

# 1st Report

Due: **May 20th(Mon) 17:00**

Place: **S5-204A** (A box will be prepared)

- Note :
- A4 paper, both side printing
  - the following 6 items should be written on the cover

**Subject Name, Report Number, Department & Course,  
Name, Student ID and Submission Date**

## Computer Access:



Install Guide

<http://www.t3.gsic.titech.ac.jp/matlab>

## Office Hour (Technical Support):

Place: S5-204A      e-mail: tateam\_at\_hfg.sc.e.titech.ac.jp  
Time: Friday 16:30-17:30

## Lecture Information:

<http://www.hfg.sc.e.titech.ac.jp/course/ROC/index.html>

# 1st Report

Please write your report in accordance with the following format.

Reports should be **simple** and **clear**.

The report example (format) can be downloaded by

[http://www.hfg.sc.e.titech.ac.jp/course/ROC/handouts/ex/19Report1\\_ex.pdf](http://www.hfg.sc.e.titech.ac.jp/course/ROC/handouts/ex/19Report1_ex.pdf)

Please follow it.

**Robust Control 1st Report Example**  
Department&Course : Student ID : Name  
2018/4/\*\*

**1 Nominal Stability and Nominal Performance**  
We address the following range of frequency.  
 $1.0 \times 10^{-2} \leq \omega \leq 1.0 \times 10^2$  rad/min

**1.1 Nominal Plant**  
Consider the nominal model  $P(s)$  as

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{s-100}{s^2+100} & \frac{100+100s}{s^2+100} \\ \frac{100+100s}{s^2+100} & \frac{s-100}{s^2+100} \end{bmatrix}$$

The  $\sigma$ -plot of  $P(s)$  is shown in Fig. 1.

Fig. 1:  $\sigma$ -plot of  $P(s)$

Next, we consider the following inverse-based controller  $K_{inv}(s)$ .

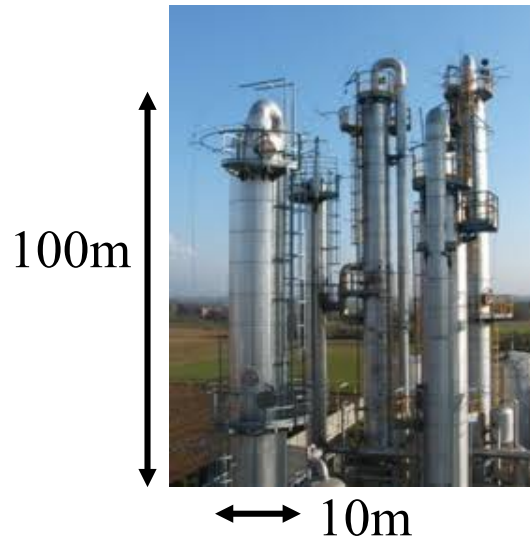
$$K_{inv}(s) = P^{-1}(s) \begin{bmatrix} \frac{1}{s^2+100} & 0 \\ 0 & \frac{1}{s^2+100} \end{bmatrix} = \begin{bmatrix} 4.4554(s-100) & 44.554(s+1) \\ -44.554(s+1) & 4.4554(s-100) \end{bmatrix}$$

**1.2 Nominal Stability (NS)**  
The plant fulfills Nominal Stability the reason why is because...

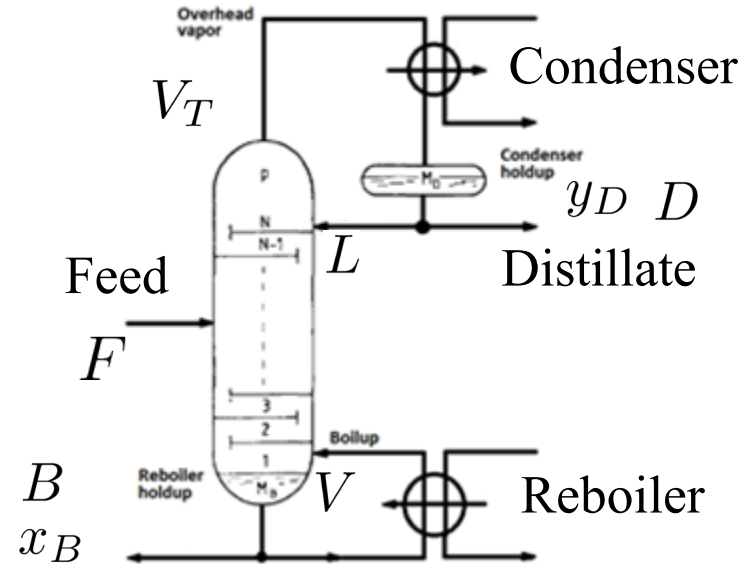
- Write your answer clearly...
- When you display figure, note that...
- etc...

# Report\*: Distillation Process [SP05, pp. 100, 509-514]

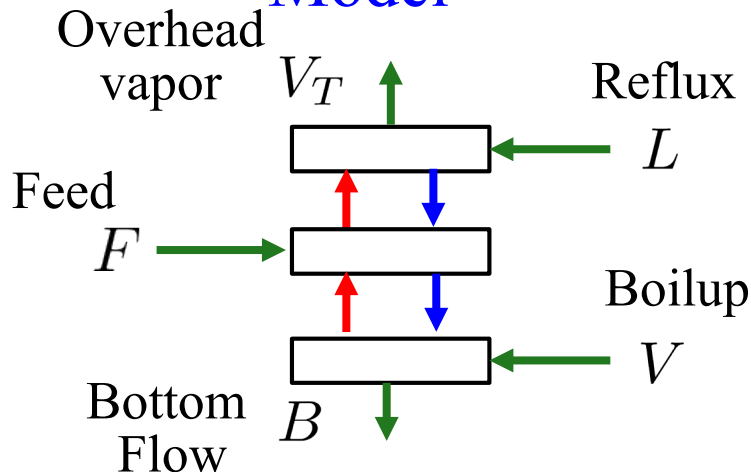
## STEP 1. Real Physical System



## STEP 2. Ideal Physical Model



## STEP 3. Ideal Mathematical Model



## STEP 4. Reduced Mathematical Model

2-Input 2-Output System

$$\begin{bmatrix} dL & dV \end{bmatrix}^T = \begin{bmatrix} dy_D & dx_B \end{bmatrix}^T$$

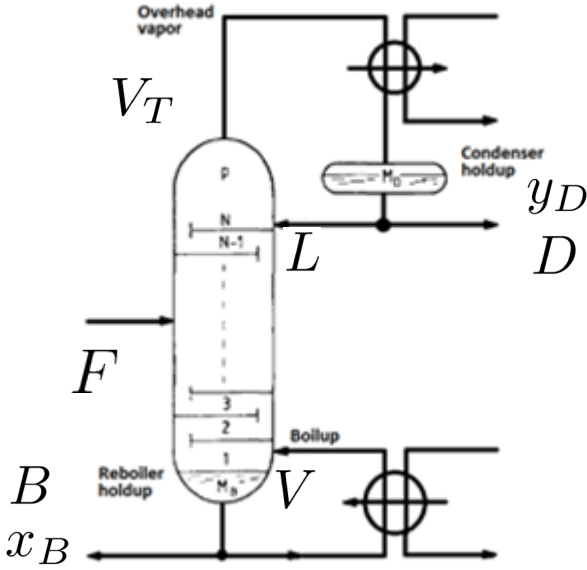
$$P(s) = \begin{bmatrix} \frac{87.8}{75s+1} & \frac{-86.4}{75s+1} \\ \frac{108.2}{75s+1} & \frac{-109.6}{75s+1} \end{bmatrix}$$

# Distillation Process (Time Delay System) [SP05, pp. 100, 509-514]

## Inputs and Outputs

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} dL \\ dV \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} dy_D \\ dx_B \end{bmatrix}$$



**Controlled Variables**

- $y_D$  : top composition
- $x_B$  : bottom composition

**Manipulated Inputs**

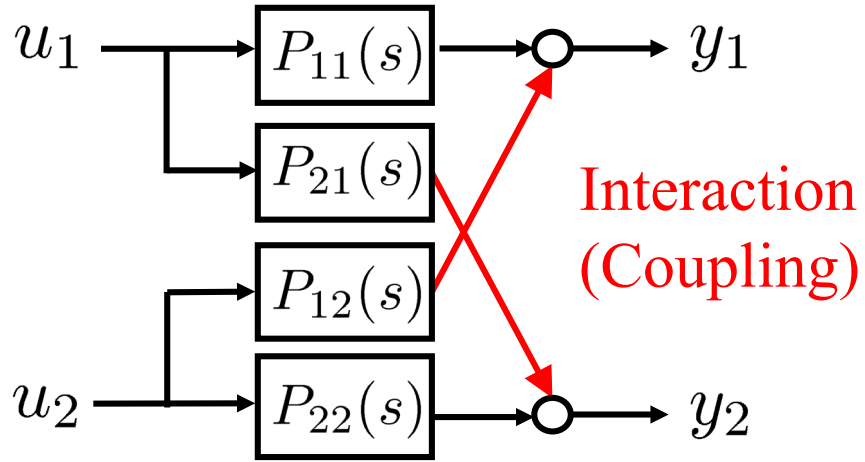
- $L$  : reflux     $D$  : distillate
- $V$  : boilup
- $B$  : bottom flow
- $V_T$  : overhead vapor

## Nominal Model

$$\begin{bmatrix} dy_D \\ dx_B \end{bmatrix} = P(s) \begin{bmatrix} dL \\ dV \end{bmatrix}$$

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{87.8}{75s+1} & \frac{-86.4}{75s+1} \\ \frac{108.2}{75s+1} & \frac{-109.6}{75s+1} \end{bmatrix}$$



Time constant 75 [min]

# [1] Nominal Stability and Nominal Performance (15 pts.)

Throughout all 3 reports, consider the following frequency range.

$$1.0 \times 10^{-3} \leq \omega \leq 1.0 \times 10^2 \text{ rad/min}$$

```

MATLAB Command
sv = sigma(Pnom, w);
figure;
semilogx(w, mag2db(sv));
xlabel(' Frequency [rad/min] ');

```

## 1.1 Nominal Plant

Show  $\sigma$ -plot of the nominal plant  $P(s)$ .

Consider the following inverse-based controller:

$$K_{inv}(s) = P^{-1}(s) \begin{bmatrix} \frac{0.7}{s} & 0 \\ 0 & \frac{0.7}{s} \end{bmatrix} = 0.7 \frac{75s + 1}{s} \begin{bmatrix} 0.3994 & -0.3149 \\ 0.3943 & -0.3200 \end{bmatrix}$$

## 1.2 Nominal Stability (NS)

Confirm that the closed-loop system is internally stable by checking the Gang of Four.

```

MATLAB Command
F = loopsens(Pnom, KI);
S = F.So;
Si = F.Si;
PS = F.PSi;
KS = F.CSo;

```

## 1.3 Step Response

Consider the closed-loop system with  $K_{inv}(s)$ .

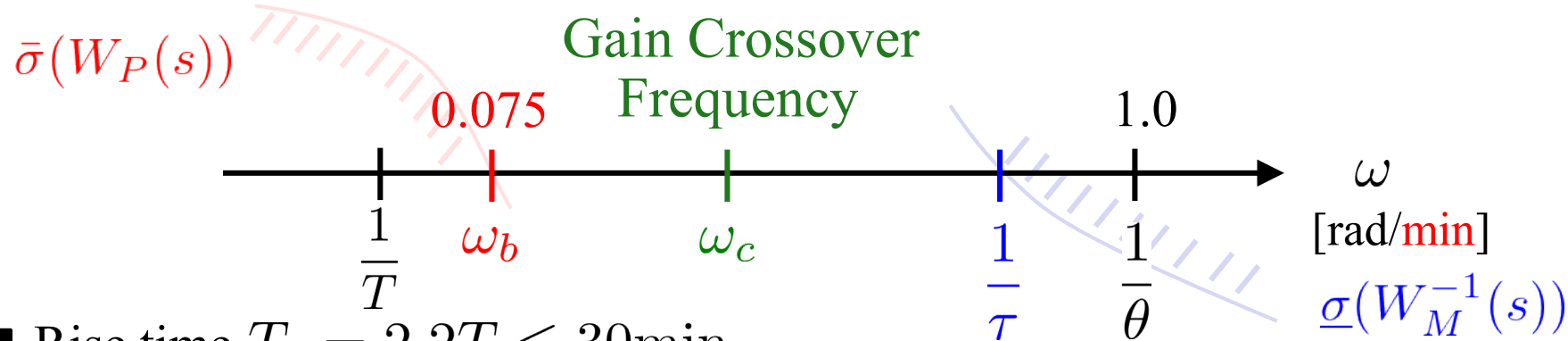
Show the step response of output  $y$  from reference signal  $r = [1.0 \ 0]^T$ .

```

MATLAB Command
time = 0:0.1:15; step_ref = ones(1,length(time)); ref = [step_ref, zeros(1,length(time))];
figure; hold on; grid on; FI = loopsens(Pnom,Kinv); [yhi,t] = lsim(FI.To,ref,time);
plot(t,yhi,'r-','LineWidth',2); plot(time,ref,'b-','LineWidth',1.5);

```

# Distillation Process: Performance Specifications



■ Rise time  $T_r = 2.2T \leq 30\text{min}$   
 (First-order System)

➔  $\frac{2.2}{30} \leq \frac{1}{T} \leq \omega_b = 0.075\text{rad/min}$

■ Steady state error  $< 0.01$   $A = 0.01$

■ Performance Weight

$\omega_c > \omega_b = 0.075\text{rad/min}$

$A = 0.01$   $M_s = 2$

$$W_P(s) = w_P(s)I_2$$

$$w_P(s) = \frac{0.5s + 0.075}{s + 0.00075}$$

■ Delay Margin:  $\theta \leq 1$  min

➔  $\omega_c < 1/\theta = 1$  rad/min

■ Gain Margin:  $\pm 20\%$ , 2dB

■ Uncertainty Weight

$\omega_c < 1/\tau \leq 1/\theta = 1$  rad/min

$$W_M(s) = w_M(s)I_2$$

$$w_M(s) = \frac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1}$$

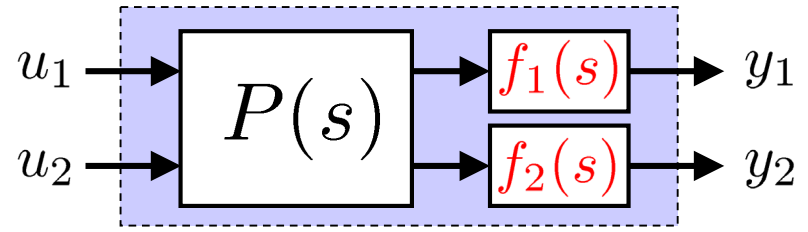
➔ 2.1

# [2] Uncertainty Weight for Real Plant

## Perturbed Plant Model

$$\tilde{P}(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix} P(s)$$

$$f_i(s) = k_i \frac{-\frac{\theta_i}{2}s + 1}{\frac{\theta_i}{2}s + 1}, \quad i = 1, 2$$



Gain Margin:  $0.8 \leq k_i \leq 1.2$   
(±20%, 2dB)

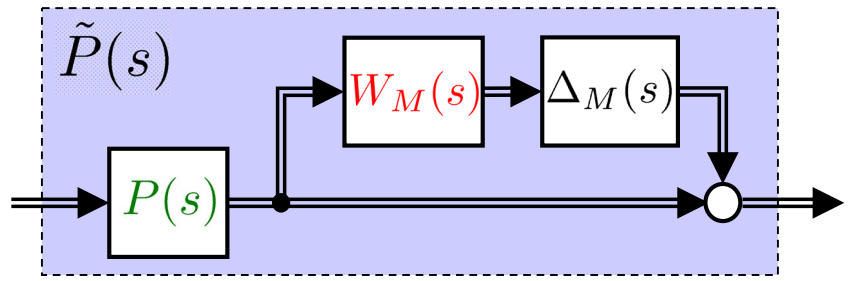
Delay Margin:  $0 \leq \theta_i \leq 1$  [min]

## Multiplicative (Output) Uncertainty

$$\Pi_0 = \{ \tilde{P}(s) \mid \tilde{P}(s) = (I + \Delta_M(s)W_M(s))P(s), \|\Delta_M\|_\infty \leq 1 \}$$

$$W_M(s) = w_M(s)I_2$$

$$w_M(s) = \frac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1}$$



## [2] Nominal Performance and Robust Stability (30 pts.)

see the programs in 3rd lecture & Use controller  $K_{inv}$  given in [1]

### 2.1 Nominal Performance (NP)

Show the NP Test graph of the closed-loop system and answer if NP is satisfied.

### 2.2 Uncertainty Weight

Design an uncertainty weight  $W_M$  with order 1 using the MATLAB command “`ucover`” and write  $W_M$  and the parameters  $1/\tau$ ,  $r_\infty$  and  $r_0$ .

After this, use the uncertainty weight  $W_M$  in 2.2.

### 2.3 Robust Stability (RS)

Show the RS Test graph of the closed-loop system and answer if RS is satisfied.

### 2.4 Set of time responses

Show the step responses of the closed-loop system with the perturbed plant model  $\tilde{P}(s)$  and  $K_{inv}(s)$  from the reference signal  $u = [1.0 \ 0]^T$  to the outputs.