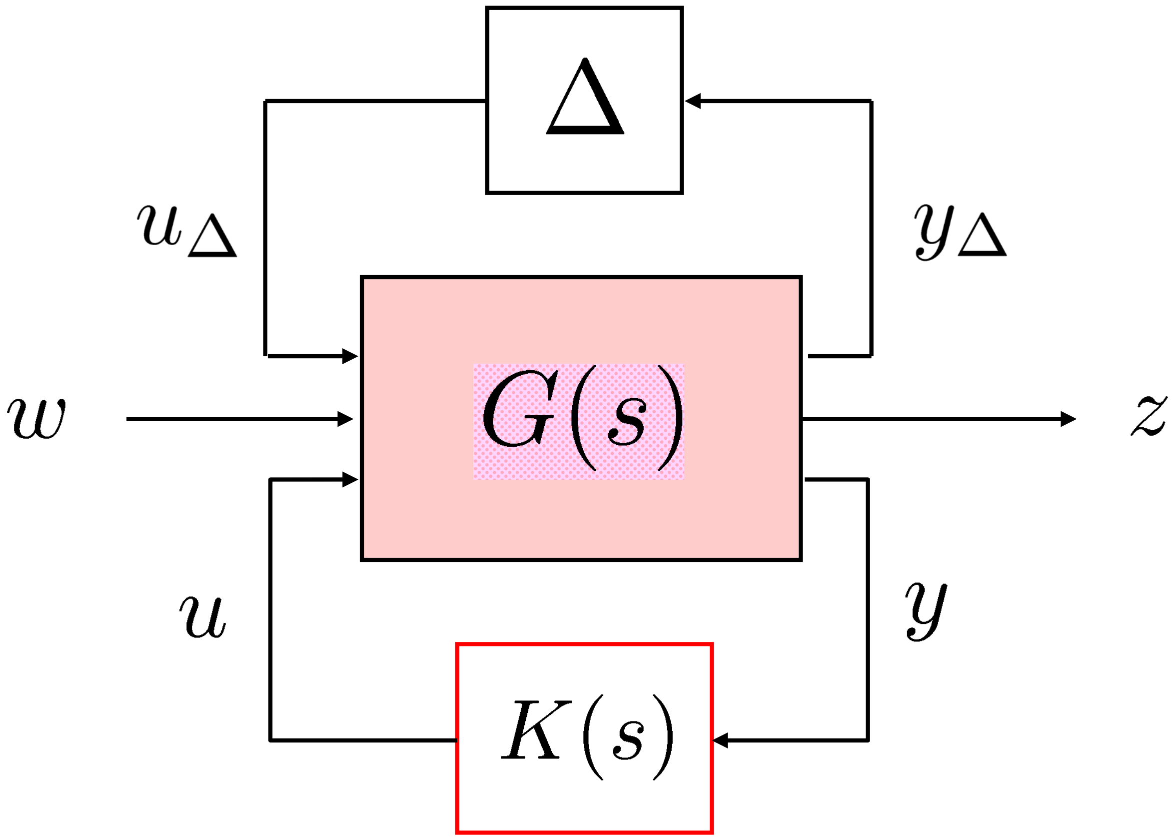




Structured Uncertainty



Big Picture [SP05, pp. 12, 289]



$$\|\Delta\|_\infty \leq 1$$

$G(s)$: Generalized Plant

$K(s)$: Controller

3. Robustness and Uncertainty

- ✓ 3.1 Why Robustness? [SP05, Sec. 4.1.1, 7.1, 9.2]
- ✓ 3.2 Representing Uncertainty [SP05, Sec. 7.2, 7.3, 7.4]
- ✓ 3.3 Uncertain Systems [SP05, Sec. 8.1, 8.2, 8.3]
- ✓ 3.4 Systems with Structured Uncertainty [SP05, Sec. 8.2]

Reference:

[SP05] S. Skogestad and I. Postlethwaite, ***Multivariable Feedback Control; Analysis and Design,*** Second Edition, Wiley, 2005.

4. Robust Stability and Loop Shaping

4.1 Robust Stability and Robust Stabilization

[SP05, Sec. 7.5, 8.4, 8.5]

4.2 Mixed Sensitivity and Loop Shaping

[SP05, Sec. 2.6, 2.8, 9.1]

4.3 1st Report

Reference:

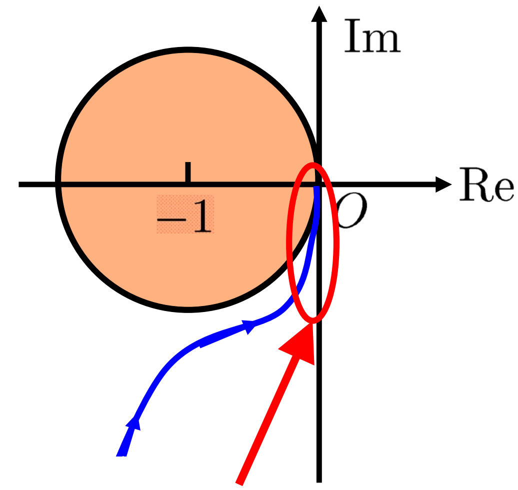
[SP05] S. Skogestad and I. Postlethwaite,
Multivariable Feedback Control; Analysis and Design,
Second Edition, Wiley, 2005.

Blind Spot of LQG Control

(i) 状態フィードバックという現実的ではない制御則が金科玉条であり，それを補う観測器も次数の点で実用性に乏しい．
(ii) 定常特性がほとんど無視されていた．たとえば，最適レギュレータはインパルス上の外乱しか処理できない．

木村, “多変数制御系の理論と応用-I,”
システムと制御, Vol. 22, No. 5, pp. 293-301, 1978

Nyquist Plot of
$-K\Phi(j\omega)$



Phase Delay -90°
(high frequencies)

Integrator $1/s$
(-20dB/dec)

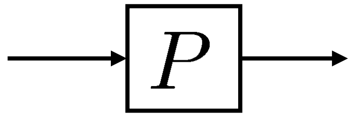
Weaker as controller in order to weaken high freq.

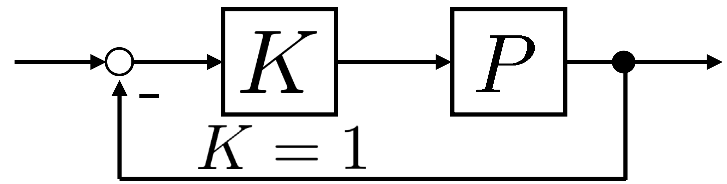
フィードバック制御系では高周波雑音を抑制するため，開ループ伝達関数の高周波特性は減衰の大きい方がよく, 実際の制御系では，必ずしも円条件を満足させないのが普通である．とはいっても，最適レギュレータの重要性は，少しも減ぜられていない．

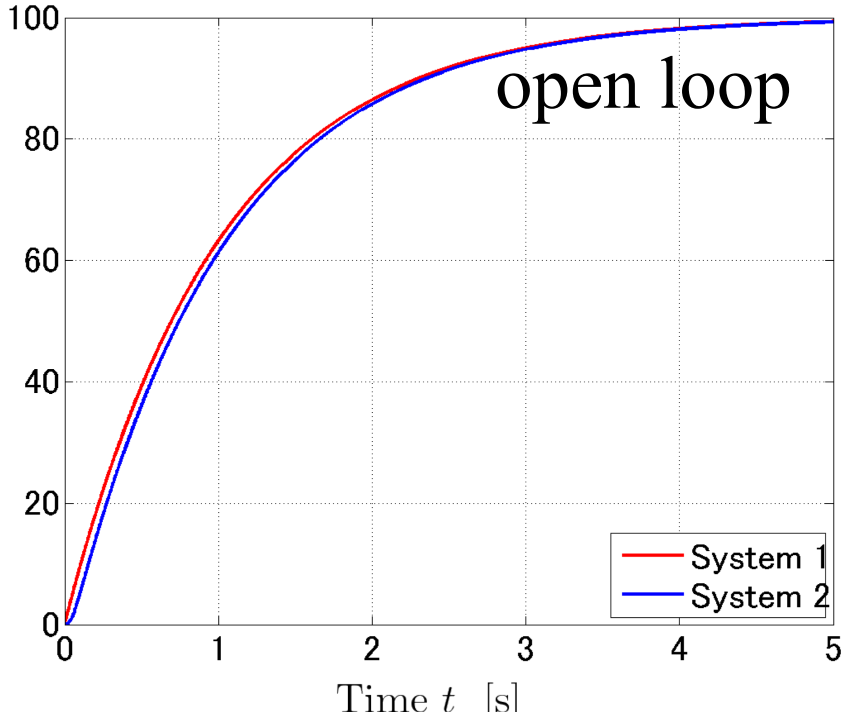
伊藤, 木村, 細江, “線形制御系の設計理論,”
計測自動制御学会編, コロナ社, 1978

When Are Two Systems Similar ? [AM09, pp. 349-352]

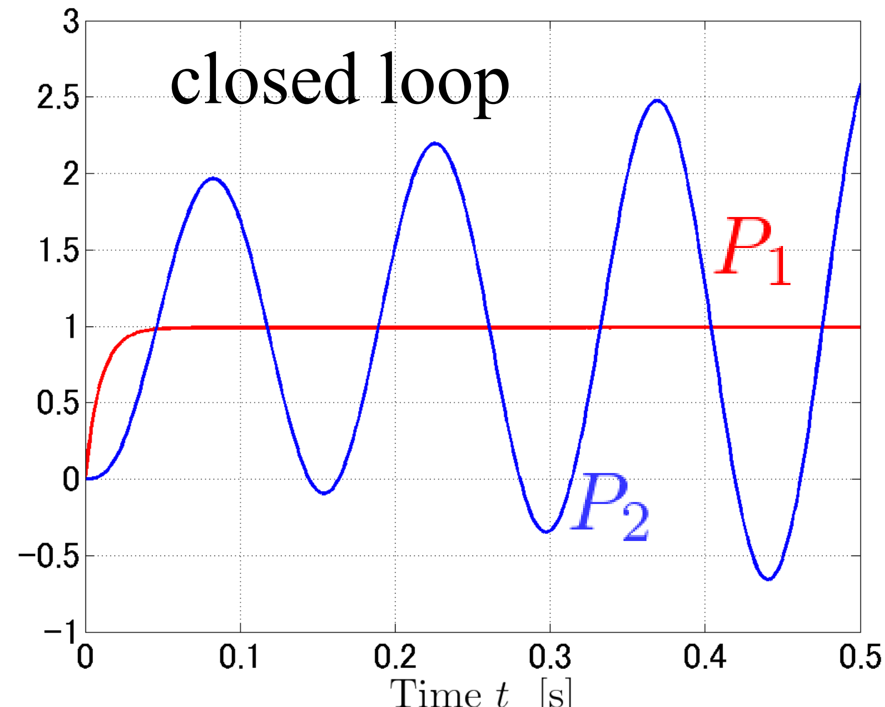
[AM09, Ex 12.2] $P_1(s) = \dfrac{100}{s+1}$, $P_2(s) = \dfrac{100}{(s+1)(0.025s+1)^2}$







(a) Step response (open loop)



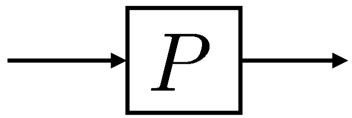
(b) Step response (closed loop)

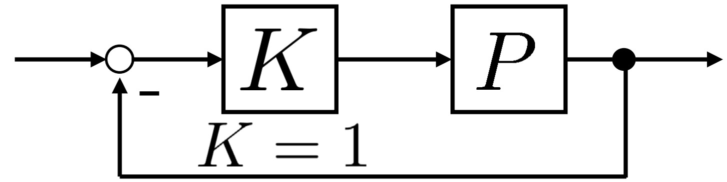
Similar in Open Loop but Large Differences in Closed Loop

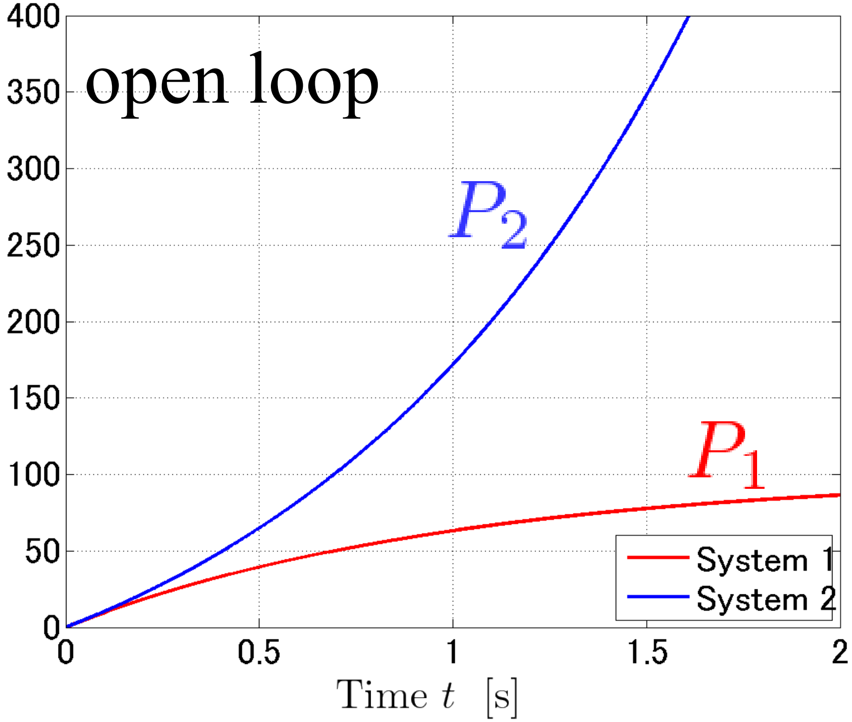
[AM09] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton Univ. Press, 2009.

When Are Two Systems Similar ? [AM09, pp. 349-352]

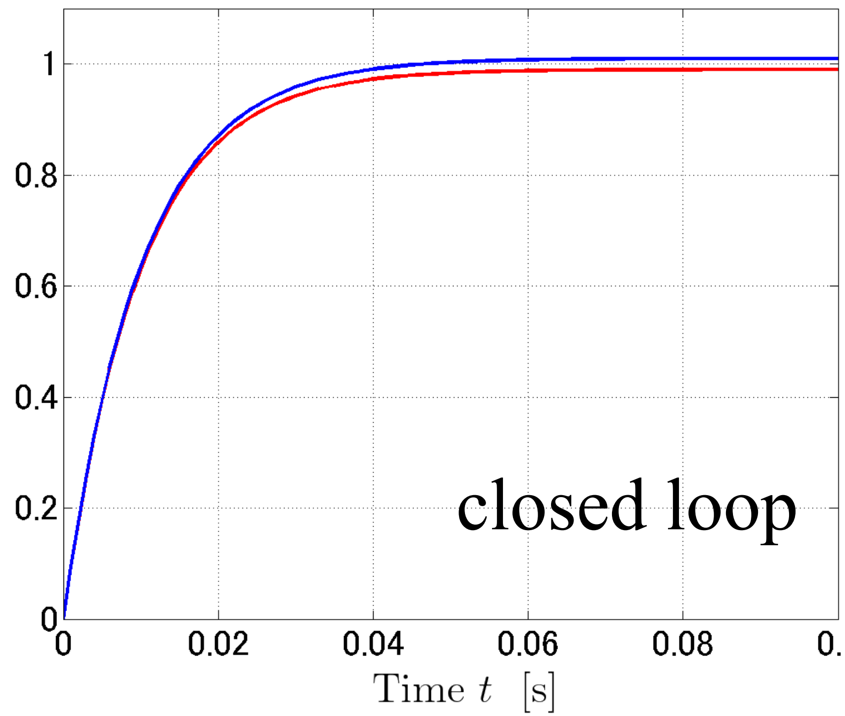
[AM09, Ex 12.3] $P_1(s) = \dfrac{k}{s+1}$, $P_2(s) = \dfrac{k}{(s-1)}$ $k = 100$







(a) Step response (open loop)



(b) Step response (closed loop)

Different in Open Loop but Similar in Closed Loop

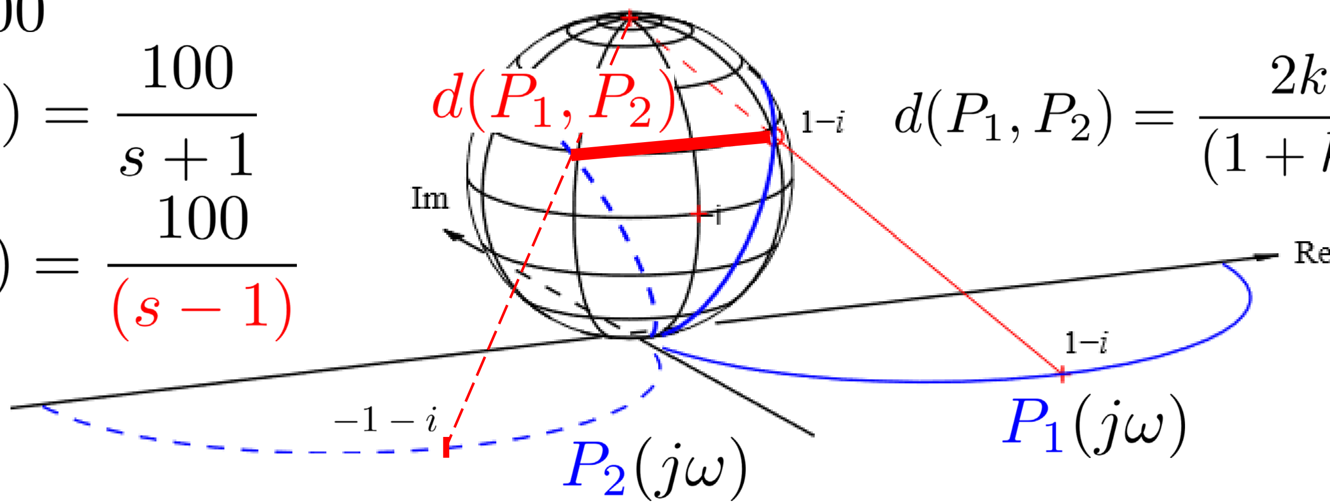
Vinnicombe Metric (ν-gap Metric)

[ZD97, Chap.17]
[AM09, pp. 349-352]

$k = 100$

$$P_1(s) = \frac{100}{s+1}$$

$$P_2(s) = \frac{100}{(s-1)}$$



$$d(P_1, P_2) = \frac{2k}{(1+k^2)} = \frac{200}{10001}$$

Vinnicombe metric (ν-gap Metric)

$$\delta_v(P_1, P_2) = d(P_1, P_2) \in [0, 1] \quad \text{if } (P_1, P_2) \in \mathcal{C}$$

G. Vinnicombe

A distance measure that is appropriate for closed loop systems

$$d(P_1, P_2) = \sup_{\omega} \frac{|P_1(j\omega) - P_2(j\omega)|}{\sqrt{(1+|P_1(j\omega)|^2)(1+|P_2(j\omega)|^2)}} \in [0, 1]$$

[AP09, Ex 12.2] $\delta_v(P_1, P_2) = 0.98$

[AP09, Ex 12.3] $\delta_v(P_1, P_2) = 0.02$

[ZD97] K. Zhou with J.C. Doyle, *Essentials of Robust Control*, Prentice Hall, 1997.

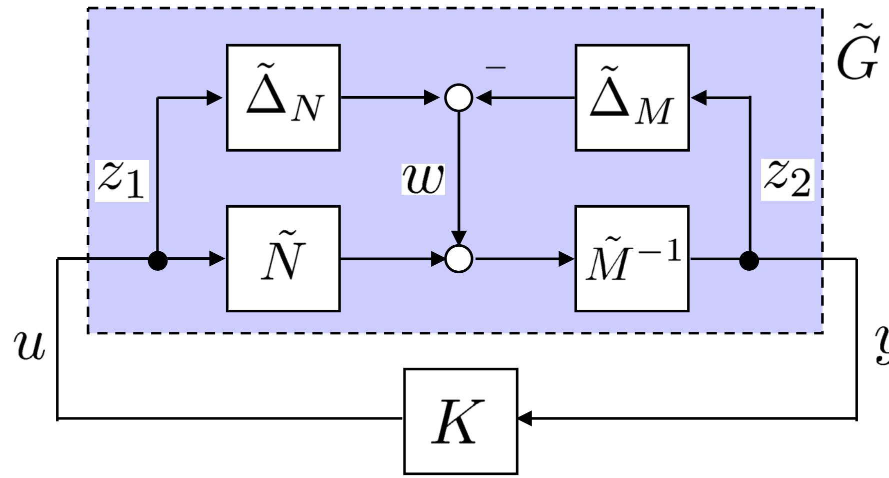
Coprime Factor Uncertainty [SP05, p. 304]

$$G = M^{-1}N$$

$$\tilde{G} = \left(\tilde{M} + \tilde{\Delta}_M\right)^{-1}\left(\tilde{N} + \tilde{\Delta}_N\right)$$

$$\|[\Delta_N \quad \Delta_M]\|_\infty \leq \epsilon$$

$$\tilde{M}, \tilde{N}, \tilde{\Delta}_M, \tilde{\Delta}_N \in \mathcal{RH}_\infty$$

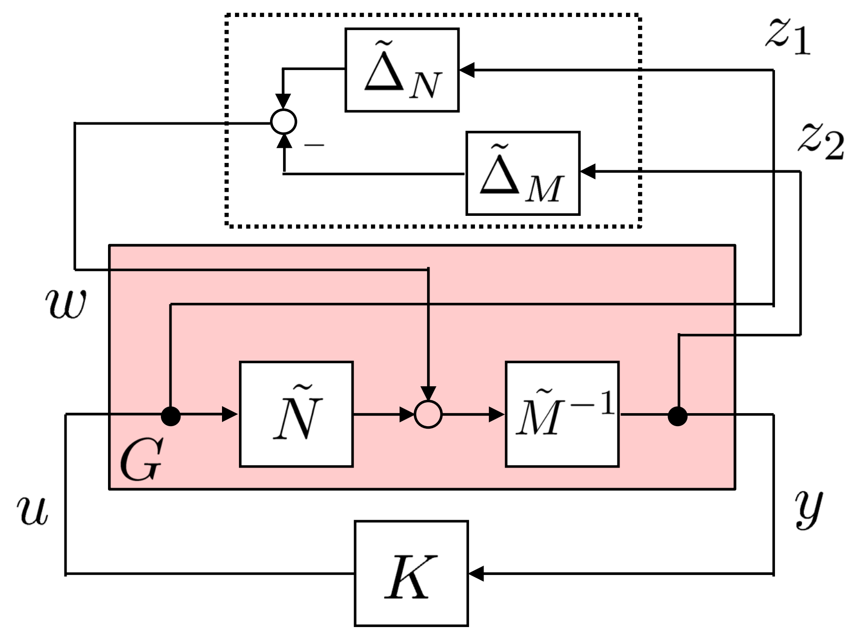


[Ex.]

$$G = \frac{1}{s+\epsilon} = \underbrace{\left[\frac{s+\epsilon}{s+1}\right]^{-1}}_{\tilde{M}^{-1}} \underbrace{\left[\frac{1}{s+1}\right]}_{\tilde{N}}$$

$$\tilde{\Delta}_M = \frac{-2\epsilon}{s+1}, \; \tilde{\Delta}_N = 0$$

$$\tilde{G} = \frac{1}{s-\epsilon} = \left[\frac{s-\epsilon}{s+1}\right]^{-1}\left[\frac{1}{s+1}\right]$$



➡ H_∞ Loop Shaping

Parametric Uncertainty: State Space [SP05, p. 292]

[Ex.]

$$A_p = \begin{bmatrix} -2-\alpha & \alpha-\beta \\ \alpha+2\beta & -\alpha \end{bmatrix} \qquad \begin{array}{ll} \alpha = 1 + w_1\delta_1 & |\delta_1| \le 1 \\ \beta = 3 + w_2\delta_2 & |\delta_2| \le 1 \end{array}$$

$$= \underbrace{\begin{bmatrix} -3 & -2 \\ 7 & -1 \end{bmatrix}}_{A} + \delta_1 \underbrace{\begin{bmatrix} -w_1 & w_1 \\ w_1 & -w_1 \end{bmatrix}}_{E_1} + \delta_2 \underbrace{\begin{bmatrix} 0 & -w_2 \\ 2w_2 & 0 \end{bmatrix}}_{E_2}$$

$$\mathrm{rank}(E_1) = 1 \qquad \mathrm{rank}(E_2) = 2$$

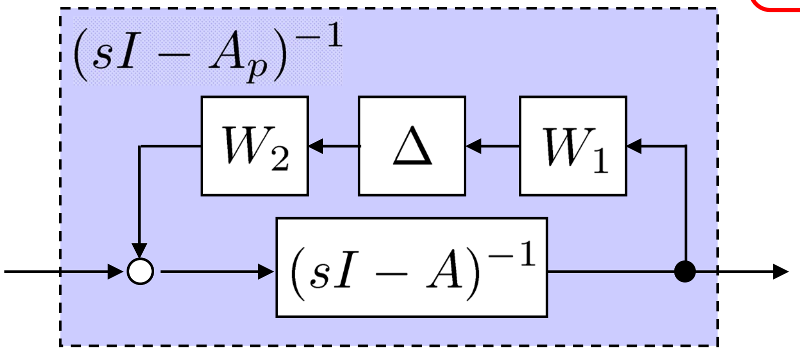
$$E_1 + E_2 = \underbrace{\left[\begin{array}{c|cc} -w_1 & 0 & -w_2 \\ w_1 & 2w_2 & 0 \end{array}\right]}_{W_2} \underbrace{\left[\begin{array}{c|cc} \delta_1 & 0 & 0 \\ \hline 0 & \delta_2 & 0 \\ 0 & 0 & \delta_2 \end{array}\right]}_{\Delta} \underbrace{\left[\begin{array}{cc} 1 & -1 \\ \hline 1 & 0 \\ 0 & 1 \end{array}\right]}_{W_1}$$

$$\Rightarrow \quad A_p = A + E_1 + E_2 = A + W_2 \Delta W_1$$

Parametric Uncertainty: State Space (Cont.) [SP05, p. 292]

$$\Phi(s) = (sI - A)^{-1}$$

$$(sI - A_p)^{-1} = (sI - A - W_2 \Delta W_1)^{-1}$$
$$= (I - \Phi(s) W_2 \Delta W_1)^{-1} \Phi(s)$$



$$\dot{x} = A_p x + B_p u$$
$$y = C_p x + D_p u$$
$$\Delta = \mathrm{diag}\{\delta_1 I, \cdots, \delta_S I\}$$

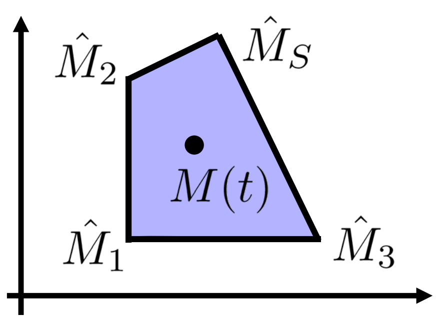
$$A_p = A + \sum_{i=1}^{S} \delta_i \hat{A}_i \quad B_p = B + \sum_{i=1}^{S} \delta_i \hat{B}_i$$
$$C_p = C + \sum_{i=1}^{S} \delta_i \hat{C}_i \quad D_p = D + \sum_{i=1}^{S} \delta_i \hat{D}_i$$

cf. Linear parameter varying (LPV) system

Polytopic-type system

Affine parameter-dependent system

➡ Gain Scheduled H_∞ Problem



Diagonal Uncertainty [SP05, pp. 289, 296, 300]

Allowed Structure

$$\boldsymbol{\Delta} = \text{diag}\{\delta_1 I_{r1}, \cdots, \delta_S I_{rS}, \Delta_1, \cdots, \Delta_F\}$$

$$= \left[\begin{array}{ccc|ccc} \delta_1 I_{r1} & & & & & \\ & \ddots & & & 0 & \\ & & \delta_S I_{rS} & & & \\ \hline & & & \Delta_1 & & \\ & 0 & & & \ddots & \\ & & & & & \Delta_F \end{array}\right]$$

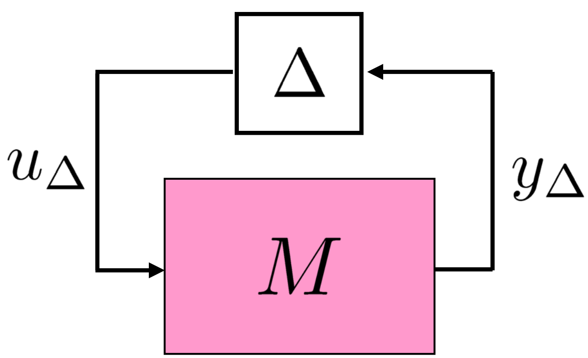
Parametric Uncertainties

$$\delta_i \in \mathcal{R}, \; i = 1, \cdots, S$$

Nonparametric Uncertainties

$$\Delta_j \in \mathcal{C}^{m_j \times m_j}, \; j = 1, \cdots, F$$

Allowed Perturbations



$$\forall \Delta \in B_{\boldsymbol{\Delta}}$$

$$B_{\boldsymbol{\Delta}} = \{\Delta \in \boldsymbol{\Delta} \mid \|\Delta\|_\infty \leq 1\}$$