

Robust Control

Spring, 2019

Instructor: Prof. Masayuki Fujita (S5-303B)

8th class

Tue., 4th June, 2019, 10:45 ~ 11:30,

S423 Lecture Room

8. Design Example

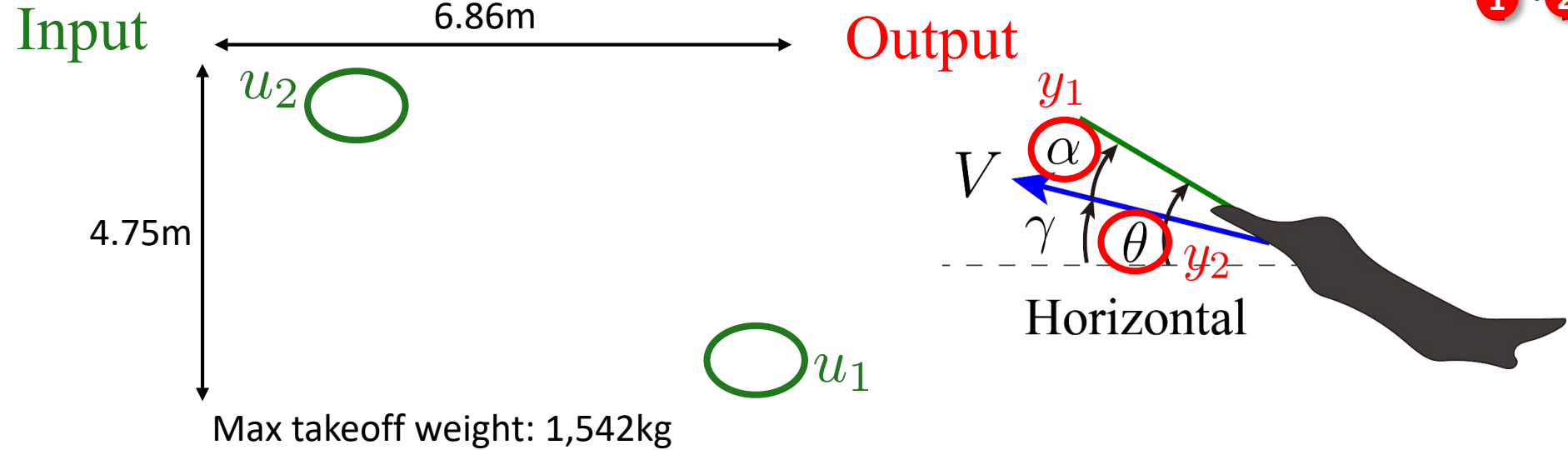
8.1 HiMAT: H_∞ Control (Highly Maneuverable Aircraft Technology)

Reference:

- [SP05] S. Skogestad and I. Postlethwaite,
Multivariable Feedback Control; Analysis and Design,
Second Edition, Wiley, 2005.
- [SLH81] M. G. Safonov, A. J. Laub and G. L. Hartmann,
Feedback Properties of Multivariable Systems:
The Role and Use of the Return Difference Matrix,
IEEE Transactions on Automatic Control, Vol. 26, No. 1, pp. 47-65, 1981.
- [ES84] M. B. Evans and L. J. Schilling,
The Role of Simulation in the Development and Flight Test of the HiMAT Vehicle,
NASA Technical Memorandum 84912, 1984.
- [CS98] R. Y. Chiang and M. G. Safonov,
Robust Control Toolbox User's Guide,
The MathWorks, 1998.



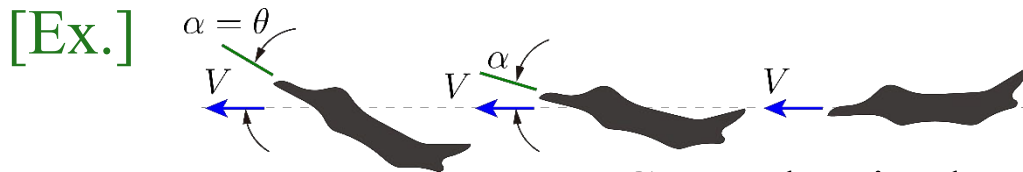
HiMAT: Attitude and flight path control



Objective

Control attitude (θ) and flight path ($\gamma = \theta - \alpha$) separately

 **MIMO System**



Control attitude without changing flight path

Specification

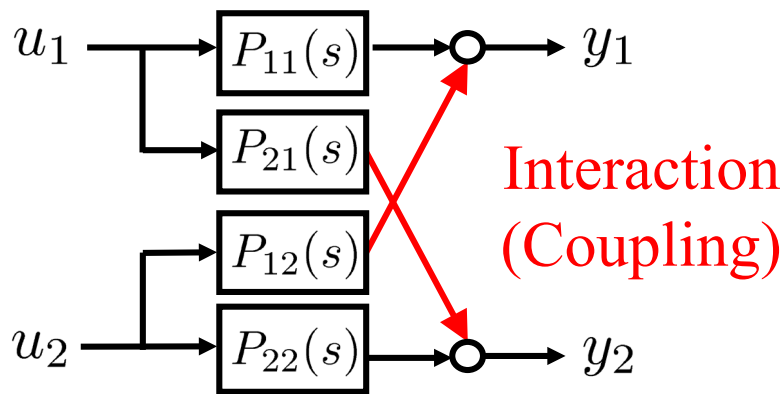
Robustness Spec.: -40dB/dec roll-off and -20dB at 100rad/s
(Because of the unmodeled low-damped structural mode)

Performance Spec.: Minimize the sens. function as much as possible 3

Modeling HiMAT u_2

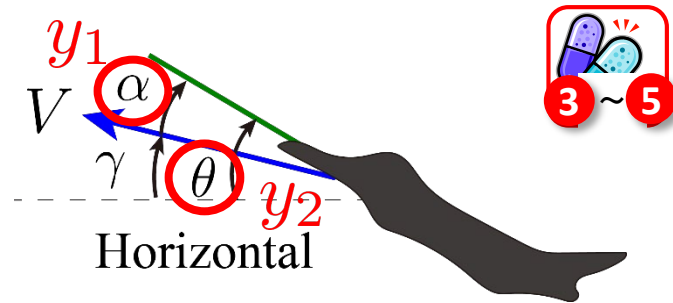
$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5.124(s+184.5)(s+0.04085)(s-0.02078)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} & \frac{-948.12(s+0.02177)(s+1.963)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} \\ \frac{-0.14896(s-4432)(s+0.06657)(s-0.047)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} & \frac{671.88(s+0.02399)(s+1.895)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} \end{bmatrix}$$



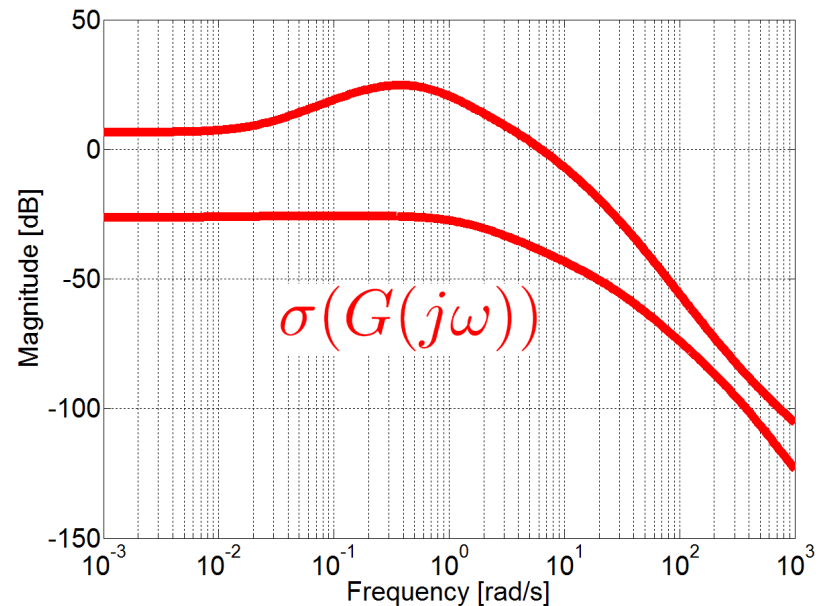
Poles $-5.6757, -0.2578,$
 $-30, -30,$
 $0.6898 \pm 0.2488j$ **Unstable**

Zeros -0.0210

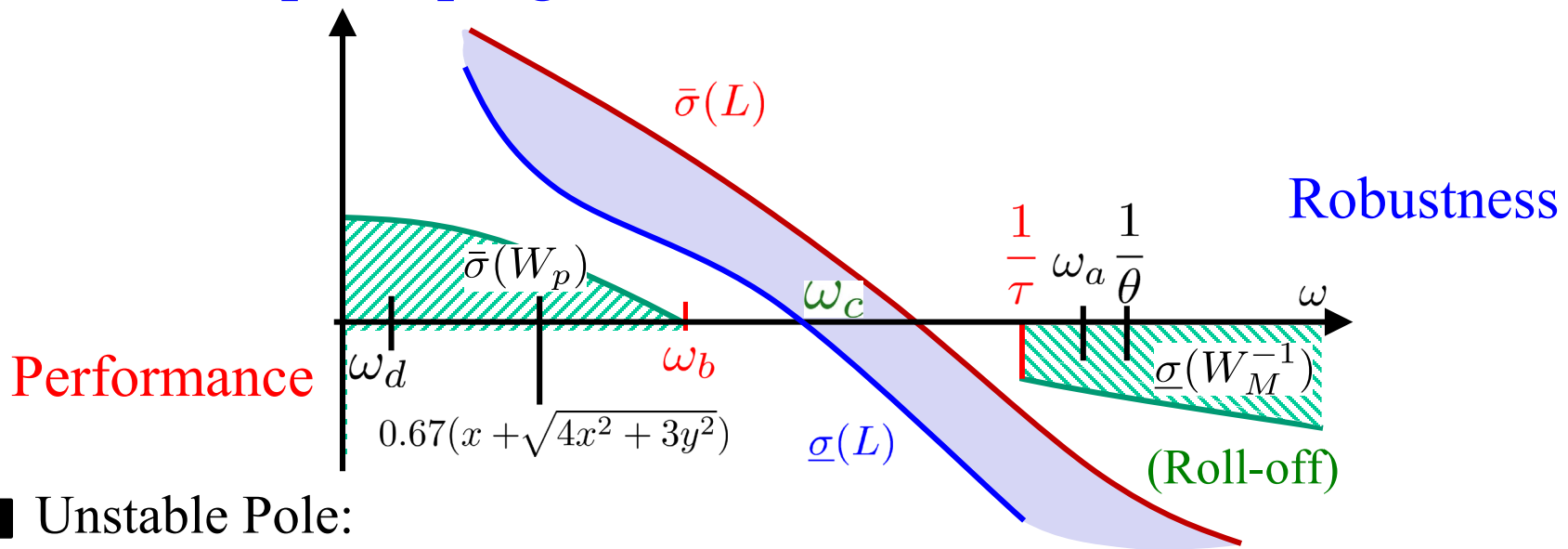


 u_1

Frequency Response



MIMO Loop Shaping [SP05, p. 343]



Unstable Pole:

$$p = x \pm yj$$

$$= 0.6898 \pm 0.2488j$$

➔ $\omega_c > 0.67(x + \sqrt{4x^2 + 3y^2})$

$$= 1.43 \text{ rad/s}$$

Disturbance (Gust):

$$\omega_c > \omega_d = 1 \text{ rad/s}$$



Performance Weight

$$\omega_c > \omega_b \approx 1.43 \text{ rad/s } W_P(s)$$

Time Delay: $\theta \leq 0.025 \text{ s}$

➔ $\omega_c < 1/\theta = 40 \text{ rad/s}$

Actuator dynamics $\omega_a = 30 \text{ rad/s}$

➔ $\omega_c < \omega_a = 30 \text{ rad/s}$



Uncertainty Weight

$$\omega_c < \frac{1}{\tau} = 30 \text{ rad/s } W_M(s)$$

HiMAT: Uncertainty Weight

Uncertain Factors:

Actuator Dynamics +
Unmodeled dynamics (High freq.)

➔ Frequency: $1/\tau \leq 30 \text{ rad/s}$

2 Oscillation mode (2nd order system)

$$95 < \omega_1 < 105 [\text{rad/s}], \quad 0.05 < \zeta_1 < 0.07$$

$$280 < \omega_2 < 320 [\text{rad/s}], \quad 0.012 < \zeta_2 < 0.018$$

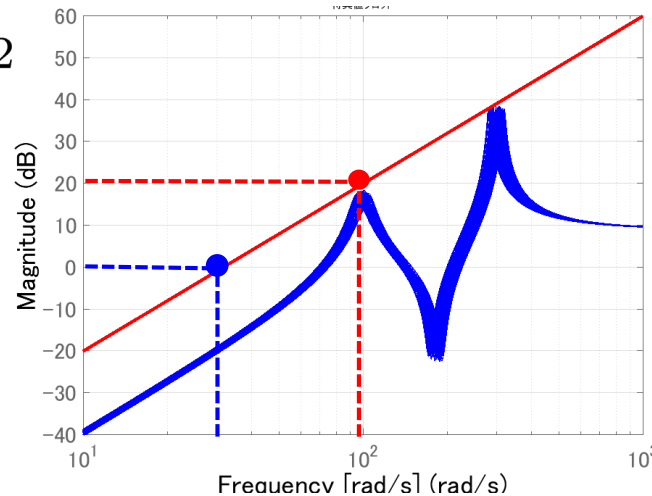
ω_1, ω_2 : Natural angular frequency

ζ_1, ζ_2 : Damping factor

$$W_M(s) = w_M(s) I_2$$

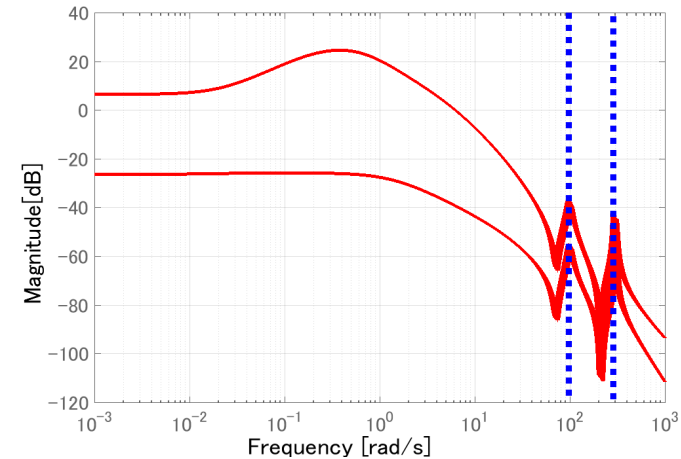
$$w_M(s) = \frac{s^2}{1000}$$

$$\left(\begin{array}{l} \tau = 0.033, \\ 1/\tau = 30 \text{ rad/s} \\ r_0 = 0, \\ r_\infty \rightarrow \infty \end{array} \right)$$

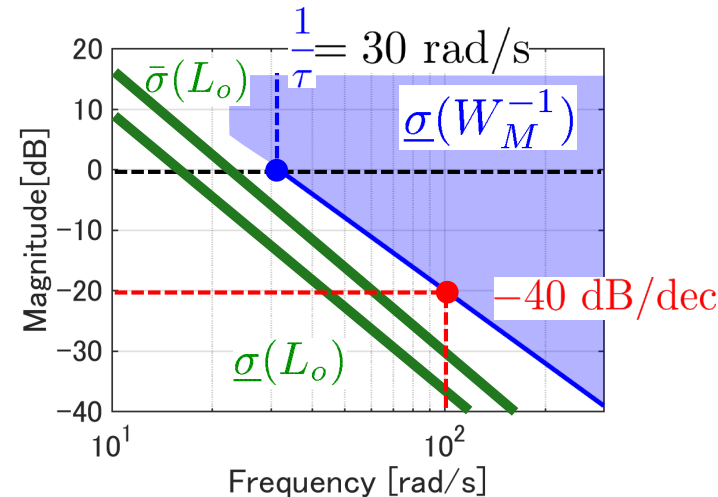


MATLAB Command

```
wM = tf([1 0 0], [1000]);
WM = eye(2)*wM;
```



Target Loop



Roll-off: -20 dB at 100 rad/s
 -40 dB/dec

HiMAT: Performance Weight

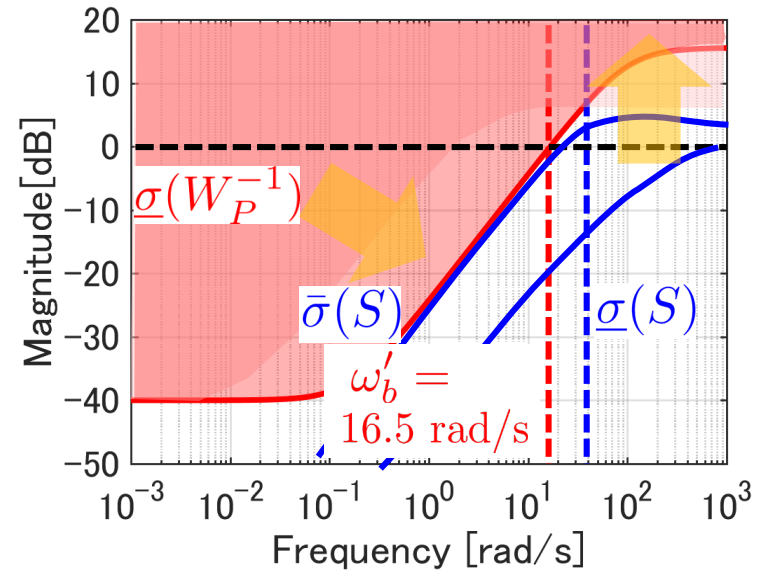
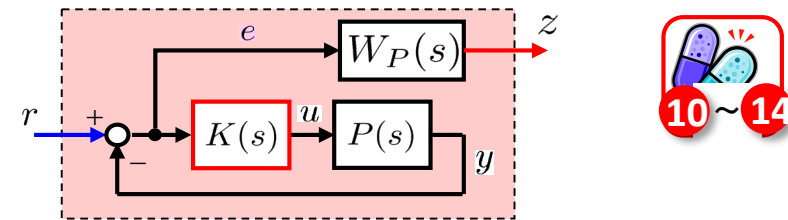
$$W_P(s) = w_p(s)I_2$$

$$w_p(s) = \frac{1}{M_s} s + \omega_b = \frac{0.5s + 1.5}{s + 0.015}$$

$$\left[\begin{array}{l} \omega_b = 1.5 (\approx 0.67(x + \sqrt{4x^2 + 3y^2})) \\ M_s = 2, A = 0.01 \end{array} \right]$$

MATLAB Command

```
Ms = 2; A = 0.01; wb = 1.5;
wP = tf([1/Ms wb], [1 wb*A]);
WP = eye(2)*wP;
```



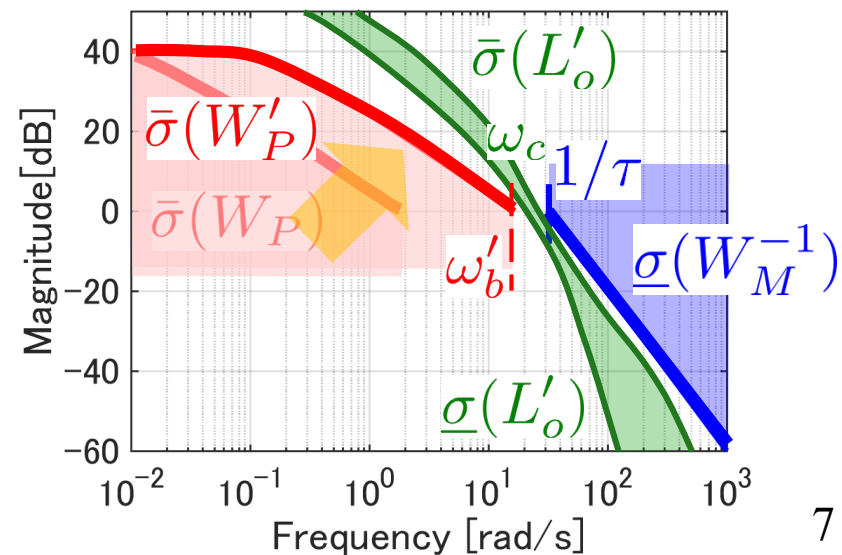
Update Larger ω_b

$$\rightarrow \omega_b = 1.5 \rightarrow 16.5$$

$$M_s = 2 \rightarrow 6 \quad (\text{Trade-off})$$

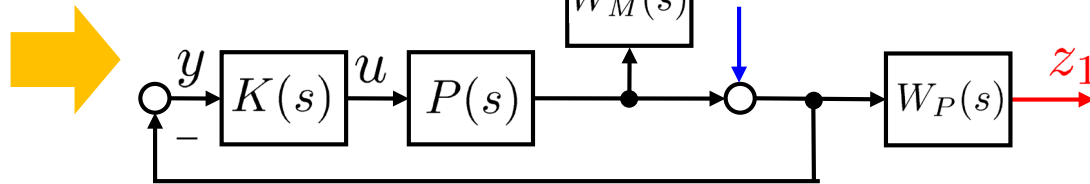
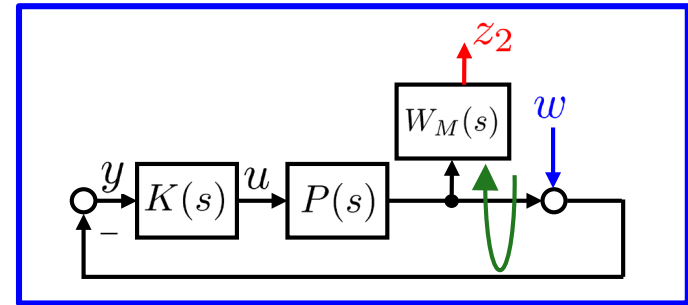
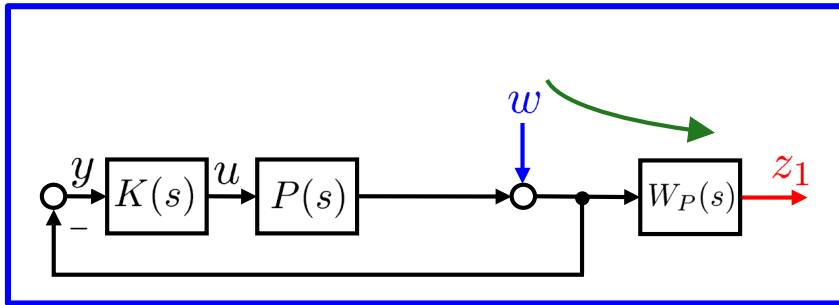
$$A = 0.01 > 0$$

$$\rightarrow w'_p(s) = \frac{0.165s + 16.5}{s + 0.01}$$

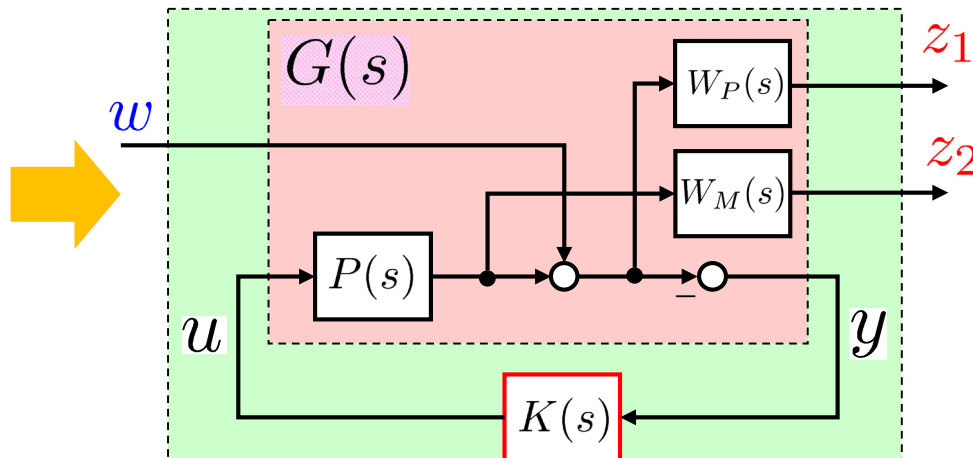


HiMAT: Control Problem Formulation

Nominal Performance $\|W_P(s)S_o(s)\|_\infty$ Robust Stability $\|W_M(s)T_o(s)\|_\infty$



Generalized Plant



MATLAB Command

```
%Generalized Plant%
systemnames = 'Pnom WP WM';
inputvar = '[w(2);u(2)]';
outputvar = '[WP;WM;-w-Pnom]';
input_to_Pnom='[u]';
input_to_WP = '[w+Pnom]';
input_to_WM = '[Pnom]';
G = sysic;
```

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_P(s)S_o(s) \\ -W_M(s)T_o(s) \end{bmatrix} w$$

$$= F_l(G, K)$$

Mixed Sensitivity

Find K such that $\|F_l(G, K)\|_\infty < \gamma$

Check 1)

Appropriately sub-optimal

(Default settings)

$$\gamma = 0.9538 < 1 \quad (\gamma_{opt} = 0.9510)$$

Check 2) $\gamma = 0.9$ No Solution

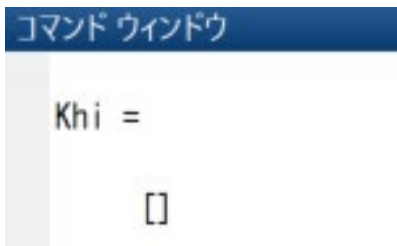
Resetting value of Gamma min based on D_11, D_12, D_21 terms

Test bounds: 0.1667 < gamma <= 0.9538

gamma	hamx_eig	xinf_eig	hamy_eig	yinf_eig	nrho_xy	p/f
0.954	2.1e-02	1.8e-05	1.6e-01	1.0e-22	0.9915	p
0.560	2.1e-02	-1.1e+06#	1.6e-01	-6.4e-20	0.2088	f
0.757	2.1e-02	-8.2e+06#	1.6e-01	-1.4e-20	0.6112	f
0.855	2.1e-02	-5.0e+07#	1.6e-01	-4.1e-20	2.9142#	f
0.905	2.1e-02	1.9e-05	1.6e-01	-4.4e-23	3.1083#	f
0.929	2.1e-02	1.8e-05	1.6e-01	-4.7e-20	1.5096#	f
0.941	2.1e-02	1.8e-05	1.6e-01	-2.5e-20	1.1979#	f
0.948	2.1e-02	1.8e-05	1.6e-01	-1.1e-19	1.0852#	f

Gamma value achieved: 0.9538

Data Structure



名前	値
CLhi	[]
Khi	[]
ghi	[]
hiinfo	[]

There is No Controller

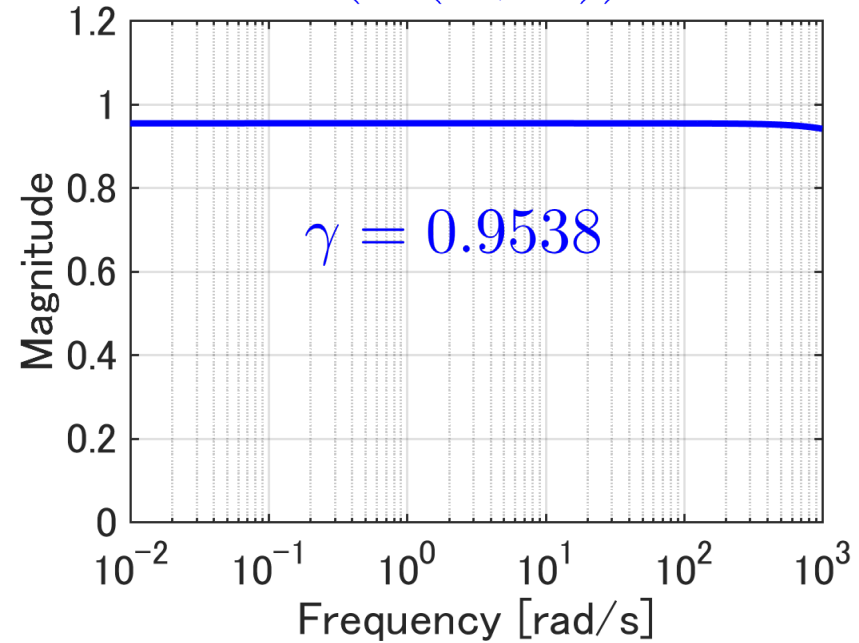
MATLAB Command

```
[Khi,CLhi,ghi,hiinfo] = ...  
hinfscn(G,nmeas,ncon,'Display', 'on');
```

MATLAB Command

```
[SV,w]=sigma(CLhi);  
figure; semilogx(w,SV)
```

Closed-loop Transfer Function $\bar{\sigma}(F_l(G, K))$



HiMAT: Controller

MATLAB Command

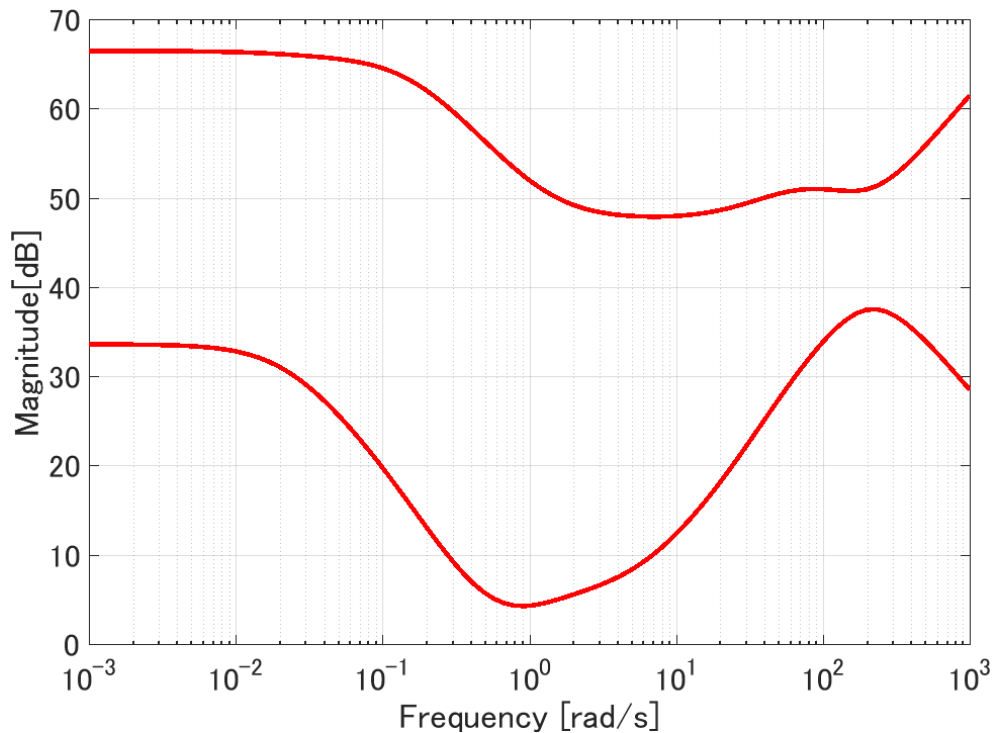
```
figure  
sigma(Khi)
```



$$K(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$

[Ex.]

$$K_{11}(s) = \frac{-2.101 \times 10^6 (s + 46.24)(s + 30)(s + 1.506)}{(s + 1.857 \times 10^4)(s + 5600)(s + 120.4)} \\ \times \frac{(s + 0.165)(s + 0.02513)(s^2 - 1453s + 1.174 \times 10^6)}{(s + 60.39)(s + 47.57)(s + 0.165)^2(s + 0.02098)}$$



Poles

$$\begin{aligned} & -1.8549 \cdot 10^4, -5.6005 \cdot 10^3 \\ & -1.2045 \cdot 10^2 \\ & -60.3871, -47.5747 \\ & -0.165, -0.165, -0.0210 \end{aligned}$$

Zeros

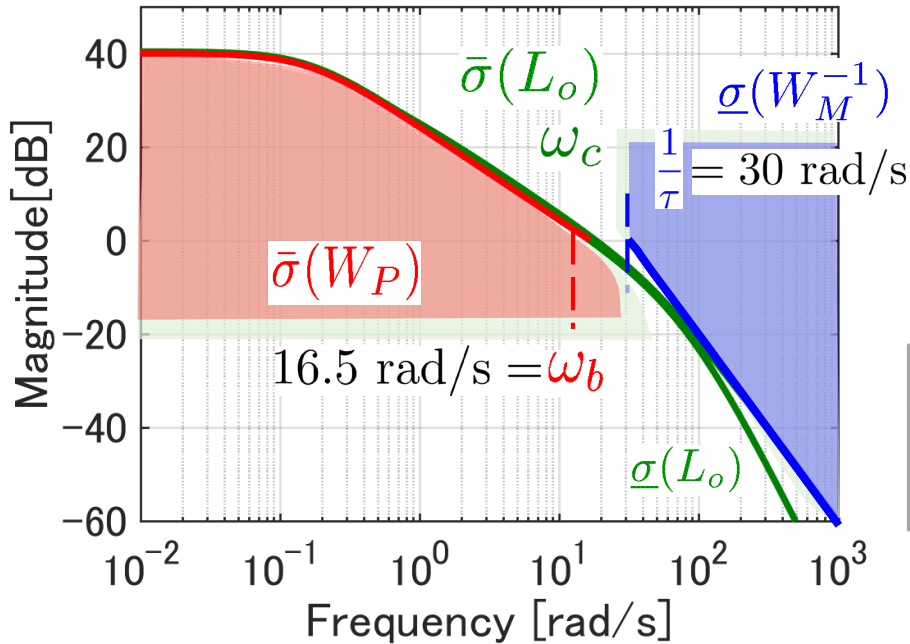
$$\begin{aligned} & -0.6127 \pm 0.3018j \\ & -30, -30 \\ & -5.6757, -0.2578 \end{aligned}$$

Order 8



Model Reduction

HiMAT: Open-loop Frequency Response



$$\gamma = 0.9538 < 1$$

corresponding maximum
stability margin

Loop Transfer Function

MATLAB Command

```
figure
sigma(Fhi.Lo,WP, inv(WM),WP/ghi,ghi*inv(WM))
```

NP/RS Test

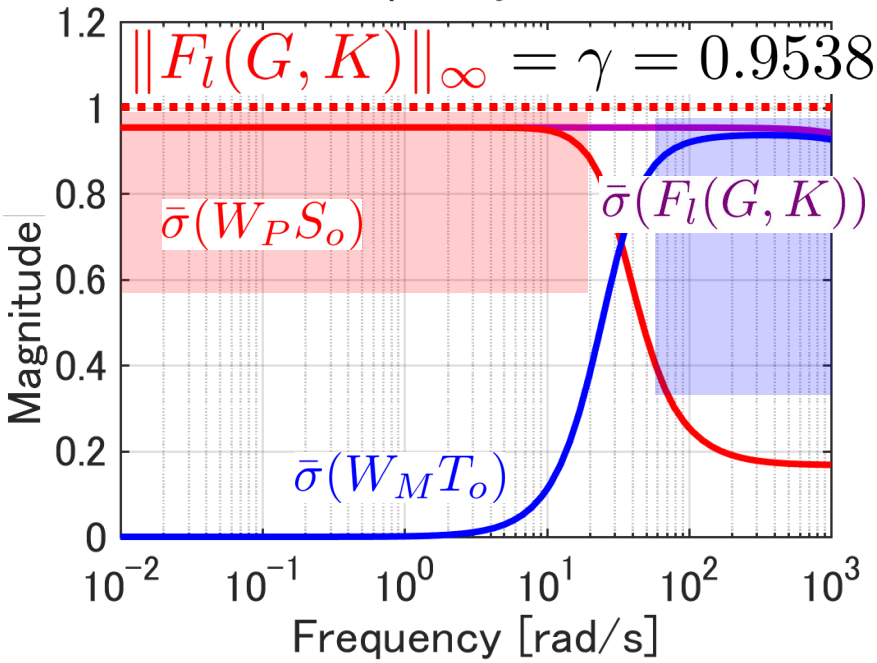
Nominal Performance (NP)

$$\|W_P S_o\|_\infty = 0.9538 < 1$$



Robust Stability (RS)

$$\|W_M T_o\|_\infty = 0.9358 < 1$$



MATLAB Command

```
[SV,w]=sigma(WP*Fhi.So);
figure; semilogx(w,SV)
[SV,w]=sigma(WM*Fhi.To);
figure; semilogx(w,SV)
```

HiMAT: Closed-loop Performance

MATLAB Command

```
pole(CLhi)  
zero(CLhi)  
figure; pzmap(CLhi)
```

Nominal Stability (NS)

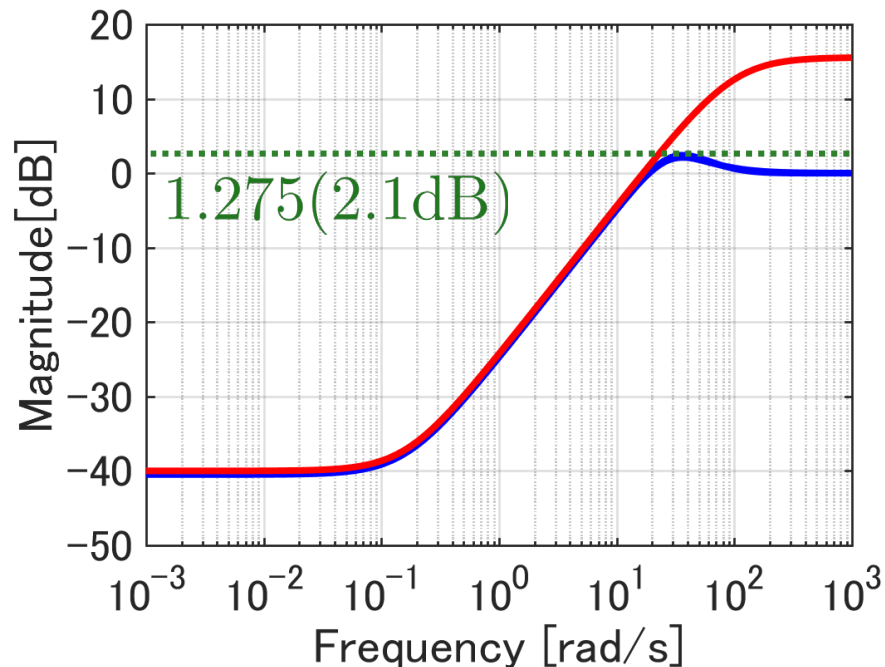
Poles of $F_l(G, K)$ 🍷

$$p = -0.165, -0.165, -0.0210, -0.2578, -5.6757, -30, -30 \\ -1.3346 \cdot 10^2, -5.6007 \cdot 10^3, -1.8549 \cdot 10^4 \\ -0.6898 \pm 0.2488j, -22.441 \pm 17.871j, -23.720 \pm 19.701j$$

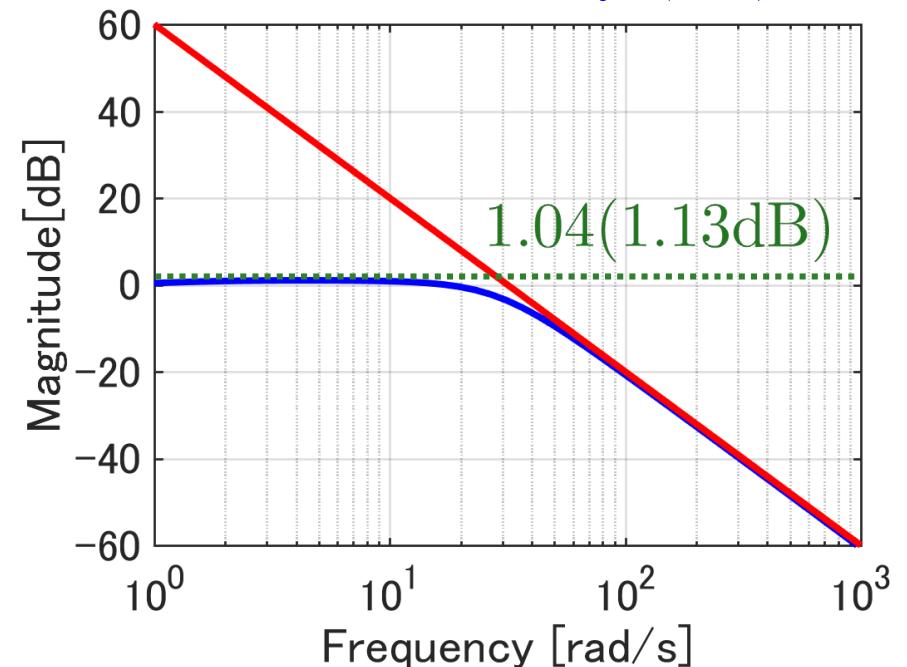
Zeros of $F_l(G, K)$ 🍷

$$-0.0210$$

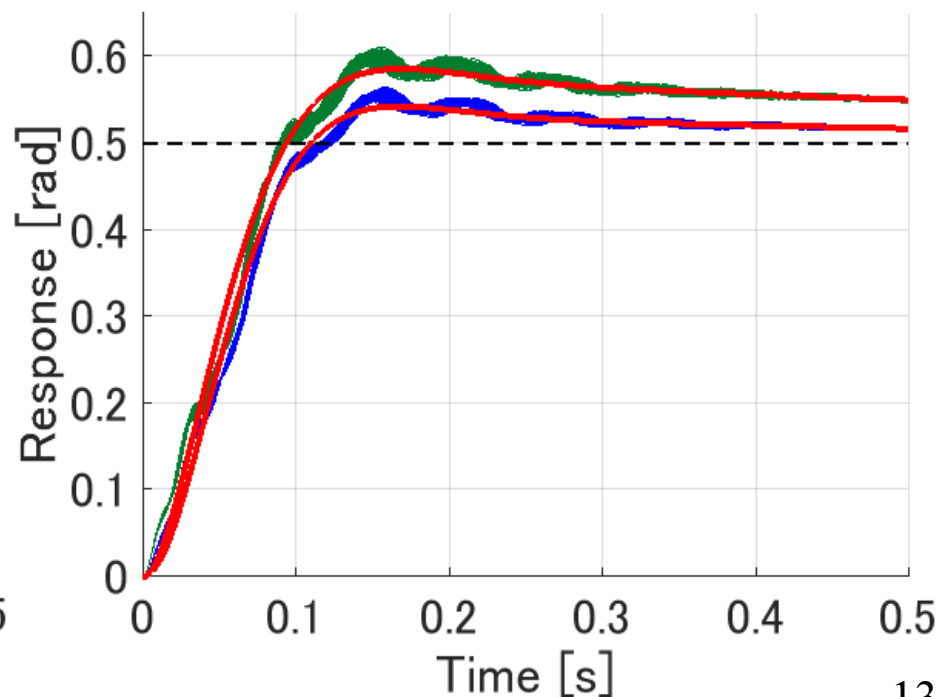
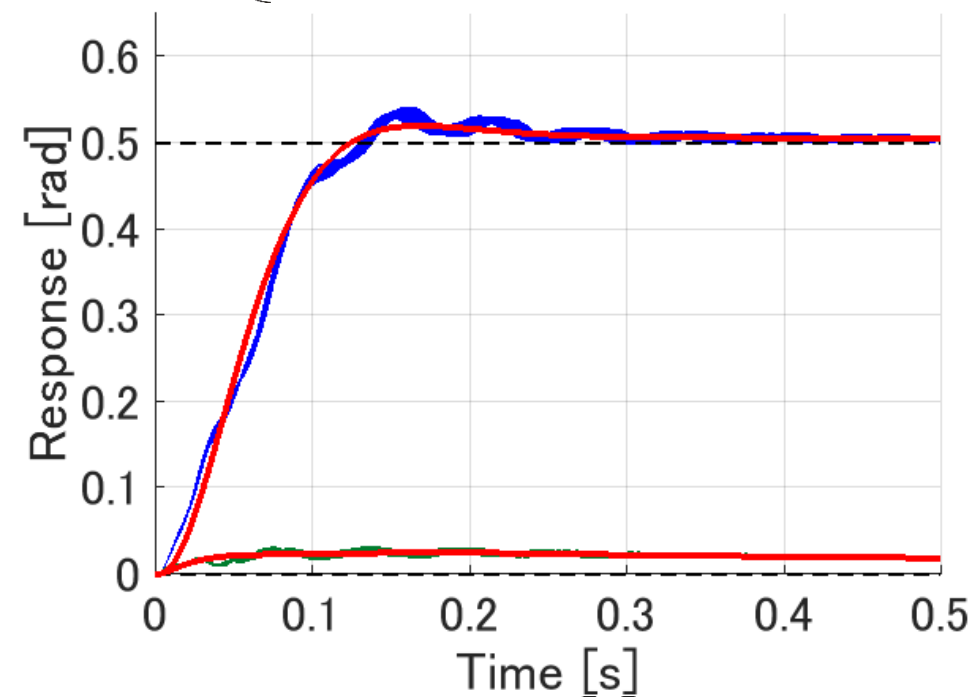
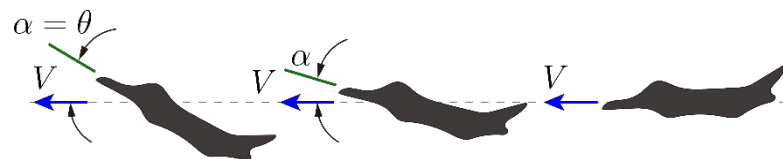
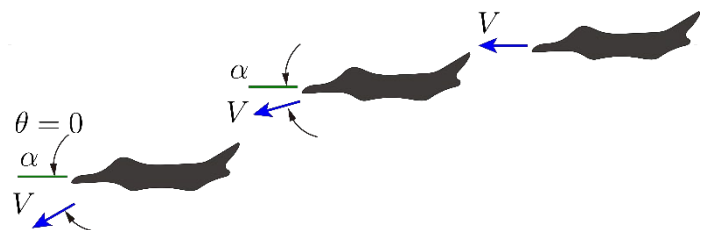
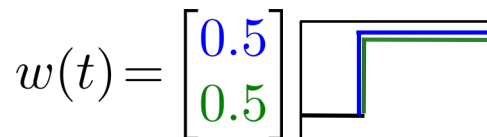
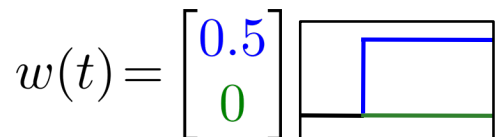
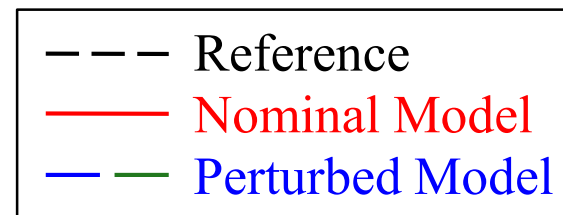
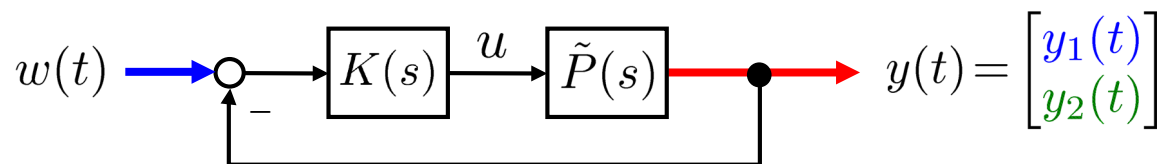
Nominal Performance (NP)



Robust Stability (RS)



HiMAT: Step Response (closed loop)



8. Design Example



8.1 HiMAT: H_∞ Control

(Highly Maneuverable Aircraft Technology)

Reference:

[SP05] S. Skogestad and I. Postlethwaite,

Multivariable Feedback Control; Analysis and Design,
Second Edition, Wiley, 2005.

[SLH81] M. G. Safonov, A. J. Laub and G. L. Hartmann,

*Feedback Properties of Multivariable Systems:
The Role and Use of the Return Difference Matrix,*

IEEE Transactions on Automatic Control, Vol. 26, No. 1, pp. 47-65, 1981.

[ES84] M. B. Evans and L. J. Schilling,

The Role of Simulation in the Development and Flight Test of the HiMAT Vehicle,
NASA Technical Memorandum 84912, 1984.

[CS98] R. Y. Chiang and M. G. Safonov,

Robust Control Toolbox User's Guide,
The MathWorks, 1998.

Class evaluation by students

Class Name : Robust Control

Code of Class : SCE.C402

Instructor : Prof. Masayuki Fujita

How to fill out questionnaire

- Use **Pencil only** (Do not use Pen)
- Mark **Carefully**





Real Product

General characteristics

Crew: None

Length: 6.86m

Wingspan: 4.75m

Height: 1.31m

Max takeoff weight: 1,542kg

Performance

Maximum speed: Mach 1.6

Maneuvering: 8G performance



Real Product

Testing Product

UAV(Unmanned Aerial Vehicle)

Teleoperation

Testing high technology

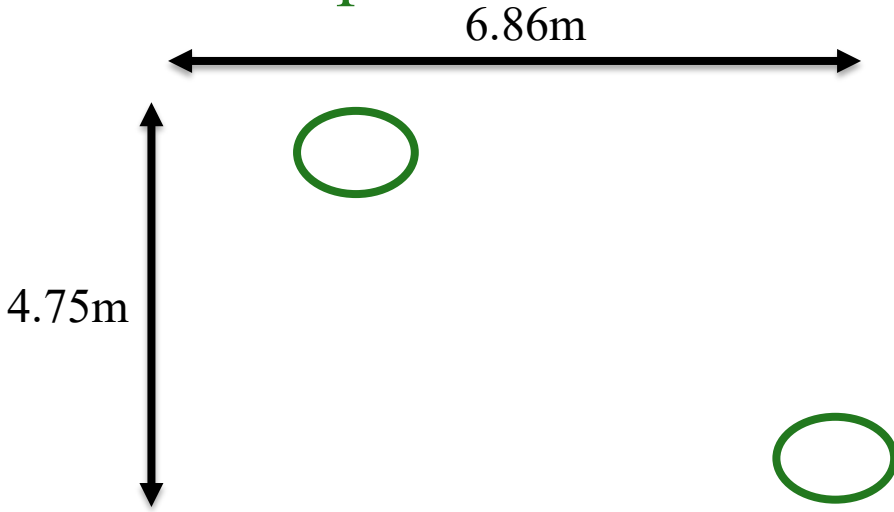
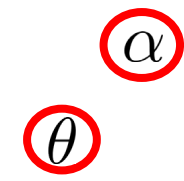
HiMAT



Modeling HiMAT

Control Input

Output



- α : angle of attack
- θ : attitude angle
- δ_e : elevon actuator
- δ_c : canard actuator
- x_e } states to represent
- x_c } actuator dynamics

$$x^T = (\dot{\alpha} \quad \alpha \quad \dot{\theta} \quad \theta \quad x_e \quad x_c)$$

$$u^T = (\delta_e \quad \delta_c) \quad y^T = (\alpha \quad \theta)$$

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5.124s^3 - 945.7s^2 - 18.98s + 0.8025}{s^5 + 34.55s^4 + 130.4s^3 - 184.4s^2 + 35.95s + 23.6} & \frac{-948.1s^2 - 1881s - 40.51}{s^5 + 34.55s^4 + 130.4s^3 - 184.4s^2 + 35.95s + 23.6} \\ \frac{-0.149s^3 + 660.1s^2 + 12.92s - 2.066}{s^5 + 34.55s^4 + 130.4s^3 - 184.4s^2 + 35.95s + 23.6} & \frac{671.9s^2 + 1290s + 30.55}{s^5 + 34.55s^4 + 130.4s^3 - 184.4s^2 + 35.95s + 23.6} \end{bmatrix}$$



HIMAT: Nominal Plant Model

State Space Form (Matrix Representation)

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \quad \begin{array}{l} x^T = (\dot{\alpha} \quad \alpha \quad \dot{\theta} \quad \theta \quad x_e \quad x_c) \\ u^T = (\delta_e \quad \delta_c) \quad y^T = (\alpha \quad \theta) \end{array}$$

$$A = \begin{pmatrix} -0.023 & -36.62 & -18.90 & -32.09 & 3.251 & -0.763 \\ 0.000 & -1.900 & -0.983 & -0.000 & -0.171 & -0.005 \\ 0.012 & 11.72 & -2.632 & 0.000 & -31.60 & 22.40 \\ 0 & 0 & 1.000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30.00 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30.00 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 30 & 0 \\ 0 & 30 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

MATLAB Command

```
% NASA HiMAT model G(s)
ag = [ -2.2567e-02 -3.6617e+01 -1.8897e+01 -3.2090e+01 3.2509e+00 -7.6257e-01;
       9.2572e-05 -1.8997e+00 9.8312e-01 -7.2562e-04 -1.7080e-01 -4.9652e-03;
       1.2338e-02 1.1720e+01 -2.6316e+00 8.7582e-04 -3.1604e+01 2.2396e+01;
        0          0 1.0000e+00          0          0          0;
        0          0          0          0 -3.0000e+01          0;
        0          0          0          0          0 0 -3.0000e+01];
bg = [ 0 0; 0 0; 0 0; 0 0; 30 0; 0 30];
cg = [ 0 1 0 0 0 0; 0 0 0 1 0 0];
dg = [ 0 0; 0 0];
G=ss(ag,bg,cg,dg);
```



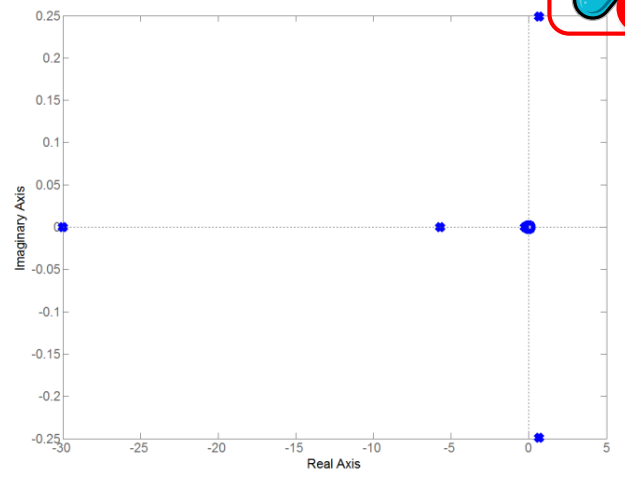
HiMAT: Nominal Plant Model

Controllability 

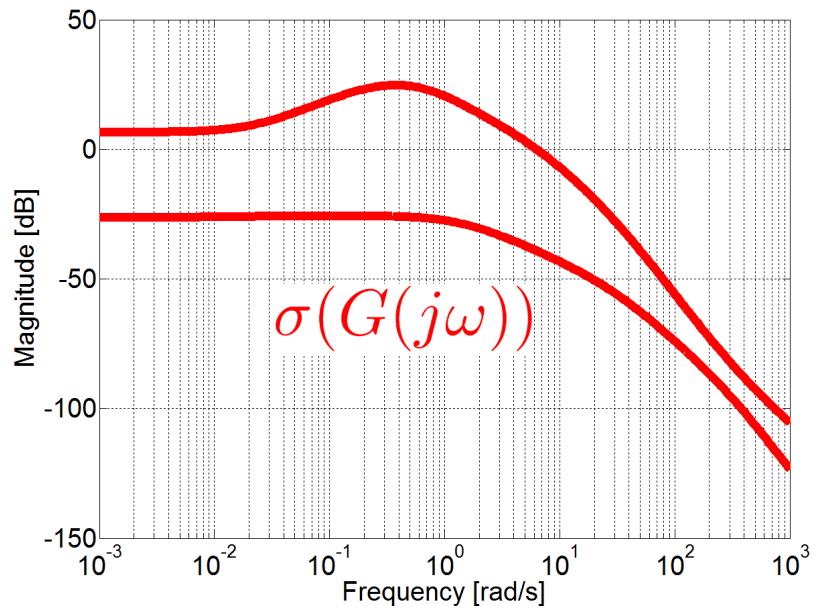
Observability 

Poles (Stability) $-5.6757, -0.2578, -30, -30,$
 $0.6898 \pm 0.2488i$
Unstable

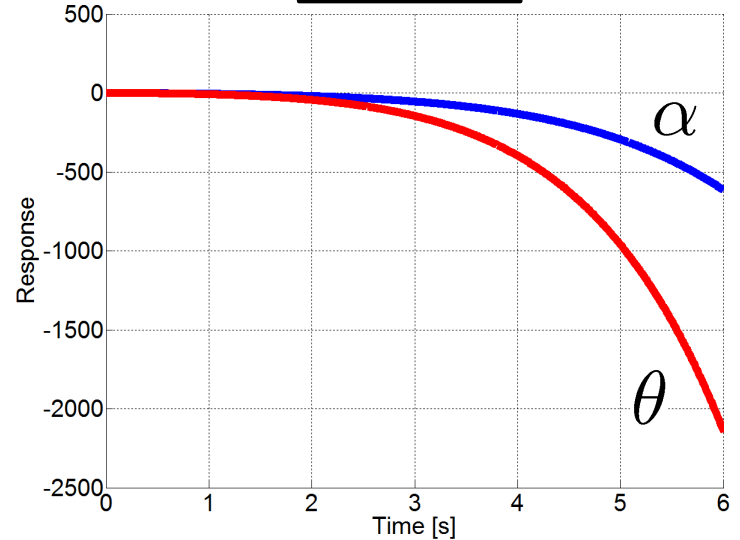
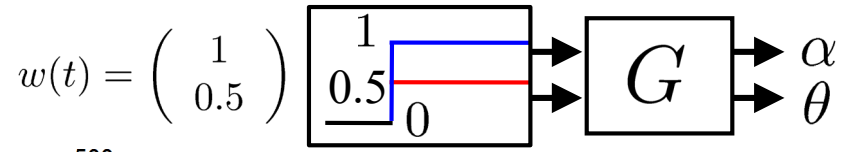
Zeros -0.0210



Frequency Response



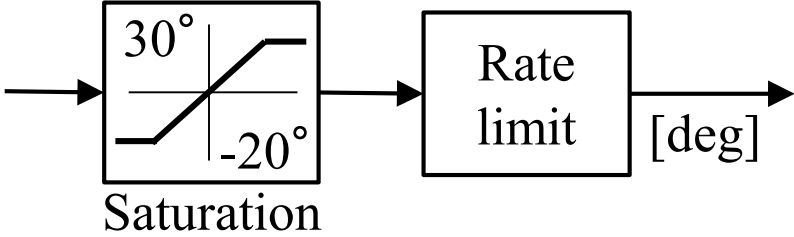
Step Response for Nominal Plant Model



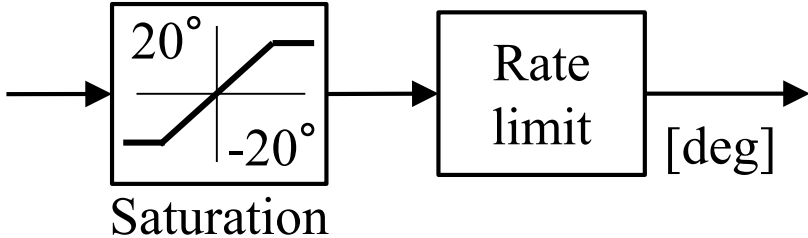


Actuators: Hydraulic serve system

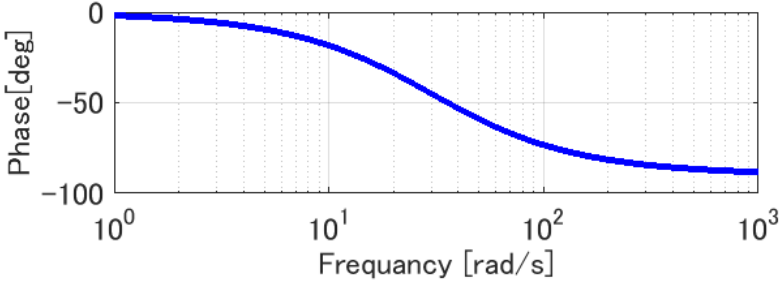
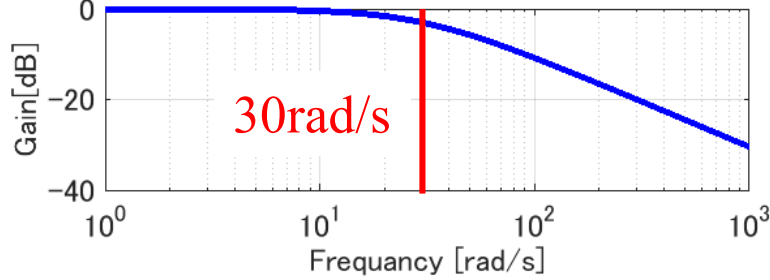
Block diagrams of elevon



Block diagrams of canard



Bode diagram of actuators

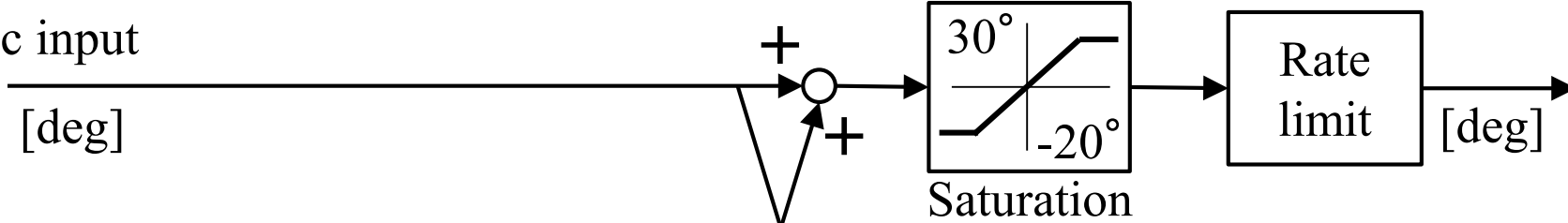




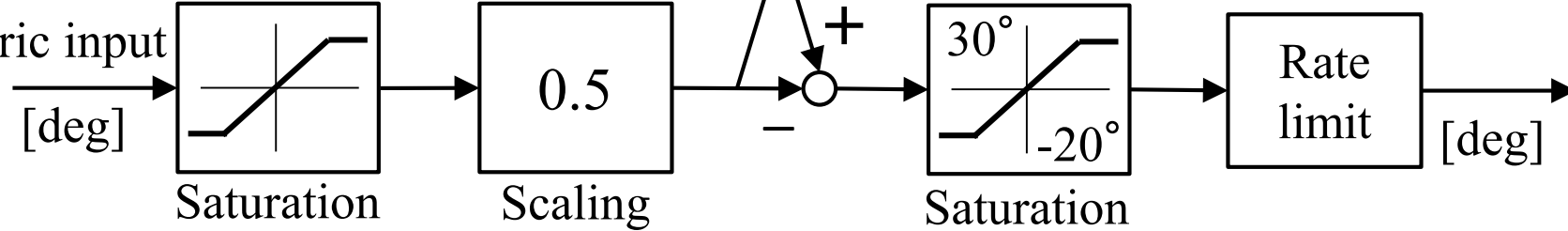
Actuators: Hydraulic serve system

Block diagrams of elevon

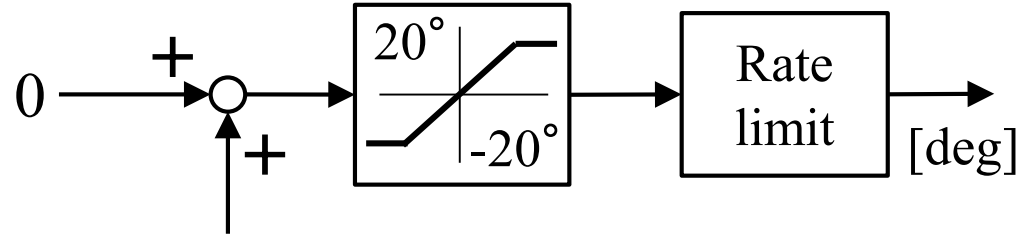
Symmetric input



Asymmetric input

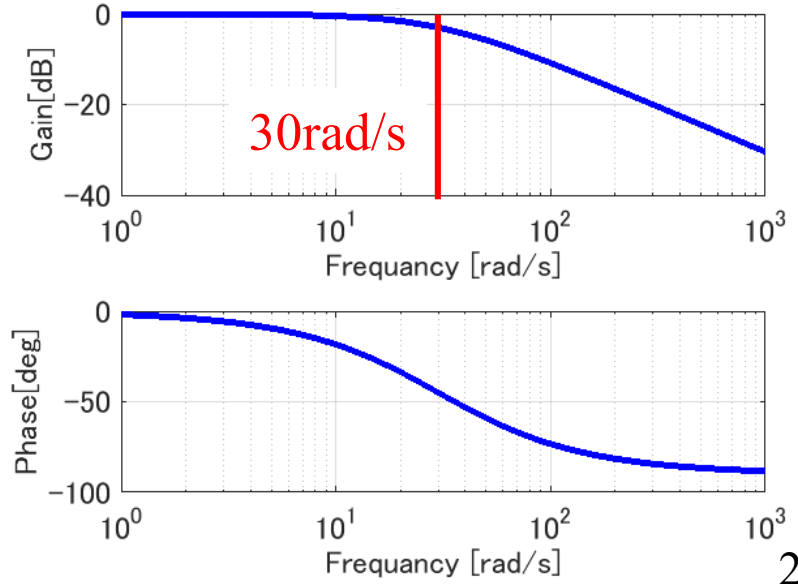


Block diagrams of canard



Symmetric canard pulse

Bode diagram of actuators

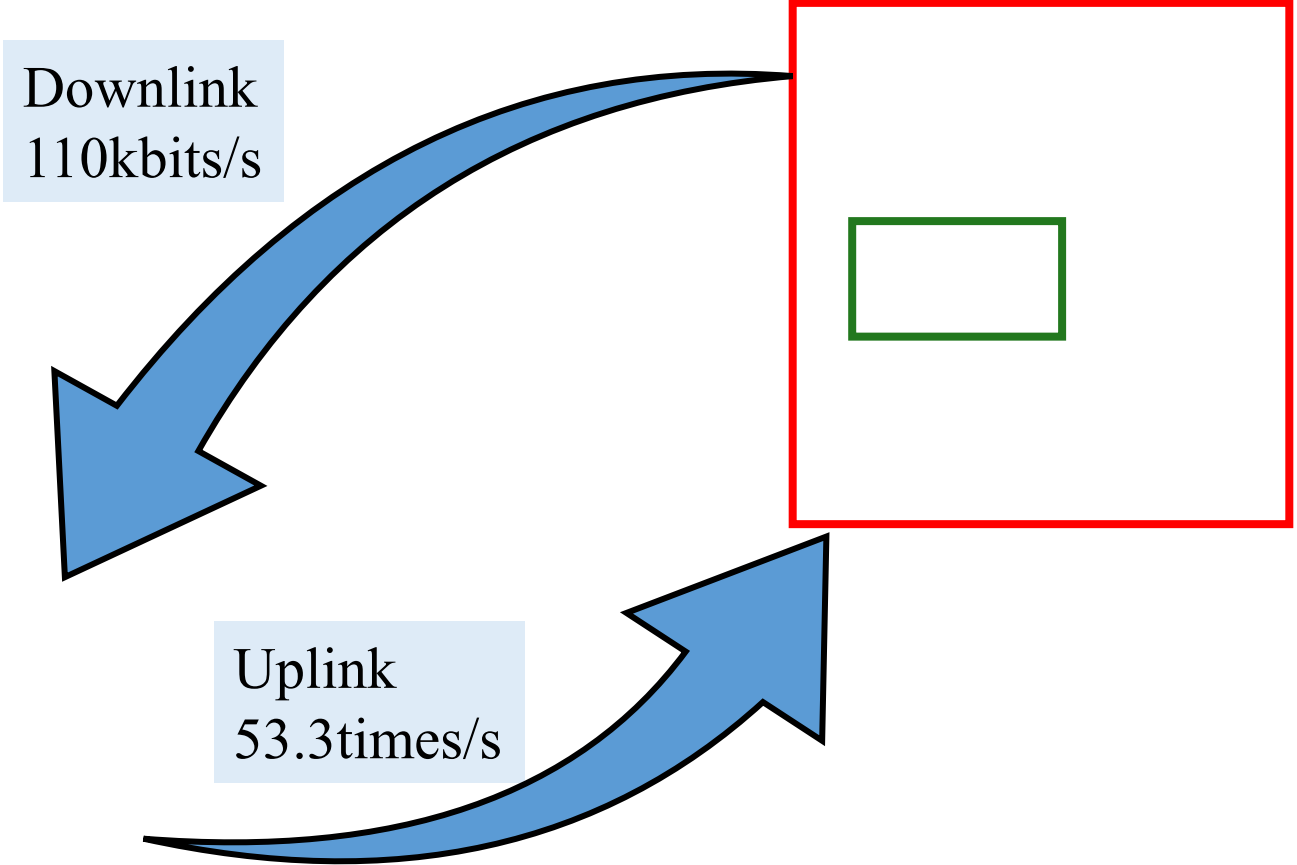




Loop Time: Control Input time

Onboard computer

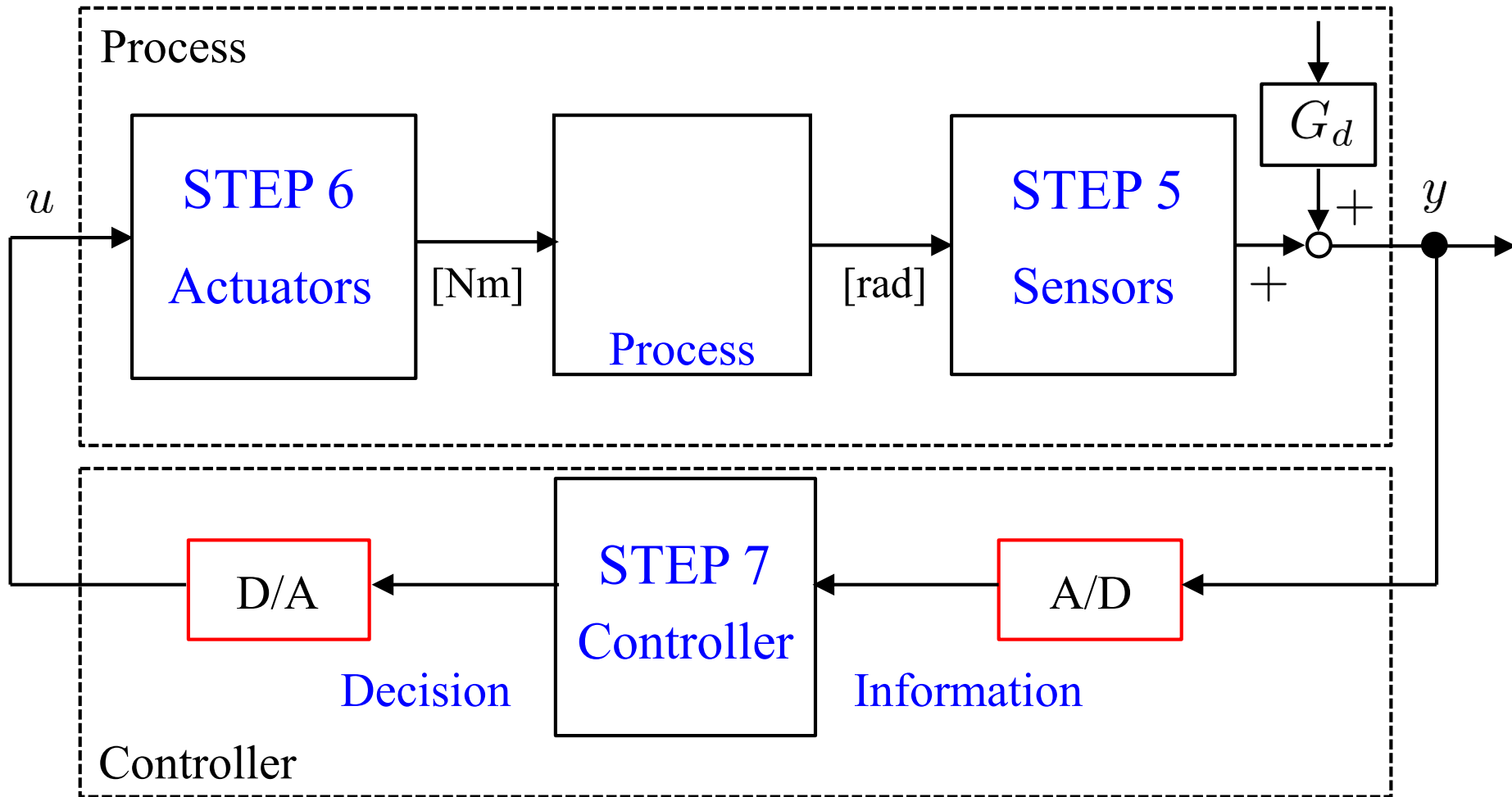
Processing Time	25 ms (40Hz)
-----------------	--------------



Models: Understand Performance Specifications



G_d : Process Noise





HiMAT: Closed-loop Performance (without update W_P)

Nominal Stability (NS)

Poles of $F_l(G, K)$ ○

$$p = -0.0150, -0.0150, -0.0210, -0.2578, -2.9920, -3.0011$$

$$-5.6757, -30, -30, -45.8984, -5.7180 \cdot 10^2, -7.5812 \cdot 10^2$$

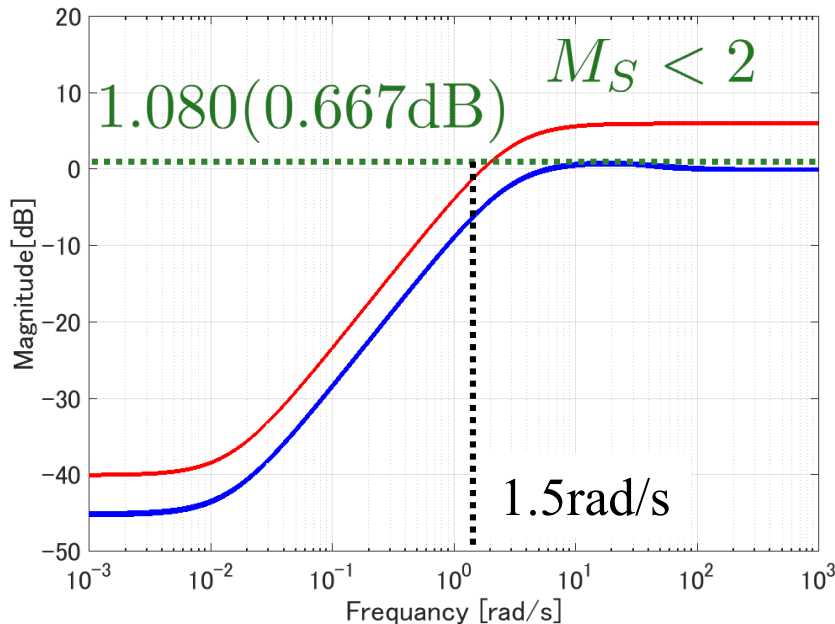
$$-0.6898 \pm 0.2488j, -34.1085 \pm 28.5745j$$

Zeros of $F_l(G, K)$ ○

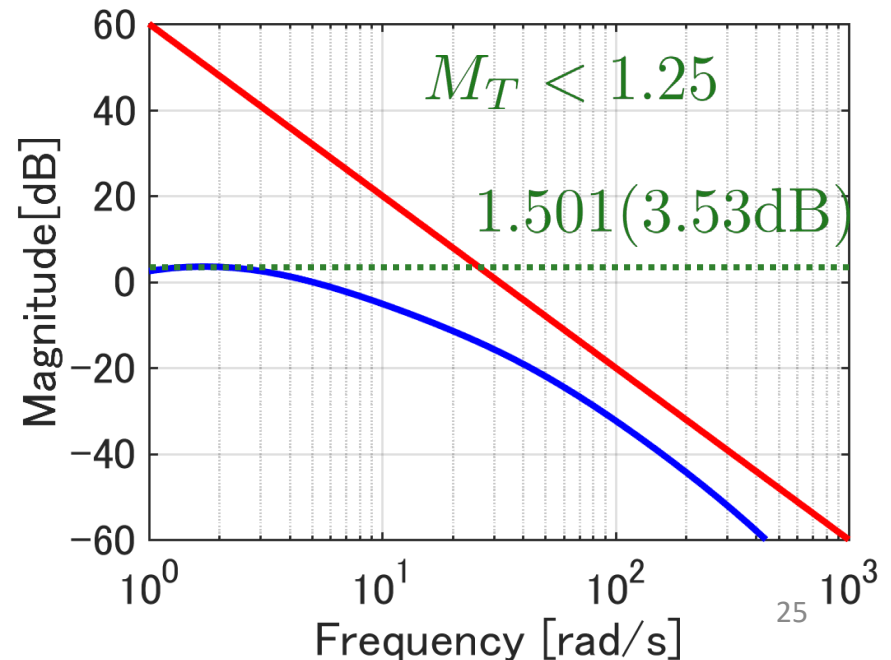
Nothing

⇒ Pole/Zero Cancellations

Nominal Performance (NP)



Robust Stability (RS)



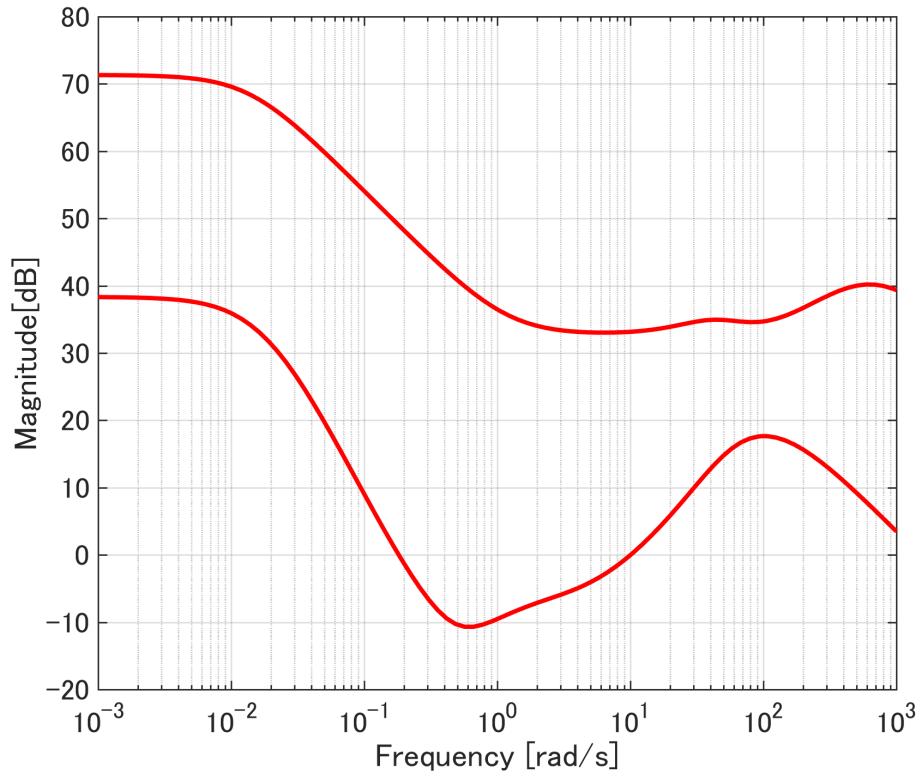
HiMAT: Controller (without update W_P)

MATLAB Command

```
figure  
sigma(Khi)
```



$$K(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$



Poles

$-7.5904 \cdot 10^2, -5.7002 \cdot 10^2$
 $-35.6460 \pm 30.3447j$
 $-52.3996, -0.0210$
 $-0.0148, -0.0148$

Zeros

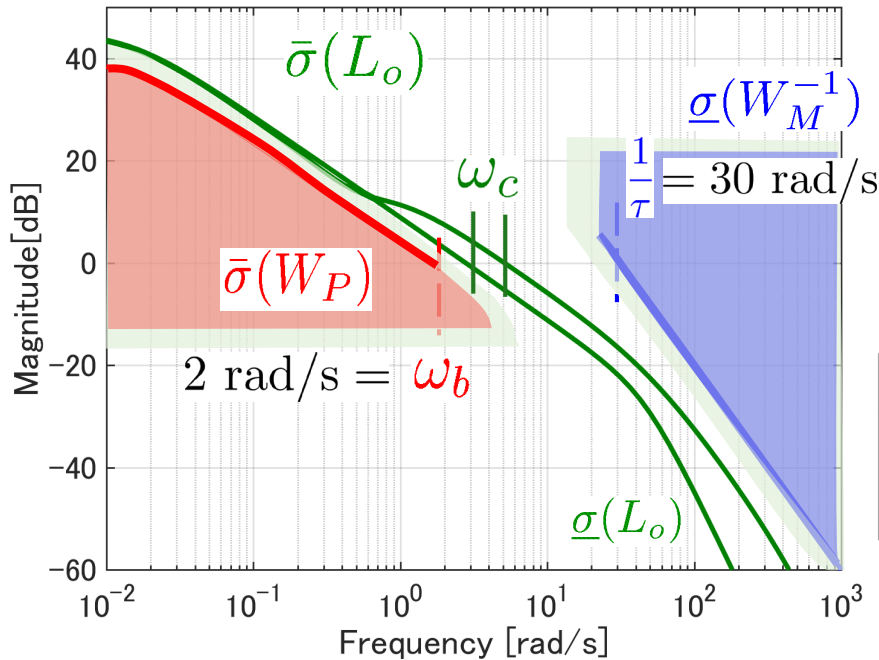
$-0.3584 \pm 0.3654j$
 $-30, -30$
 $-5.6757, -0.2578$

Order 8

Numerical problems or inaccuracies may be caused too high order

➔ Difficult to implement

HiMAT: Open-loop Frequency Response (without update W_P)



$$\gamma = < 1$$

corresponding maximum
stability margin




Loop Transfer Function

MATLAB Command


```
figure
sigma(Fhi.Lo,WP, inv(WM),WP/ghi,ghi*inv(WM))
```

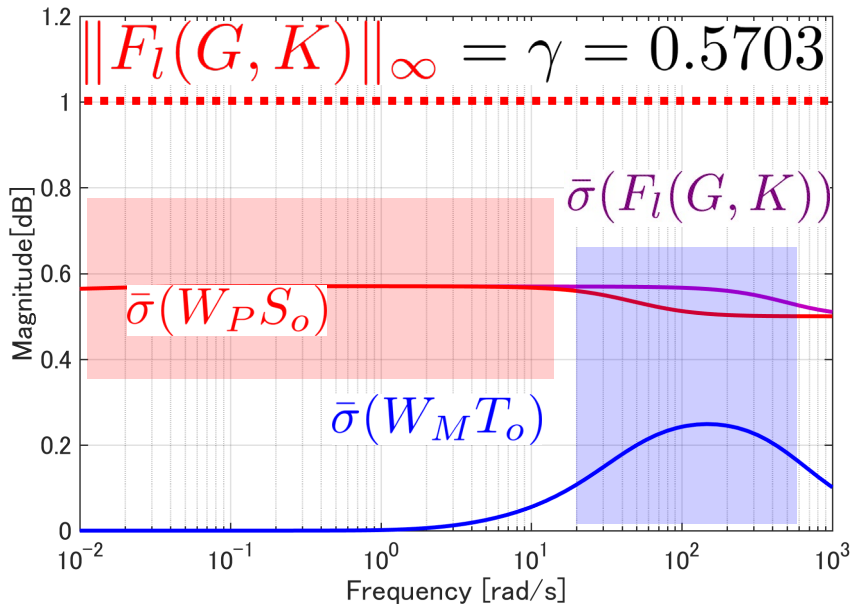
NP/RS Test

Nominal Performance (NP)

$$\|W_P S_o\|_\infty = 0.570 < 1$$


Robust Stability (RS)

$$\|W_M T_o\|_\infty = 0.249 < 1$$




MATLAB Command

```
[SV,w]=sigma(WP*Fhi.So);
figure; semilogx(w,SV)
[SV,w]=sigma(WM*Fhi.To);
figure; semilogx(w,SV)
```

HiMAT: γ -iteration to obtain H_∞ Controller (without update W_P)

Find K such that $\|F_l(G, K)\|_\infty < \gamma$

[SP05, p. 358]



Check 1)

Appropriately sub-optimal

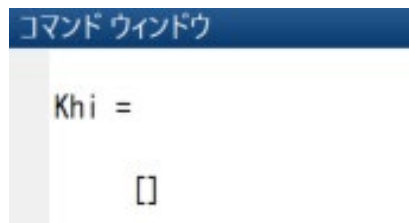
(Default settings)

$$\gamma = 0.5703 < 1 \quad (\gamma_{opt} = 0.5438)$$

Check 2) $\gamma = 0.5$ No Solution

Test) Tune W_M and sub-optimality

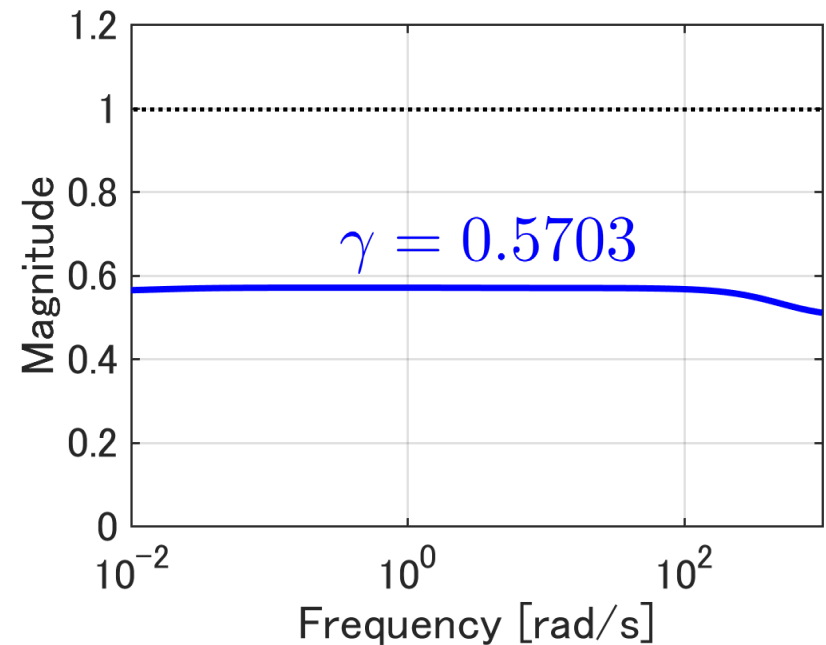
Data Structure



名前	値
CLhi	[]
Khi	[]
ghi	[]
hiinfo	[]

There is No Controller

Closed-loop Transfer Function
 $\bar{\sigma}(F_l(G, K))$



“interested” frequency range

MATLAB Command

```
[Khi,CLhi,ghi,hiinfo] = ...  
hinfscn(G,nmeas,ncon,'Gmax',100,'Gmin',100);
```

MATLAB Command

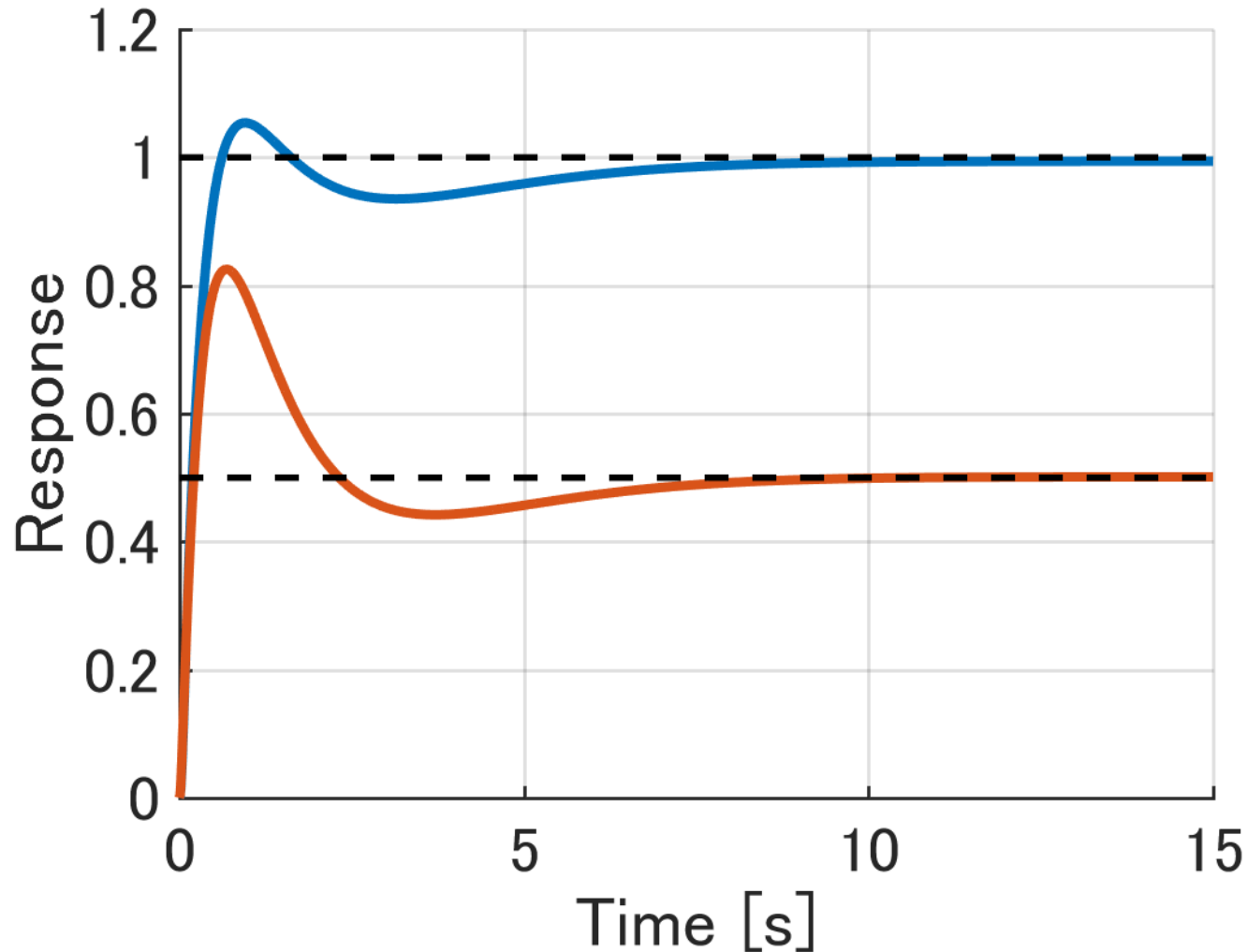
```
[SV,w]=sigma(CLhi);  
figure; semilogx(w,SV)
```

HiMAT: Step Response of closed loop (without update W_P)



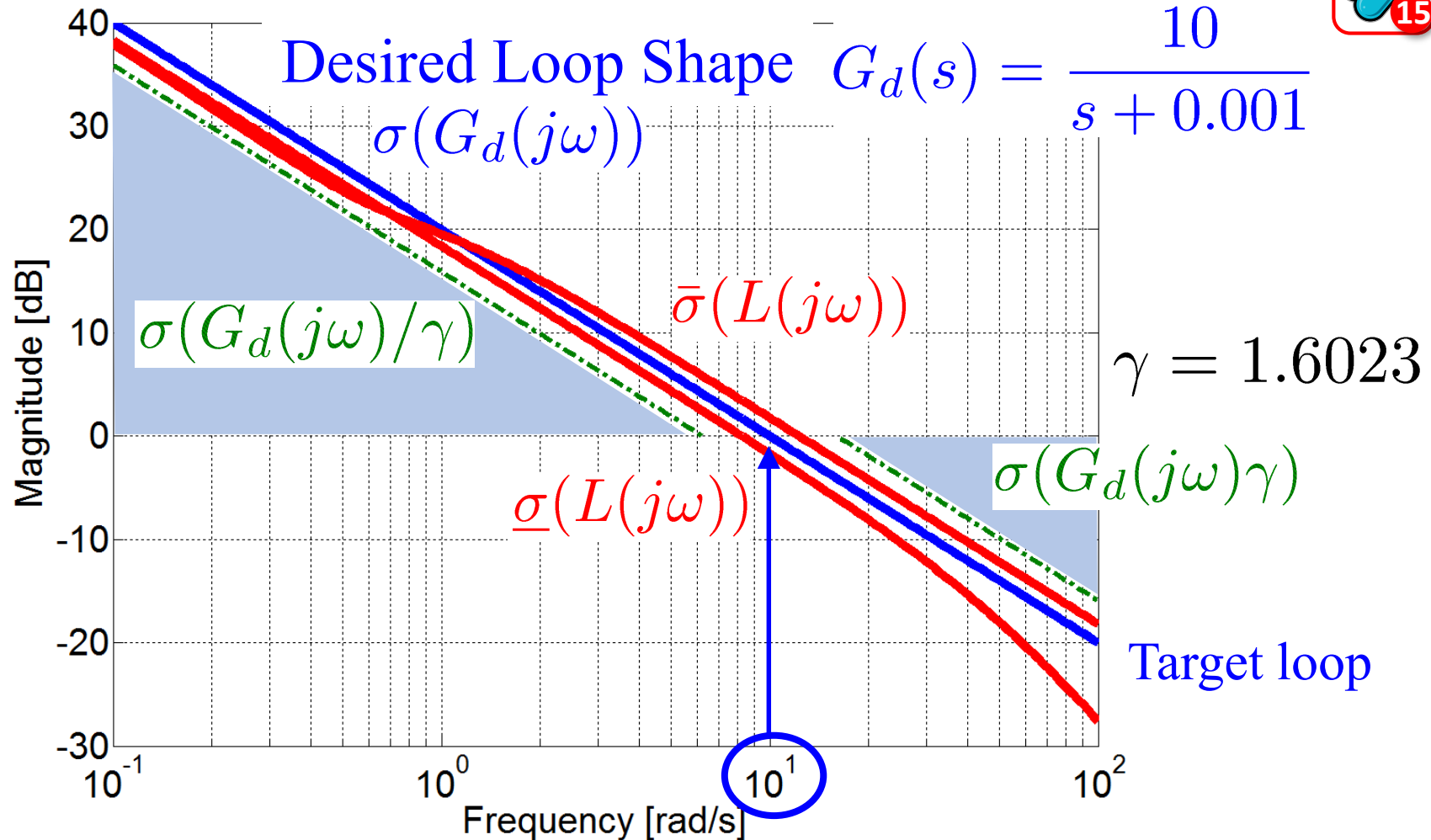
$$w(t) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$

A step function plot for $w(t)$. The vertical axis has tick marks at 0, 0.5, and 1. A blue line starts at 0 and jumps to 1 at $t=0$. A red line starts at 0 and jumps to 0.5 at $t=0$.



HIMAT: Specifications and Open-loop (Loop Shaping Synthesis)

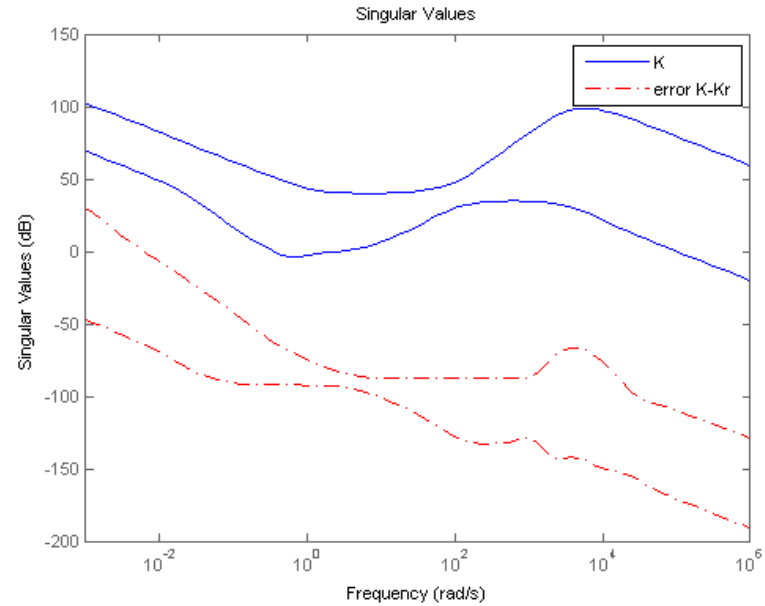
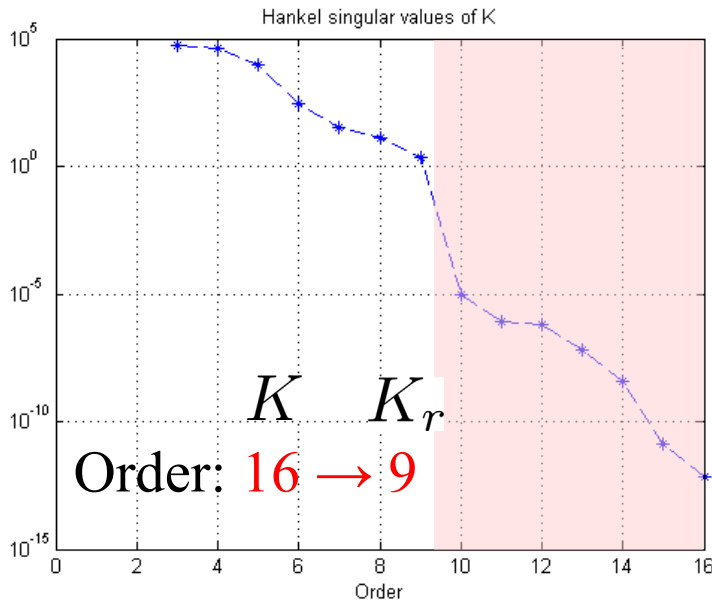
Approximately a bandwidth of 10 rad/s



MATLAB Command

```
Gd= tf( 10, [1 .001] );
[K,CL,GAM,INFO]=loopsyn(G,Gd);
sigma(Gd,'b',G*K,'r',Gd/GAM,'g:',Gd*GAM,'g:',{.1,100})
```

HIMAT: Controller Model Reduction (Loop Shaping Synthesis)



Step responses

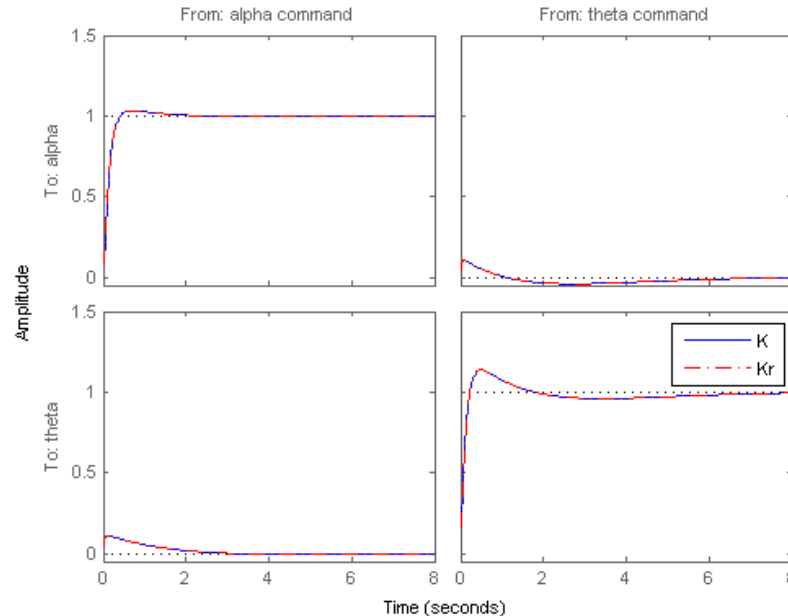
MATLAB Command

```

size(K)
hsv = hankelsv(K);
semilogy(hsv,'*--'), grid
title('Hankel singular values of K'),
xlabel('Order')

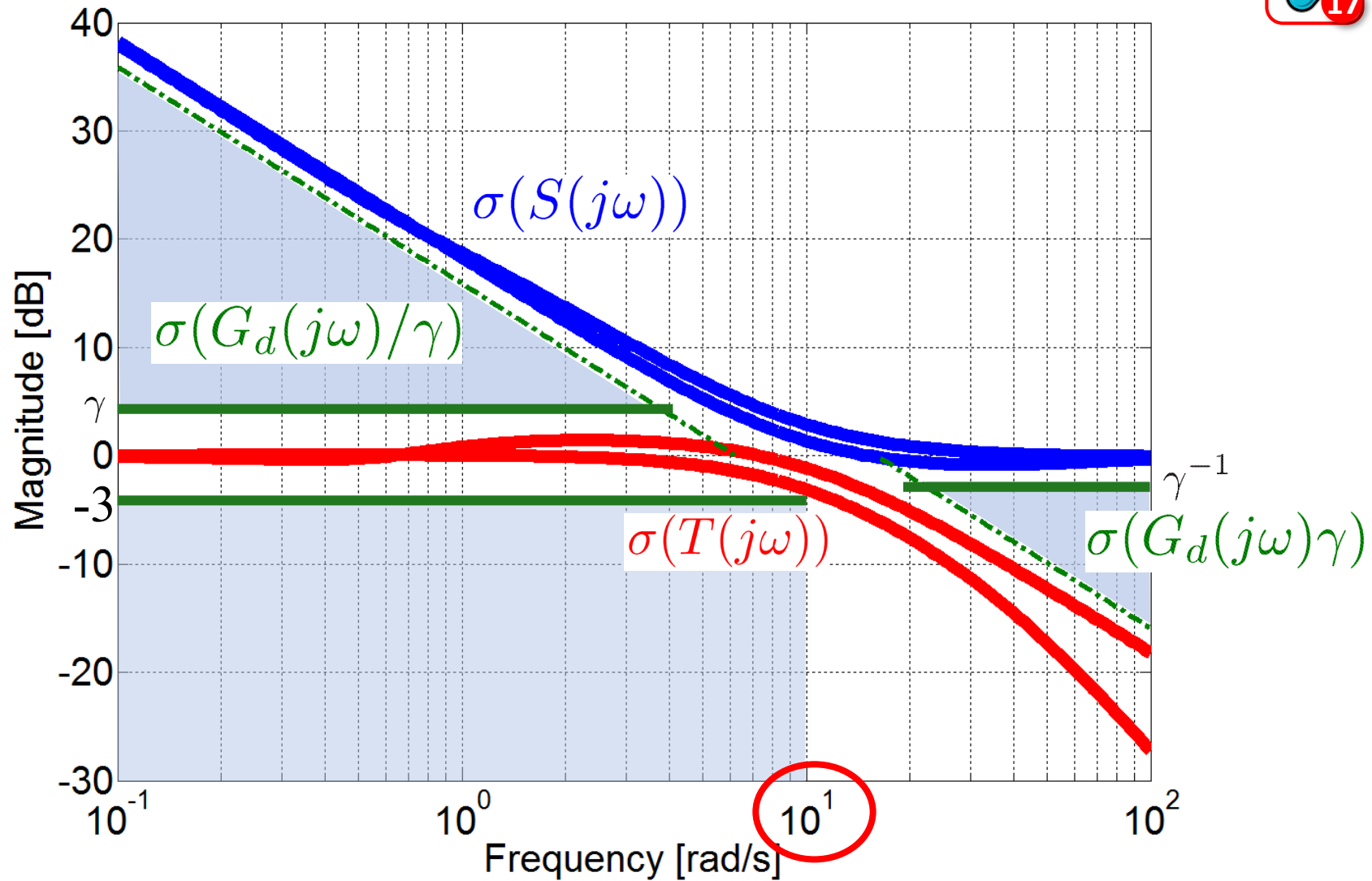
Kr = reduce(K,9); order(Kr)

sigma(K,'b',K-Kr,'r-.')
legend('K','error K-Kr')
    
```





HIMAT: Sensitivity (Loop Shaping Synthesis)

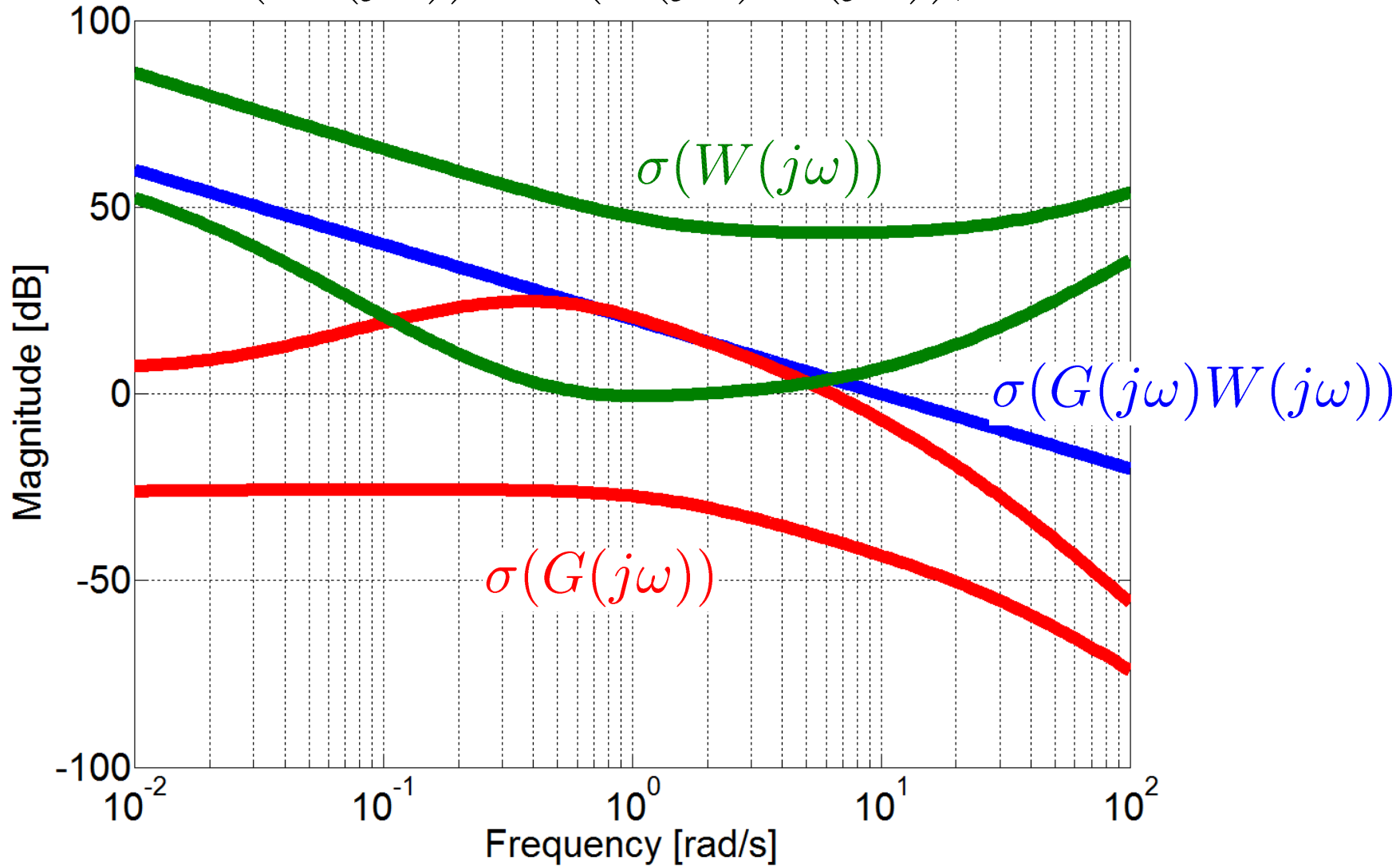


MATLAB Command

```
T = feedback(G*K,eye(2));  
S = eye(2)-T;  
sigma(inv(S),'m',T,'g',L,'r--',Gd,'b',Gd/GAM,'b:',Gd*GAM,'b:',{.1,100})
```


HIMAT: Designed Weight (Loop Shaping Synthesis)

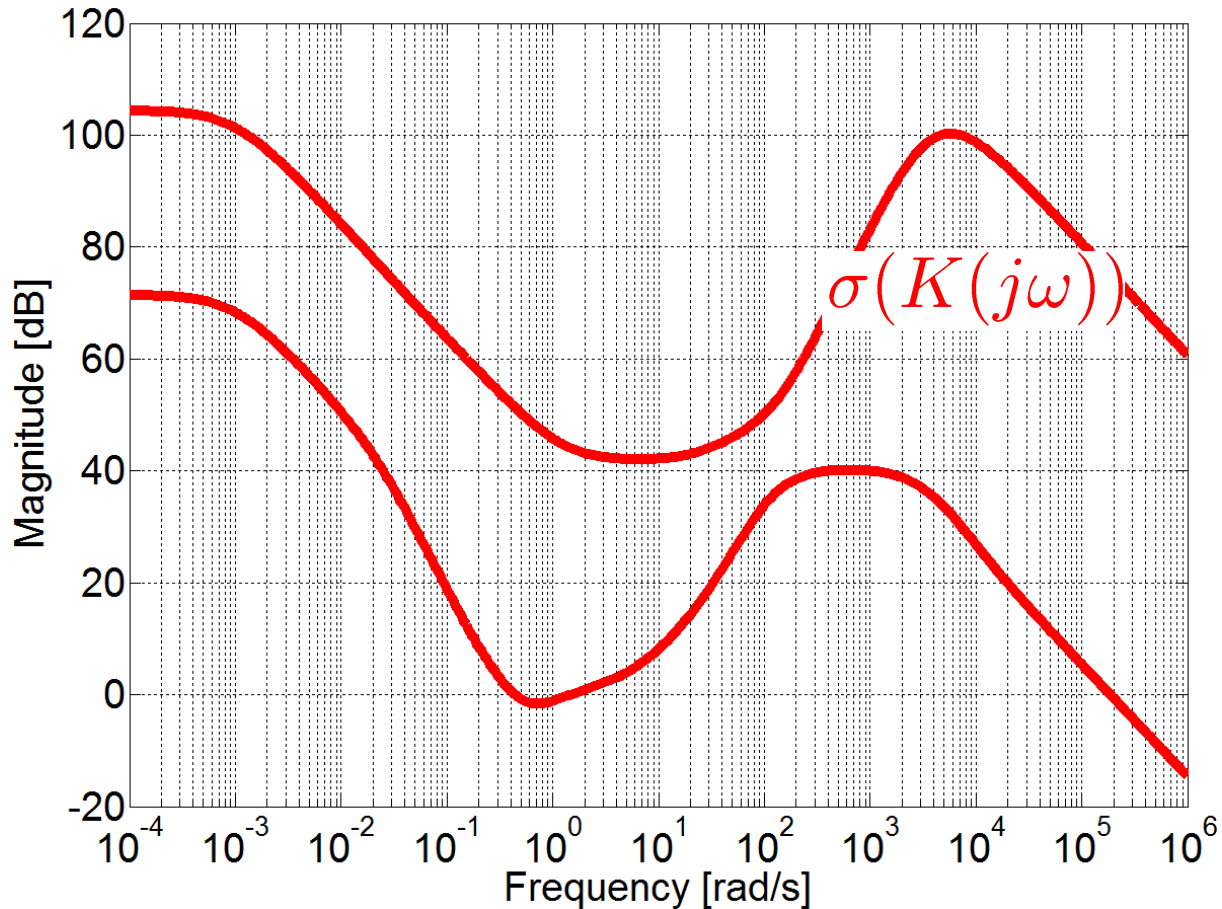
$$\sigma(G_d(j\omega)) \approx \sigma(G(j\omega)W(j\omega)), \forall \omega$$



MATLAB Command

```
sigma(INFO.Gs,'b',G,'r',INFO.W,'g',{.01,100});
```

HIMAT: Controller (Loop Shaping Synthesis)



MATLAB Command

```
sigma(K);
```

Order: 16

Numerical problems or inaccuracies may be caused too high order

➔ Difficult to implement

HIMAT: Step Response of closed loop (Loop Shaping Synthesis)



$$w(t) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix} \begin{matrix} 1 \\ 0.5 \\ 0 \end{matrix}$$

