Robust and Optimal Control, Spring 2015

Instructor: Prof. Masayuki Fujita (S5-303B)

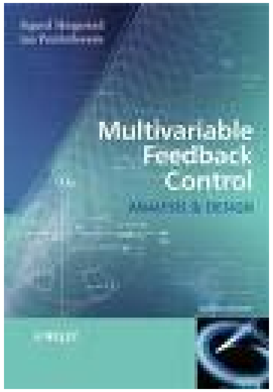
B. Robust Performance

B.1 μ-synthesis and DK-iteration [SP05, Sec. 8.12]

B.2 Spinning Satellite: μ-synthesis

Reference:

[SP05] S. Skogestad and I. Postlethwaite,
Multivariable Feedback Control; Analysis and Design,
Second Edition, Wiley, 2005.



μ-synthesis [SP05, p. 328]

$$\min_K \max_\omega \mu_{\hat{\Delta}}\left(N(j\omega)\right)$$

"At present there is no direct method to synthesize a μ-optimal controller"

$$\Rightarrow \min_K \max_\omega \min_{D_\omega \in \mathcal{D}} \bar{\sigma}\left(D_\omega N(j\omega) D_\omega^{-1}\right)$$

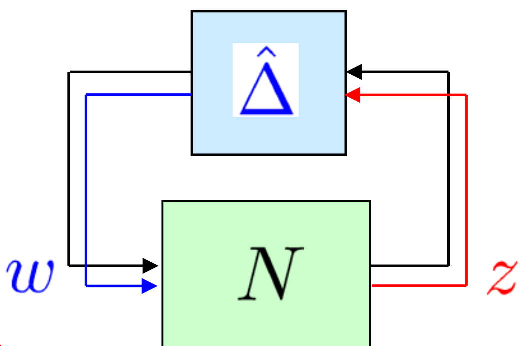
$$\Rightarrow \min_K \left(\min_{D \in \mathcal{D}} \| D N(j\omega) D^{-1} \|_\infty \right)$$

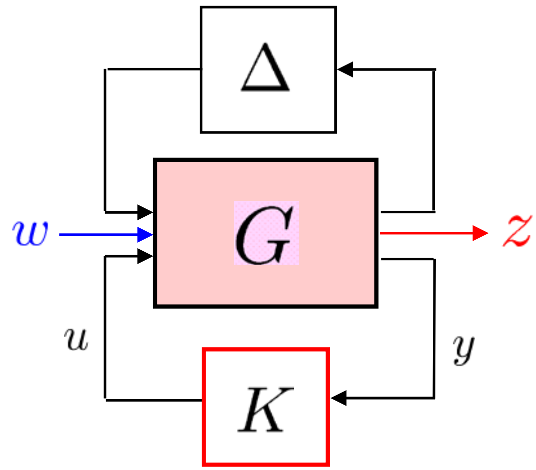
Scaled H_∞ Control

(D-Step)

μ-analysis

Structured Uncertainty

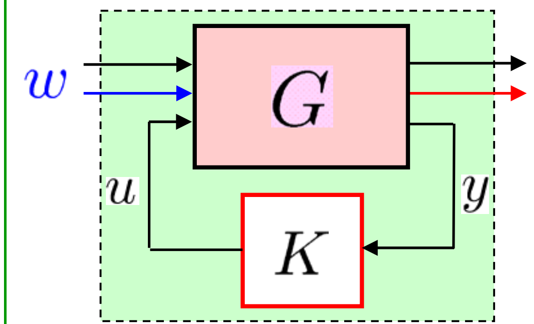




DK-iteration

(K-Step)

H_∞-synthesis



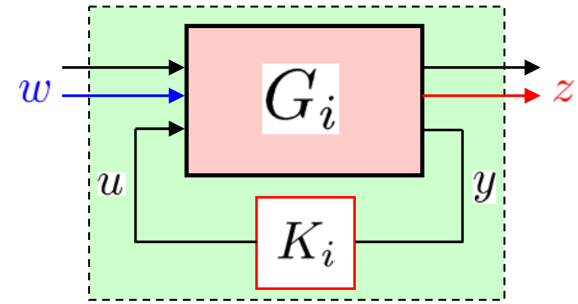
$z = F_l(G, K)w$

$N = F_l(G, K)$

The iterations may converge to a local optimum. However, practical experience suggests that the method work well in most cases.

DK-iteration [SP05, p. 328]

Step 1: Set $i = 1, G_i := G, D_i = I$.

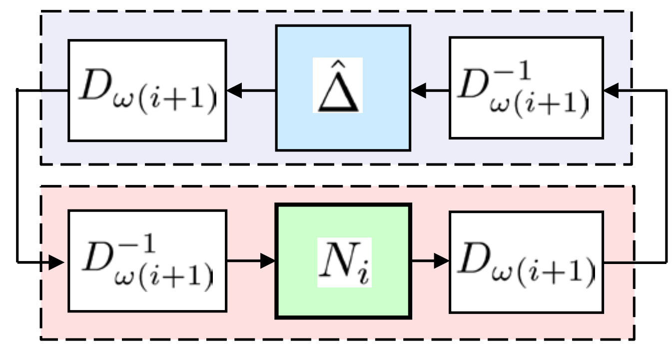


Step 2: (K-Step) H_∞-synthesis

Synthesize an $\mathcal{H}_\infty$ controller K_i which minimizes $\|N_i\|_\infty = \|F_l(G_i, K_i)\|_\infty$

$$z = F_l(G_i, K_i)w$$

Step 3: (D-Step) μ-analysis



Derive $D_{\omega(i+1)}(j\omega)$ which minimizes $\bar{\sigma}(D_{\omega(i+1)} N_i D^{-1}_{\omega(i+1)})$ at each frequency.

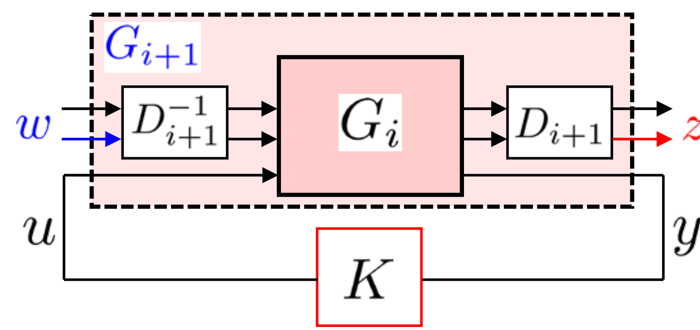
If $\max_\omega \bar{\sigma}(D_{\omega(i+1)} N_i D^{-1}_{\omega(i+1)}) < 1$, finish it.

If not, go to Step 4.

Step 4: Fit the magnitude of each element of $D_{\omega(i+1)}(j\omega)$-scales to a stable and minimum-phase transfer function $D_{i+1}(s)$.

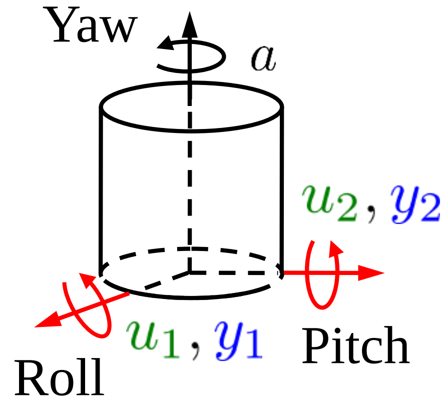
Step 5: Set the new generalized plant

$$G_{i+1} = \begin{bmatrix} D_{i+1} & 0 \\ 0 & I \end{bmatrix} G_i \begin{bmatrix} D^{-1}_{i+1} & 0 \\ 0 & I \end{bmatrix} .$$



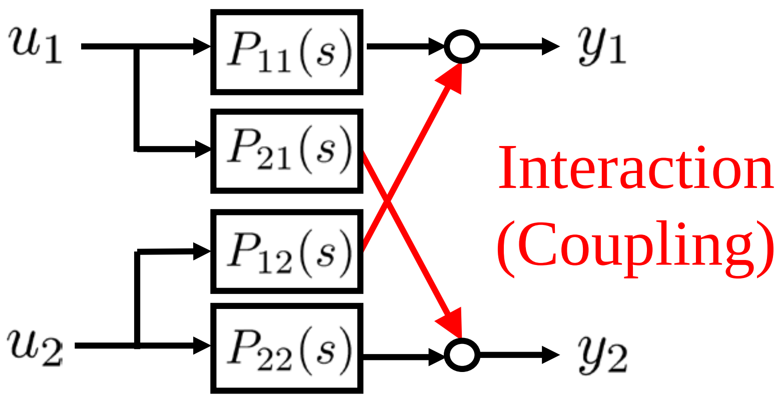
Then, set $i := i + 1$ and go to Step 2.

Spinning Satellite: Nominal Plant Model*



Transfer Function Matrix Form

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix} = \begin{bmatrix} \frac{s-100}{s^2+100} & \frac{10s+10}{s^2+100} \\ \frac{-10s-10}{s^2+100} & \frac{s-100}{s^2+100} \end{bmatrix}$$



MATLAB Command

```
N = { [1 -100],[10 10];[-10 -10],[1 -100] } ;
D = [1 0 100] ;
Pnom = tf(N,D) ;
Pnom = ss(Pnom,'min') ;
```

State Space Form (Matrix Representation)

$$P(s) = C(sI - A)^{-1}B + D$$

$$A = \begin{bmatrix} 0 & 10 \\ -10 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$P = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \qquad \begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned}$$

$$C = \begin{bmatrix} 1 & 10 \\ -10 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

No state feedback

MATLAB Command

```
sysA = [0 10; -10 0]; sysB = eye(2);
sysC = [1 10; -10 1]; sysD = zeros(2);
Pnom = ss(sysA, sysB, sysC, sysD);
```

Spinning Satellite: Characteristics of Nominal Plant Model*

Controllability $\mathrm{rank}\,[B\ \ AB] = 2$ ○

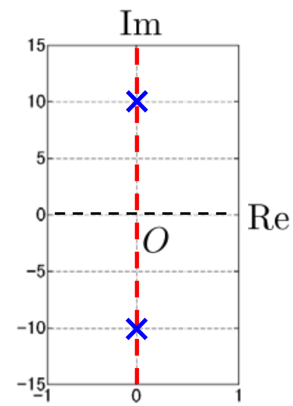
Observability $\mathrm{rank}\begin{bmatrix} C \\ CA \end{bmatrix} = 2$ ○

MATLAB Command
rank(ctrb(Pnom)) rank(obsv(Pnom))

Poles (Stability)

$p = \pm 10j$ (at Imaginary axis)

Vibrational System

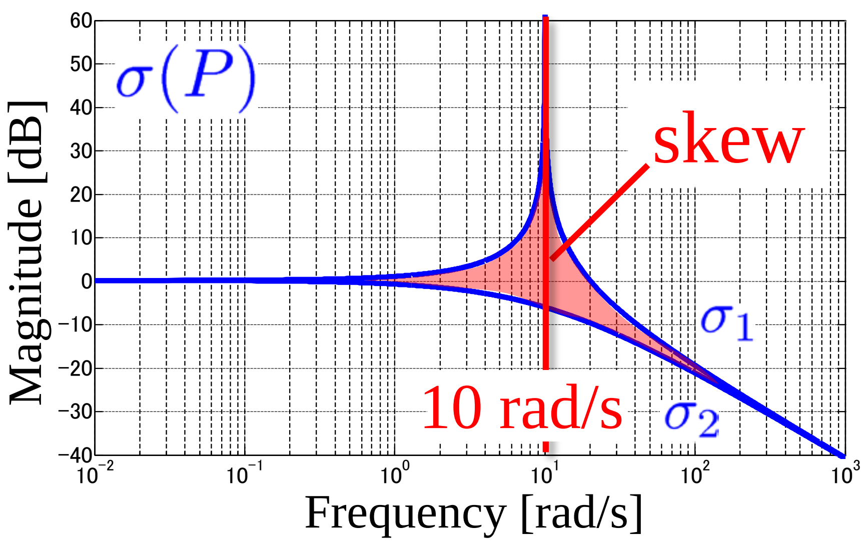


MATLAB Command
pole(Pnom) zero(Pnom)

MATLAB Command
figure pzmap(Pnom)

Multivariable Zeros None

Frequency Response



MATLAB Command
sigma(Pnom)

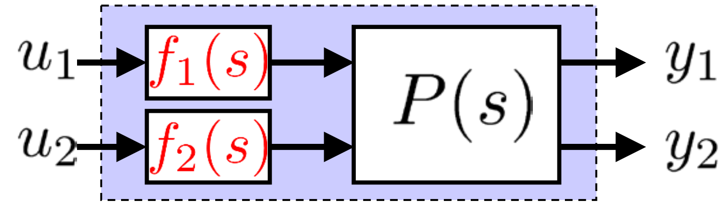
Condition Number

$$\gamma(P) := \frac{\bar{\sigma}(P)}{\underline{\sigma}(P)}$$ [SP05, p. 82]

Operating Frequency Range

Spinning Satellite: Plant Model with Input Uncertainty*

Uncertain Plant Model (Real System)



$$\tilde{P}(s) = P(s) \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix}$$

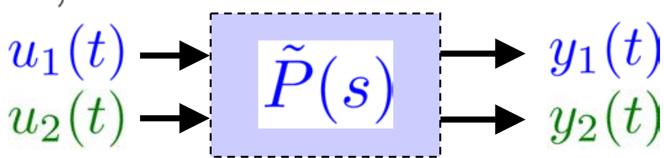
$$f_i(s) = k_i \frac{-\frac{\theta_i}{2} s + 1}{\frac{\theta_i}{2} s + 1}, \quad i = 1, 2$$

$(GM \geq 2\text{dB})$

Gain Margin: $0.8 \leq k_i \leq 1.2 \;\; (\pm 20\%)$

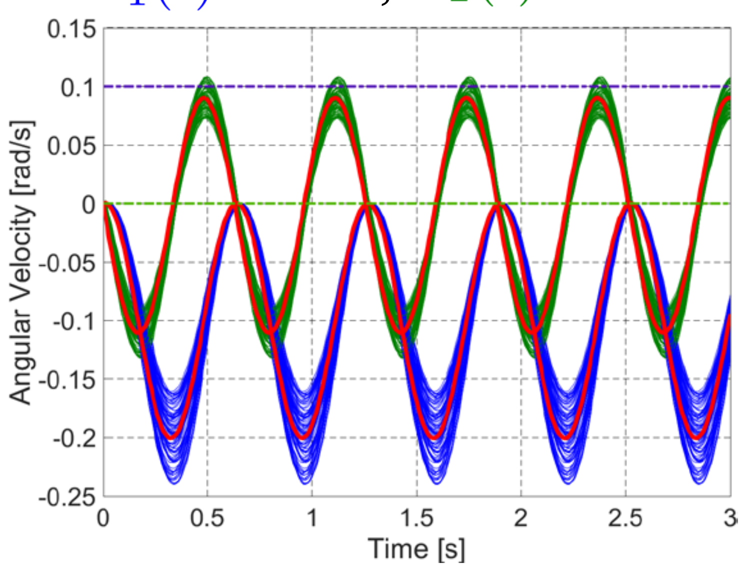
Delay Margin: $0 \leq \theta_i \leq 0.02$

Time Responses

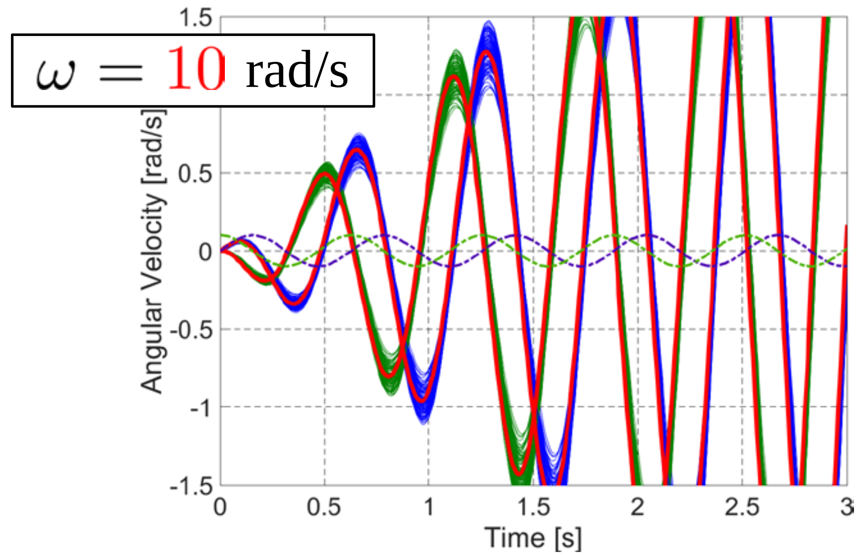


Reference
Nominal Model
Uncertain Model

$u_1(t) = 0.1$, $u_2(t) = 0$



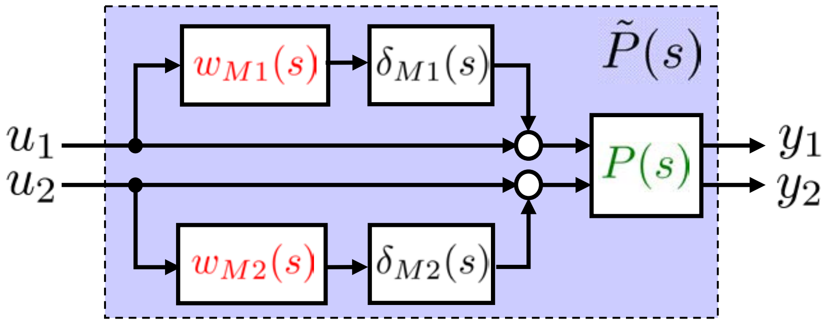
$u_1(t) = 0.1\sin(\omega t)$, $u_2(t) = 0.1\cos(\omega t)$



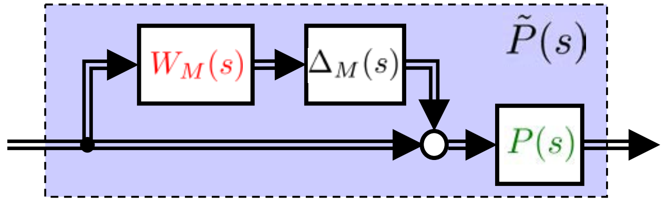
Spinning Satellite: Multiplicative Uncertainty*

Multiplicative (Input) Uncertainty [SP05, p. 296]

$$\Pi_0' = \{\tilde{P}(s) \mid \tilde{P}(s) = P(s)(I + \Delta_M(s) W_M(s)),\ \|\Delta\|_\infty \le 1\}$$



$$\|\delta_{M1}\|_\infty \le 1, \|\delta_{M2}\|_\infty \le 1$$



$$\Delta_M = \begin{bmatrix} \delta_{M1} & 0 \\ 0 & \delta_{M2} \end{bmatrix}, \|\Delta_M\|_\infty \le 1$$

Structured Uncertainty

Uncertainty Weight: $W_M(s) = \begin{bmatrix} w_{M1}(s) & 0 \\ 0 & w_{M2}(s) \end{bmatrix}$

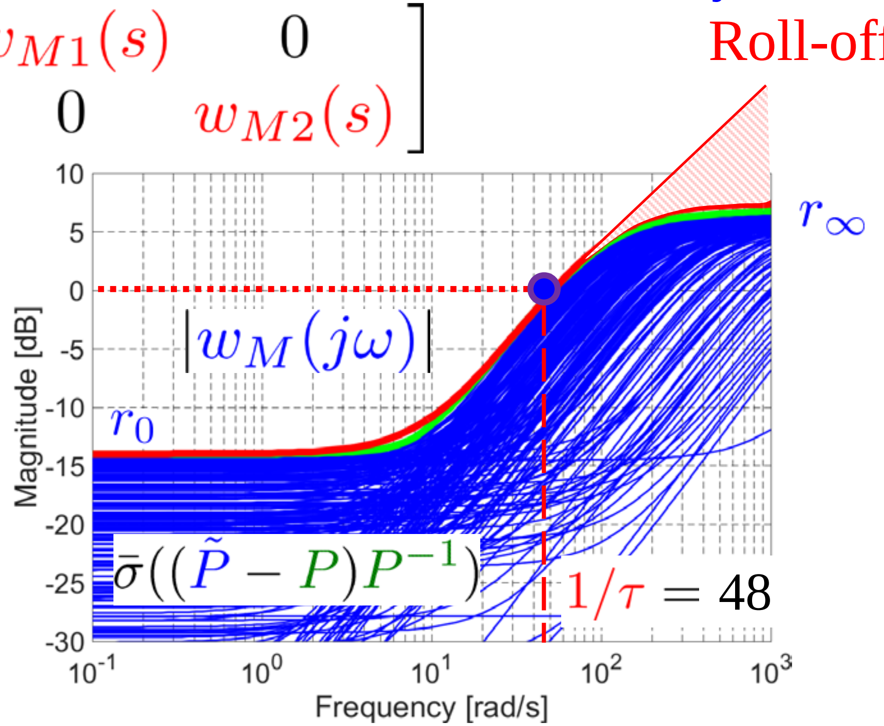
$$w_{Mi}(s) = \frac{\tau s + r_0}{\frac{\tau}{r_\infty} s + 1} = \frac{0.021s + 0.2}{0.0091s + 1}$$

$i = 1, 2$

$$\left(\begin{array}{l} \tau = 0.021,\ 1/\tau = 48 \text{ rad/s} \\ r_0 = 0.2,\ r_\infty = 2.3 \end{array} \right)$$

Update: $r_\infty = 2.3 \rightarrow 100$

$$w'_{Mi}(s) = \frac{0.021s + 0.2}{2.1 \times 10^{-4} s + 1}$$

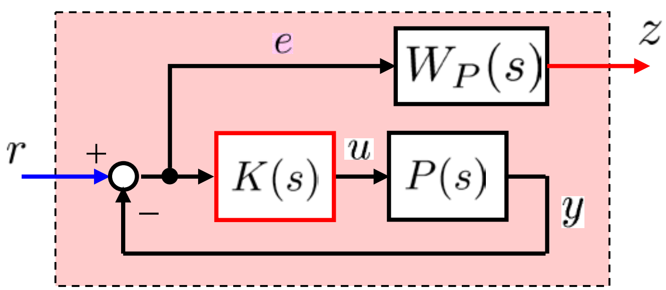


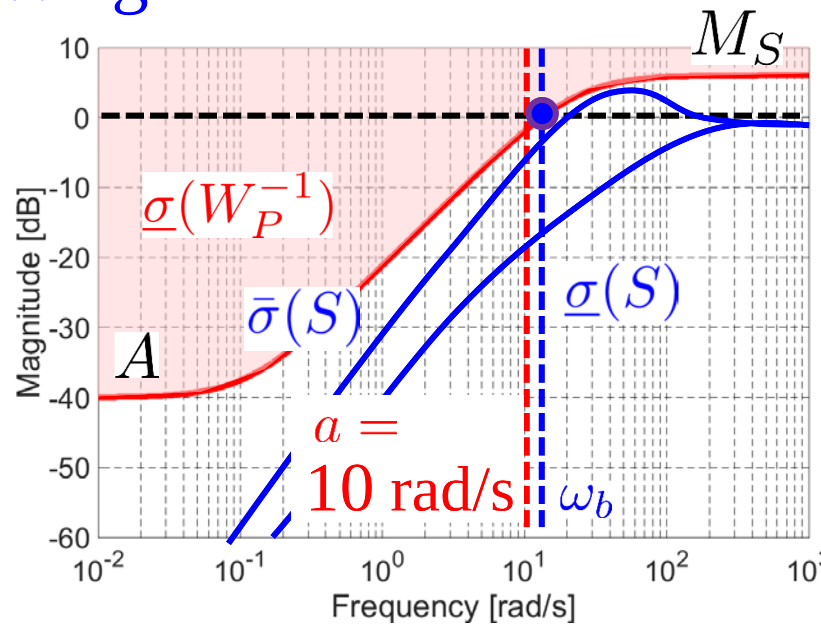
Spinning Satellite: Performance Weight*

$$W_P(s) = w_p(s) I_2,$$

$$w_p(s) = \frac{\frac{1}{M_S}s + \omega_b}{s + \omega_b A} = \frac{0.5s + 11.5}{s + 0.115}$$

$$\{\omega_b = 11.5,\, M_s = 2,\, A = 0.01\}$$





Update

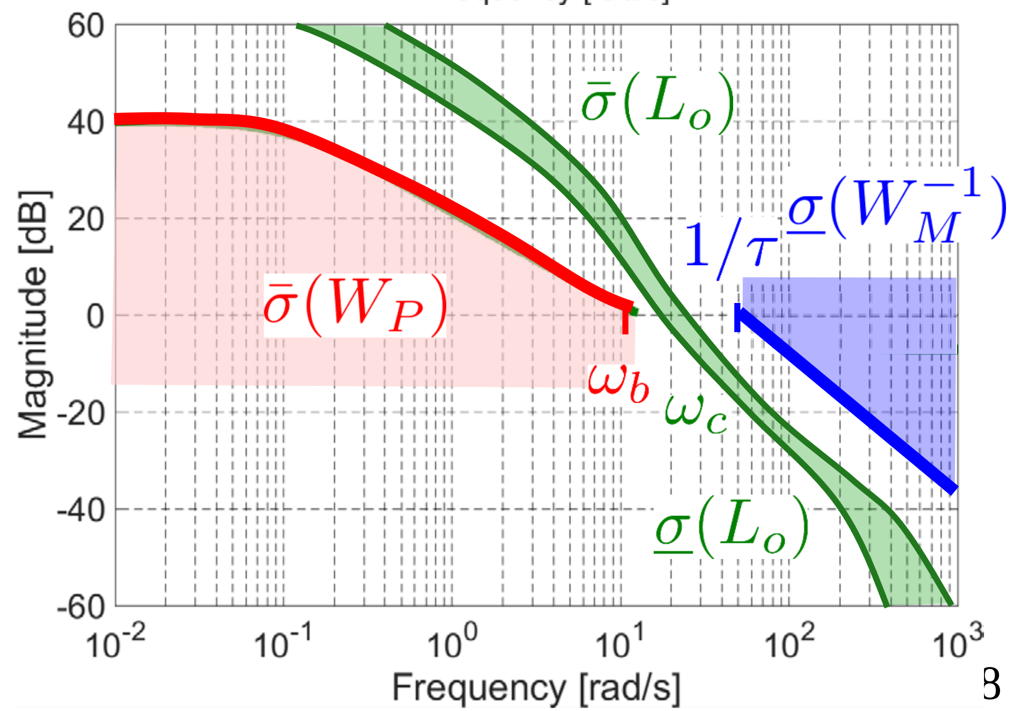
$$\omega_b \to 1/\tau$$

$\omega_b = 11.5 \to 11.5$

$M_S = 2 \to 8$ (Trade-off)

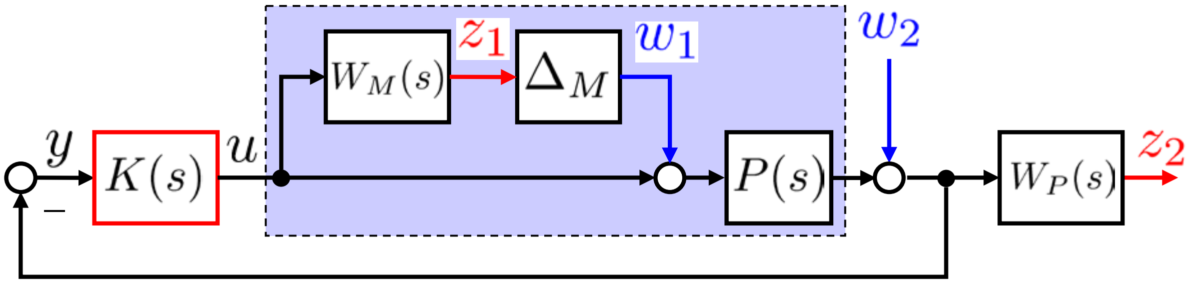
$A = 0.01 > 0$ 

$$w_p'(s) = \frac{0.125s + 11.5}{s + 0.115}$$

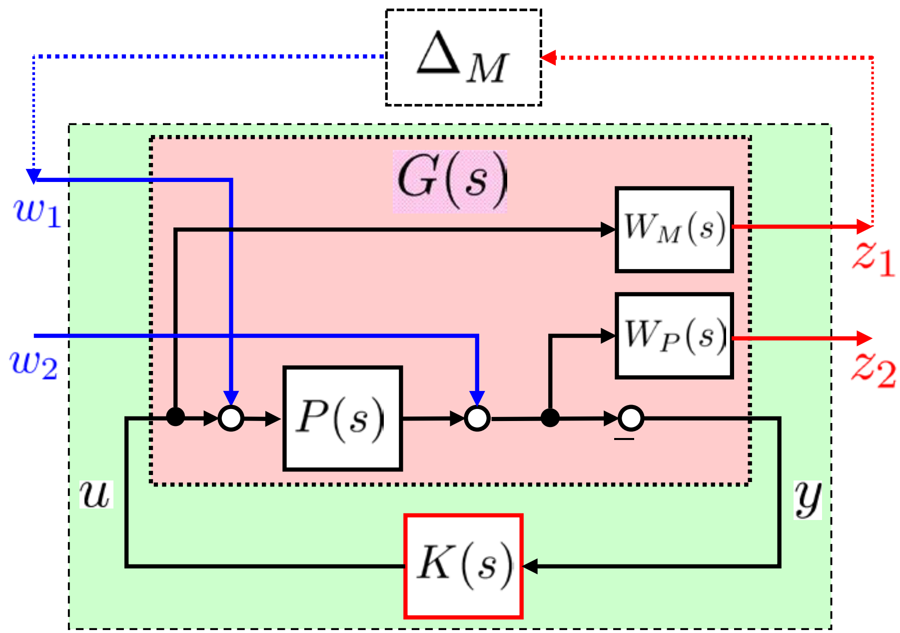


Spinning Satellite: Generalized Plant*

Input Uncertainty



Generalized Plant

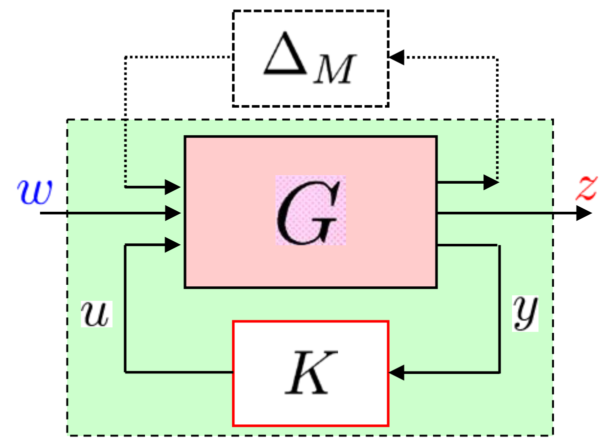


$$z = F_l(G, K)w$$

$$F_l(G, K) = \begin{bmatrix} -W_M T_i & -W_M K S_o \\ W_P S_o P & W_P S_o \end{bmatrix}$$

MATLAB Command

```
%Generalized Plant%
systemnames = 'Pnom WP WM';
inputvar = '[w1(2); w2(2); u(2)]';
outputvar = '[WM;WP;-w2-Pnom]';
input_to_Pnom = '[u+w1]';
input_to_WP = '[w2+Pnom]';
input_to_WM = '[u]';
G = sysic;
%with Structured Uncertainty%
unc1 = ultidyn('unc1',[1 1]);
unc2 = ultidyn('unc2',[1 1]);
unc = [unc1 0; 0 unc2];
Gunc = lft(unc,G);
```



$$\Delta_M = \begin{bmatrix} \delta_{M1} & 0 \\ 0 & \delta_{M2} \end{bmatrix}$$

Structured Uncertainty

MATLAB Command dksyn

[k, cl, bnd, info] = dksyn(G, nmeas, ncont, option)

Input argument

G	Generalized Plant
nmeas	Number of measurement outputs
ncont	Number of control inputs

Output argument

k	Controller
cl	Closed-loop system which consists of G and K
bnd	Upper bound of μ
info	Information of Iteration

option

Note: use dksynOptions/dkitopt to create option

FrequencyVector	(Default: []) i.e. automatically the frequency range is chosen
InitialController	(Default: [])
AutoIter	(Default: 'on')
DisplayWhileAutoIter	(Default: 'off')
StartingIterationNumber	(Default: '1')
NumberOfAutoIteration	(Default: '10')
AutoIterSmartTerminateTol	(Default '0.005')
MixedMU	(Default: 'off')
AutoScalingOrder	(Default: '5')
AutoIterSmartTerminate	(Default: 'on')

Spinning Satellite: DK -iteration

dksyn

Frequency Range $\omega : 10^{-2} \sim 10^{3}$ rad/s   Data: 200 points

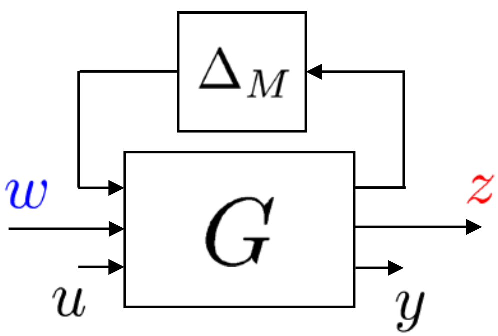
MATLAB Command

```
omega = logspace(-2,3,200);
options = dksynOptions('FrequencyVector',omega,'AutoIter','off');
ny = 2; nu = 2;
[Kdk,CLdk,gdk,dkinfo] = dksyn(Gunc,ny,nu,options);
gdk
Fdk = loopsens(Pnom,Kdk);
N = lft(G,Kdk);
```

Iteration 1 Step 1: Initialization

MATLAB Command Window

```
Iteration Number:  1
--------------------

Information about the Interconnection Structure IC:
6 個の出力 s、6 個の入力 s、6 個の状態 s をもつ状態空間モデル。
```



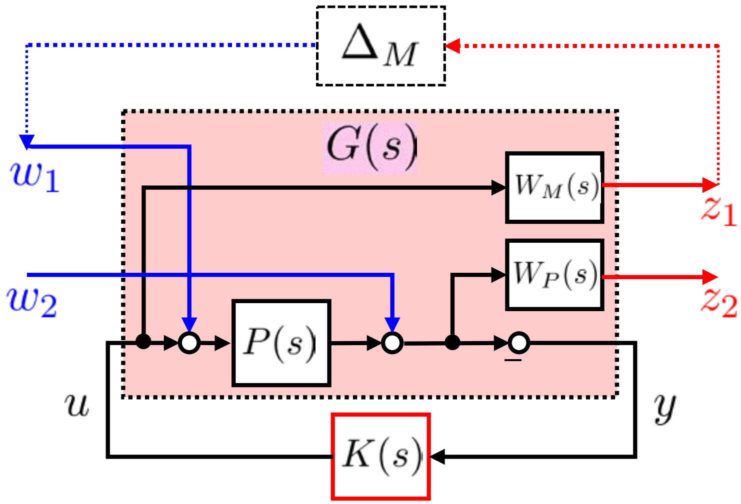
Generalized Plant : Order 6

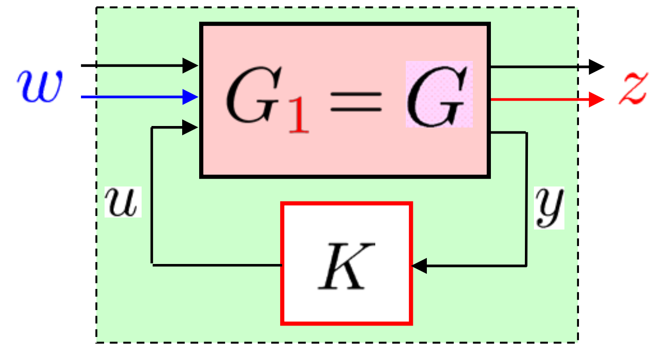
($P(s)$: Order 2, $W_M(s)$: Order 2, $W_P(s)$: Order 2)

➡ $G_1 = G$, $D_1 = I_6$

Spinning Satellite: DK-iteration

Iteration 1 Step 2: (K-Step) H_∞-synthesis

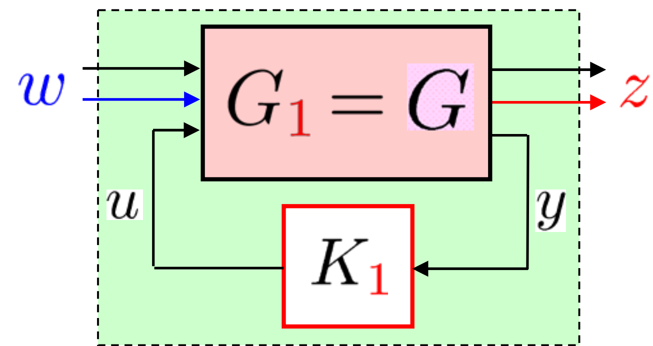




$$z = F_l(G, K)w$$

$$N = F_l(G, K) = \begin{bmatrix} -W_M T_I & -W_M K S_o \\ W_P S_o P & W_P S_o \end{bmatrix}$$

$$\min_K \|F_l(G, K)\|_\infty$$



$$z = F_l(G, K_1)w$$

H_∞ Optimal Controller

Find K_1 s.t. $\min_K \|F_l(G, K)\|_\infty < \gamma$

hinfsyn

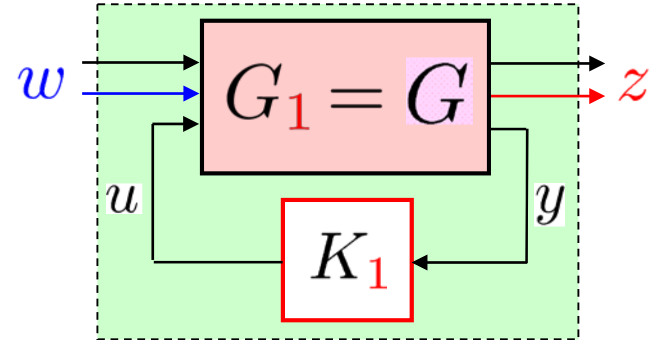
$$K_1 \approx K_\infty$$

Spinning Satellite: DK-iteration

Iteration 1 Step 2: (K-Step) H_∞-synthesis

hinfsyn

Find K_1 s.t. $\min_K \|F_l(G, K)\|_\infty < \gamma$



$z = F_l(G, K_1)w$

Initial Bounds on γ

$$\gamma_{min} = 0.1250 \leq \gamma \leq 2.2364 = \gamma_{max}$$

γ-iteration (Bisection method)

Approximately sub-optimal

$\gamma = 1.7086$ [Tolgam=1] $(\gamma_{opt} = 1.3214)$

MATLAB Command Window

Resetting value of Gamma min based on D_11, D_12, D_21 terms

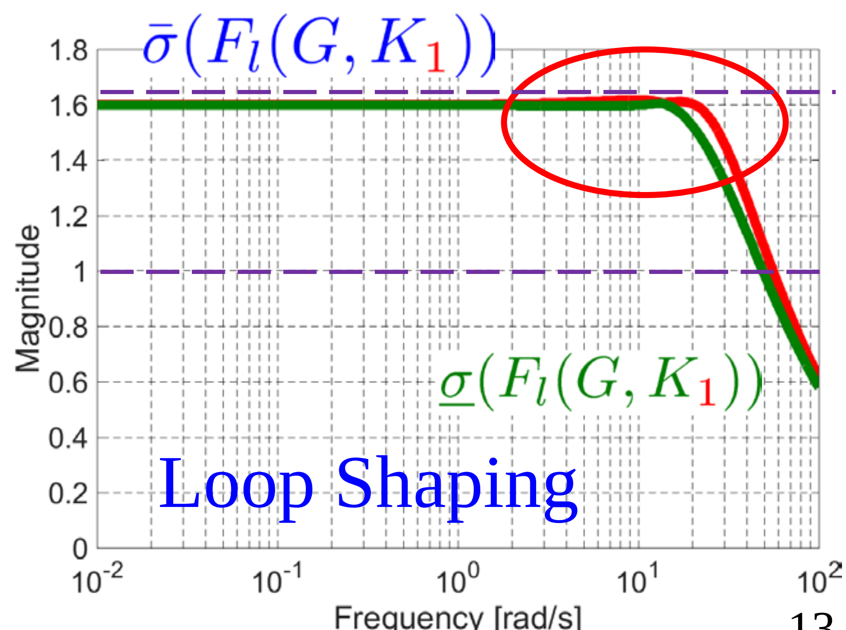
Test bounds: 0.1250 < gamma <= 2.2364

gamma	hamx_eig	xinf_eig	hamy_eig	yinf_eig	nrho_xy	p/f
2.236	8.6e+00	2.4e-04	1.2e-01	-2.0e-12	0.1855	p
1.181	7.9e+00	2.6e-04	1.2e-01	-2.0e-12	1.9463#	f
1.709	8.4e+00	2.5e-04	1.1e-01	0.0e+00	0.3858	p

Gamma value achieved: 1.7086

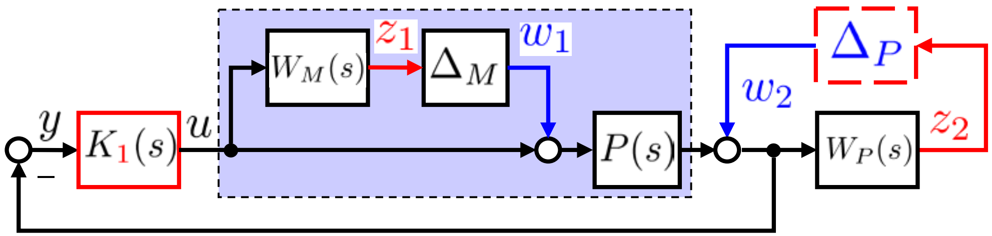
Singular Value plot of closed-loop system in GRAPHICS window
Make sure that chosen Frequency range is appropriate
Do you want to modify OMEGA_DK? (y/n): n

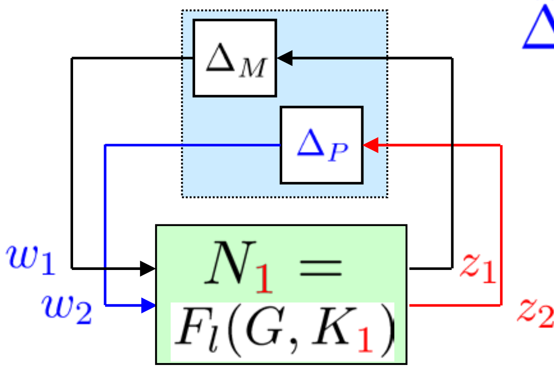
Closed-loop Transfer Function



Spinning Satellite: DK-iteration

Iteration 1 Step 3: (D-Step) μ-analysis mussv





$$\hat{\Delta} = \begin{bmatrix} \Delta_M & 0 \\ 0 & \Delta_P \end{bmatrix}$$

Structured Uncertainty

MATLAB Command Window

Iteration Summary

Iteration #	1
Controller Order	6
Total D-Scale Order	0
Gamma Acheived	1.709
Peak mu-Value	1.618 > 1

Another D-K iteration? (y/n): y

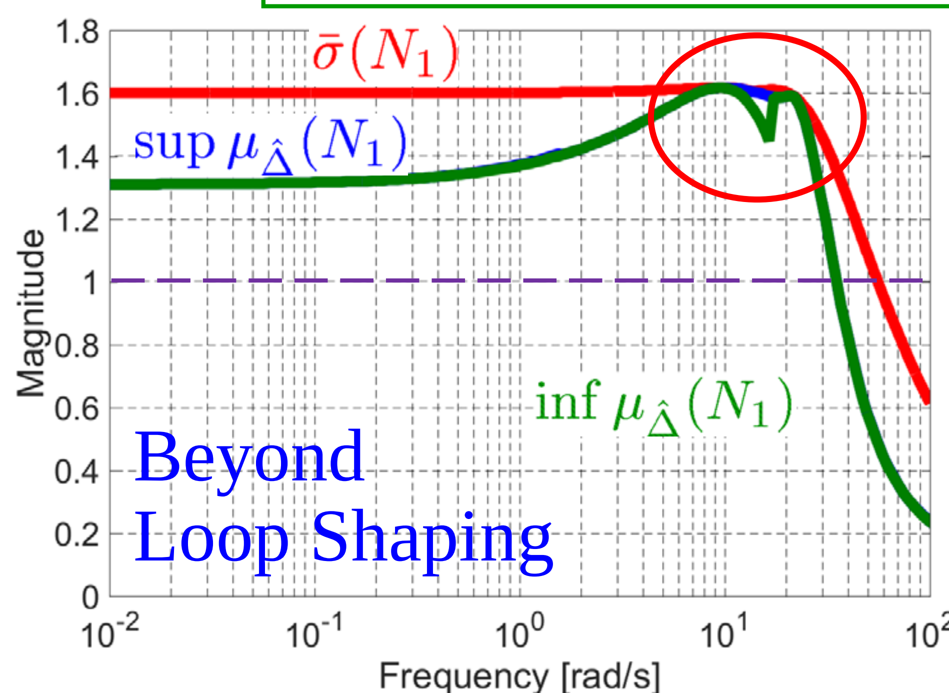
$$\mu_{\hat{\Delta}}(N_1) < \gamma, \ \gamma = 1.618 > 1$$

$$(\bar{\sigma}(\hat{\Delta}) = 0.62 \rightarrow \det(I - N_1\hat{\Delta}) = 0)$$

RP ×

Another DK-iteration

(Next MU iteration number: 2)



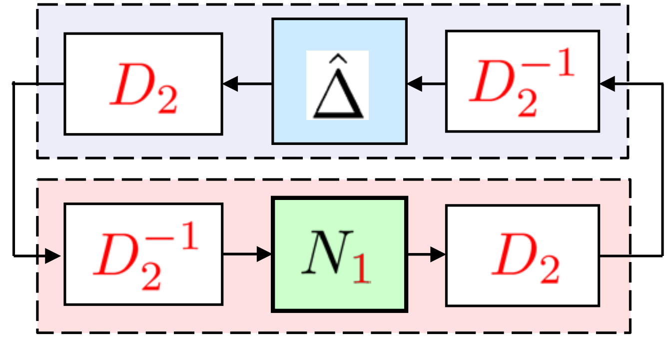
Spinning Satellite: DK-iteration

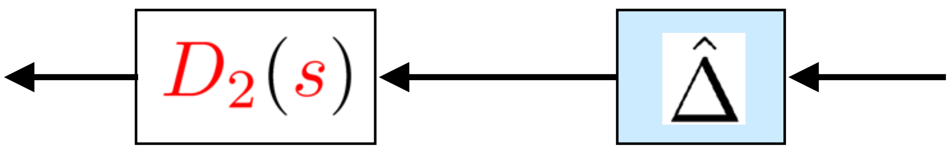
Iteration 1 Step 4: Scale-fitting

Upper Bound

$$\mu_{\hat{\Delta}}(N) \le \min_{D \in \mathcal{D}} \bar{\sigma}(DND^{-1})$$

$$\Rightarrow \min_{D_2 \in \mathcal{D}} \|D_2 F_l(G, K_1) D_2^{-1}\|_\infty$$





$$\left[\begin{array}{cc:c} d_{2,1}(s) & 0 & 0 \\ 0 & d_{2,2}(s) & 0 \\ \hdashline 0 & 0 & I_2 \end{array}\right] \left[\begin{array}{cc:c} \Delta_{M1} & 0 & 0 \\ 0 & \Delta_{M2} & 0 \\ \hdashline 0 & 0 & \Delta_P \end{array}\right]$$

Block Diagonal

$$\begin{Bmatrix} \Delta_M & : \text{Diagonal} \\ \Delta_P & : \text{Full-block} \end{Bmatrix}$$

MATLAB Command Window

Enter Choice (return for list):

Choices:

nd	Move to Next D-scaling
nb	Move to Next D-Block
i	Increment Fit Order
d	Decrement Fit Order
apf	Auto-PreFit
mx 3	Change Max-Order to 3
at 1.01	Change Auto-Prefit Tol to 1.01
0	Fit with zeroth order
2	Fit with second order
n	Fit with n'th order
e	Exit with Current Fittings
s	See Status

Spinning Satellite: DK-iteration

Iteration 1 Step 4: Scale-fitting

Data: $d_{\omega 2,i}(\omega)$ 200 points $i = 1, 2$

Fitting ("apf")

Transfer functions:

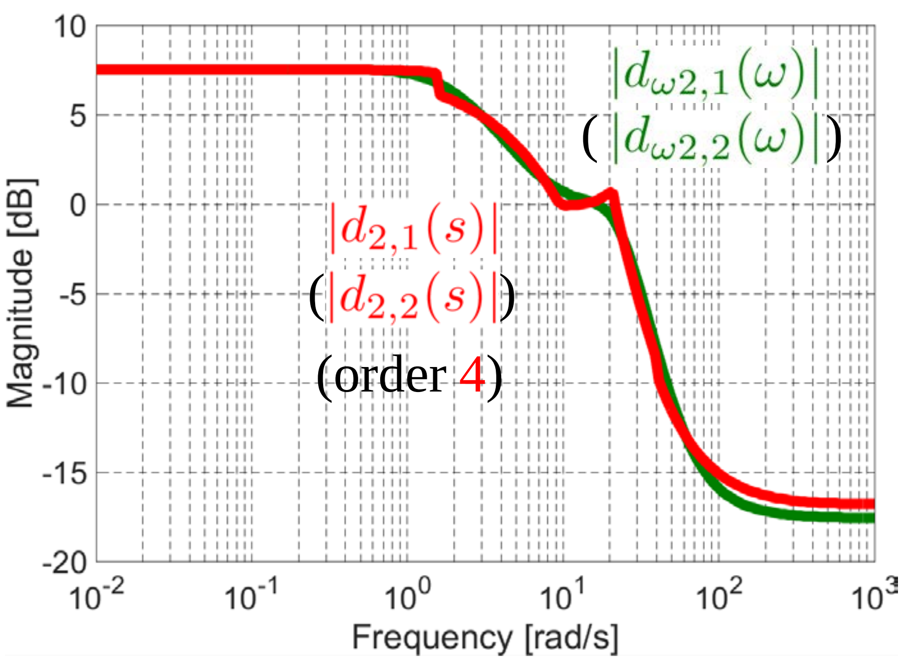
$$D_2(s) = \left[\begin{array}{cc|c} d_{2,1}(s) & 0 & 0 \\ 0 & d_{2,2}(s) & 0 \\ \hline 0 & 0 & I_2 \end{array}\right]$$

$$d_{2,1}(s) = d_{2,2}(s) = \frac{0.13207(s+7.875)(s+0.96)(s^2+98.71s+3403)}{(s+2.514)(s+1.034)(s^2+26.14s+548.7)}$$

(order 4)

MATLAB Command Window

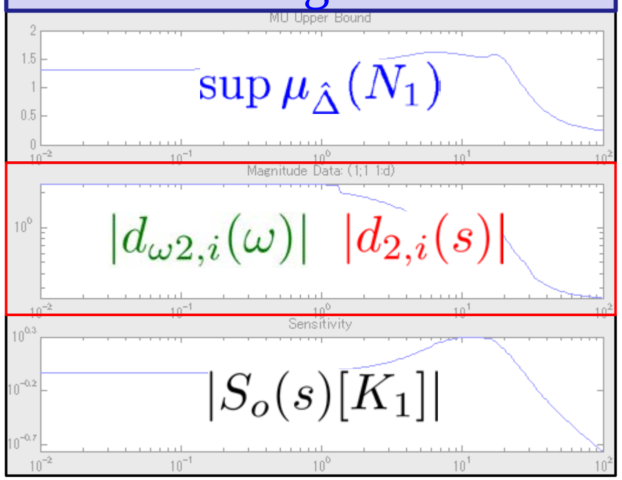
```
Enter Choice (return for list): apf
Starting Auto-PreFit...
Auto Fit in Progress
  Block 1, MaxOrder=5, Order = 0
 1  2  3  4
  Block 2, MaxOrder=5, Order = 0
 1  2  3  4
  Block 3, MaxOrder=5, Order = 0
Done
Enter Choice (return for list):  e
```



$D_2(s)$

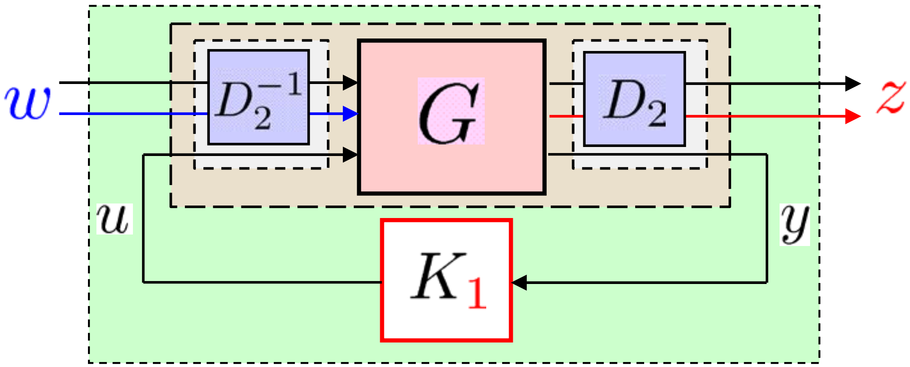
(order 8)

MATLAB Figure Window

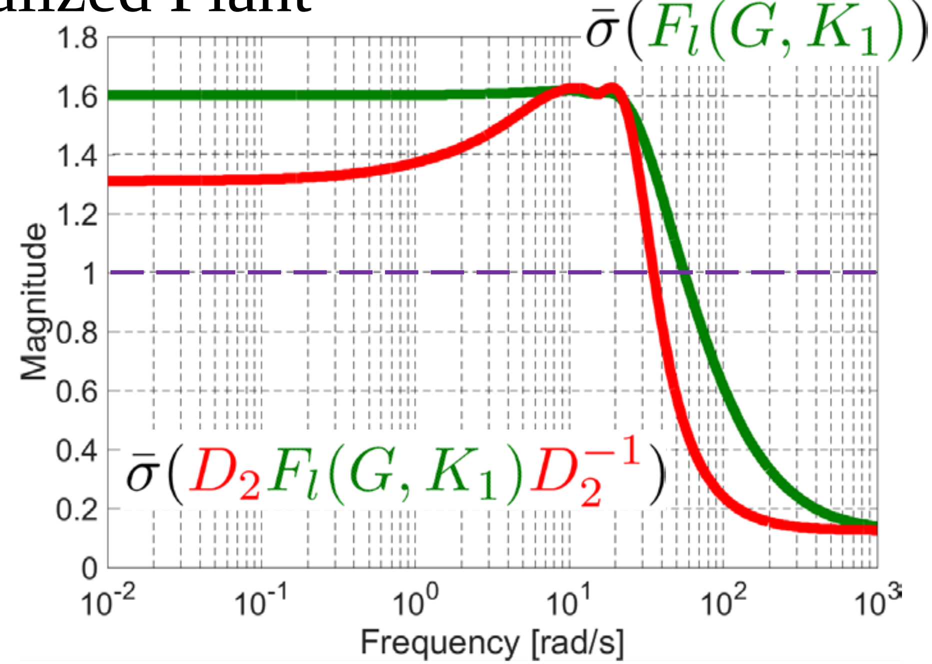


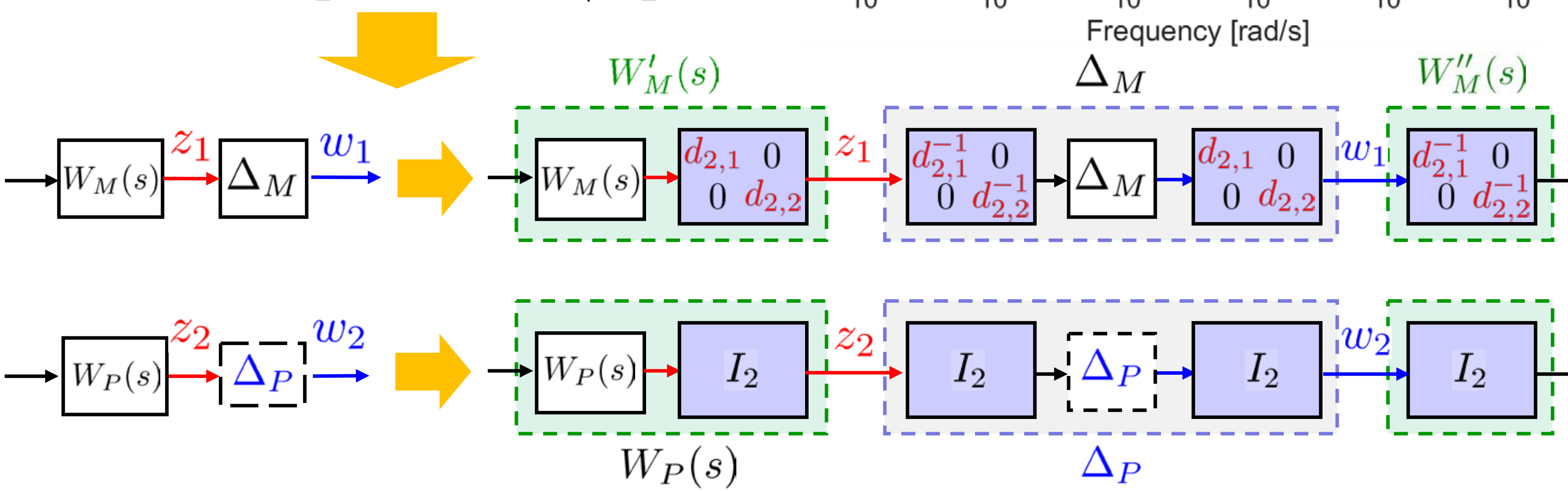
Spinning Satellite: DK-iteration

Iteration 1 Step 5: Scaled Generalized Plant



$$D_2(s) = \left[\begin{array}{cc|c} d_{2,1}(s) & 0 & 0 \\ 0 & d_{2,2}(s) & 0 \\ \hline 0 & 0 & I_2 \end{array}\right]$$

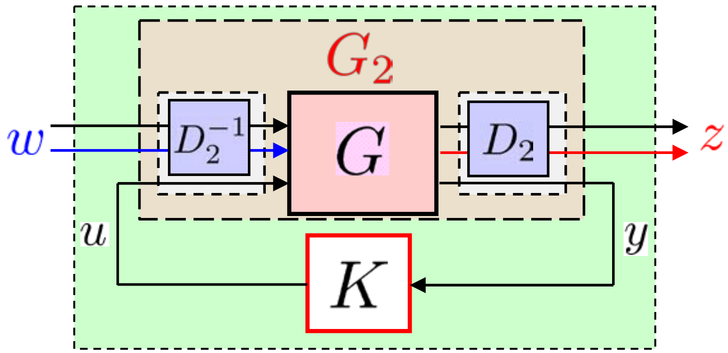


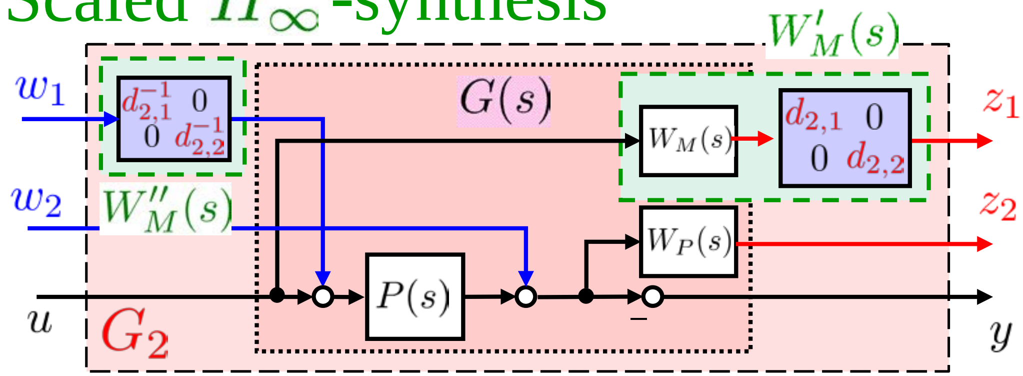


DK-iteration implies auto tuning of weights: W_M and W_P

Spinning Satellite: DK-iteration

Iteration 2 Step 2: (K-Step) Scaled H_∞-synthesis





$$N_2 = F_l(G_2, K_2)$$

$$\min_K \left\| D_2^{-1} F_l(G_2, K) D_2 \right\|_\infty$$

$$\Rightarrow \min_K \left\| F_l(G_2, K) \right\|_\infty$$

H_∞ Optimal Controller

Find K_2 s.t.

$$\min_K \left\| F_l(G_2, K) \right\|_\infty < \gamma$$

MATLAB Command Window

Iteration Number: 2

Information about the Interconnection Structure IC:
6 個の出力、6 個の入力、22 個の状態を持つ状態空間モデル。
Resetting value of Gamma min based on D_11, D_12, D_21 terms

Test bounds: 0.1250 < gamma <= 1.6506

gamma	hamx_eig	xinf_eig	hamy_eig	yinf_eig	nrho_xy	p/f
1.651	9.6e-01	-1.6e-17	1.1e-01	-3.7e-16	0.1088	p
0.888	9.6e-01	-8.5e-17	1.1e-01	-1.6e-16	3.1640#	f
1.269	9.6e-01	-5.2e-17	1.1e-01	-2.9e-16	0.2365	p
1.078	9.6e-01	-2.2e-16	1.2e-01	-3.1e-14	0.4607	p
0.983	9.6e-01	-1.7e-16	1.1e-01	-1.2e-16	0.8142	p
0.935	9.6e-01	-2.0e-16	1.1e-01	-2.3e-16	1.2975#	f
0.959	9.6e-01	-6.3e-17	1.1e-01	-2.9e-16	1.0013#	f

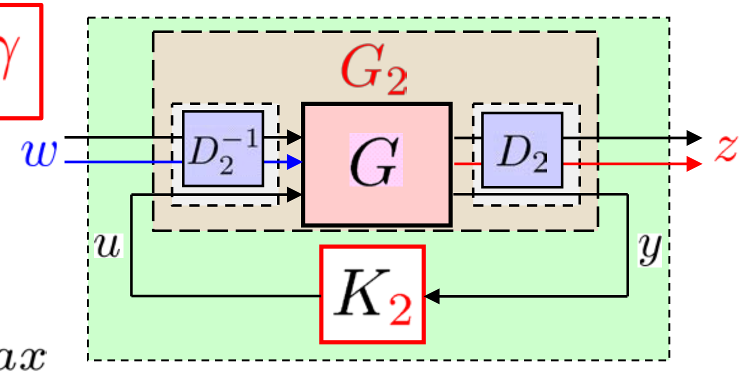
Gamma value achieved: 0.9831 < 1

Spinning Satellite: DK-iteration

Iteration 2 Step 2: (K-Step) Scaled H_∞-synthesis

Find K_2 s.t. $\min_K \|F_l(G_2, K)\|_\infty < \gamma$

D_2 : fixed



$N_2 = F_l(G_2, K_2)$

Initial Bounds on γ

$$\gamma_{min} = 0.1250 \le \gamma \le 1.6506 = \gamma_{max}$$

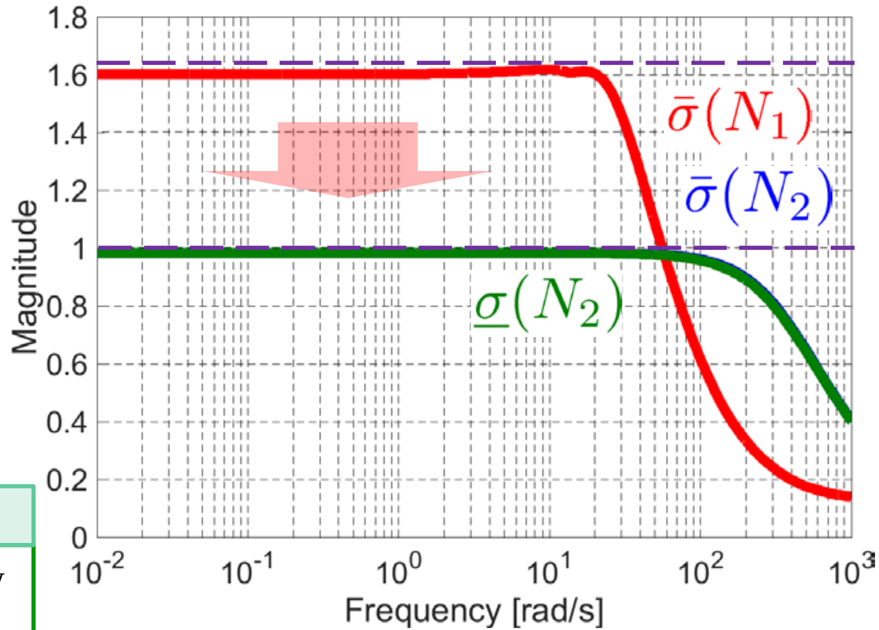
γ-iteration (Bisection method)

Approximately sub-optimal

$\gamma = 0.9831$ [Tolgam=0.05]

$(\gamma_{opt} = 0.9669)$

It is often desirable to use slightly suboptimal controller, which is 5% higher than the optimal value γ_{opt}

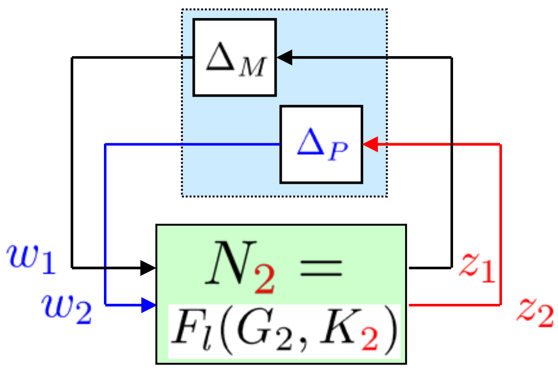


MATLAB Command Window

```
Singular Value plot of closed-loop system in GRAPHICS window
Make sure that chosen Frequency range is appropriate
Do you want to modify OMEGA_DK? (y/n):  n
```

Spinning Satellite: DK-iteration

Iteration 2 Step 3: (D-Step) μ-analysis



$$\hat{\Delta} = \begin{bmatrix} \Delta_M & 0 \\ 0 & \Delta_P \end{bmatrix}$$

Structured Uncertainty

K_2 fixed

MATLAB Command Window

Iteration Summary

Iteration #	1	2
Controller Order	6	22
Total D-Scale Order	0	16
Gamma Acheived	1.709	0.983
Peak mu-Value	1.618	0.987 < 1

Another D-K iteration? (y/n): n

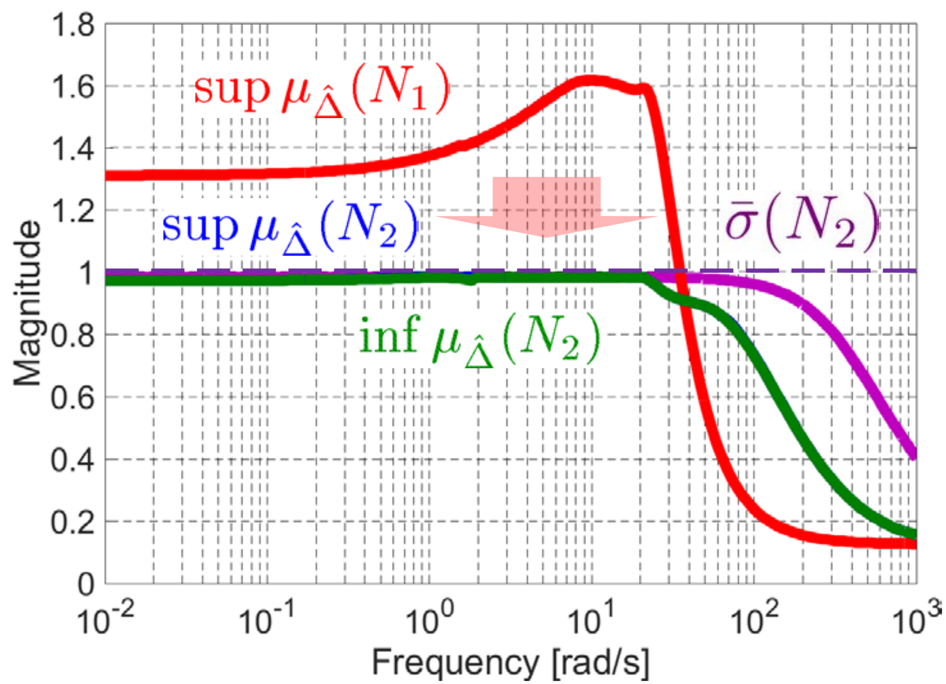
Next MU iteration number: 3

$$\mu_{\hat{\Delta}}(N_1) < \gamma, \ \gamma = 0.987 < 1$$

$$(\bar{\sigma}(\hat{\Delta}) = 1.00 \rightarrow \det(I - N_2\hat{\Delta}) = 0)$$

RP ○ $K_\mu = K_2$

(Finish DK-iteration)



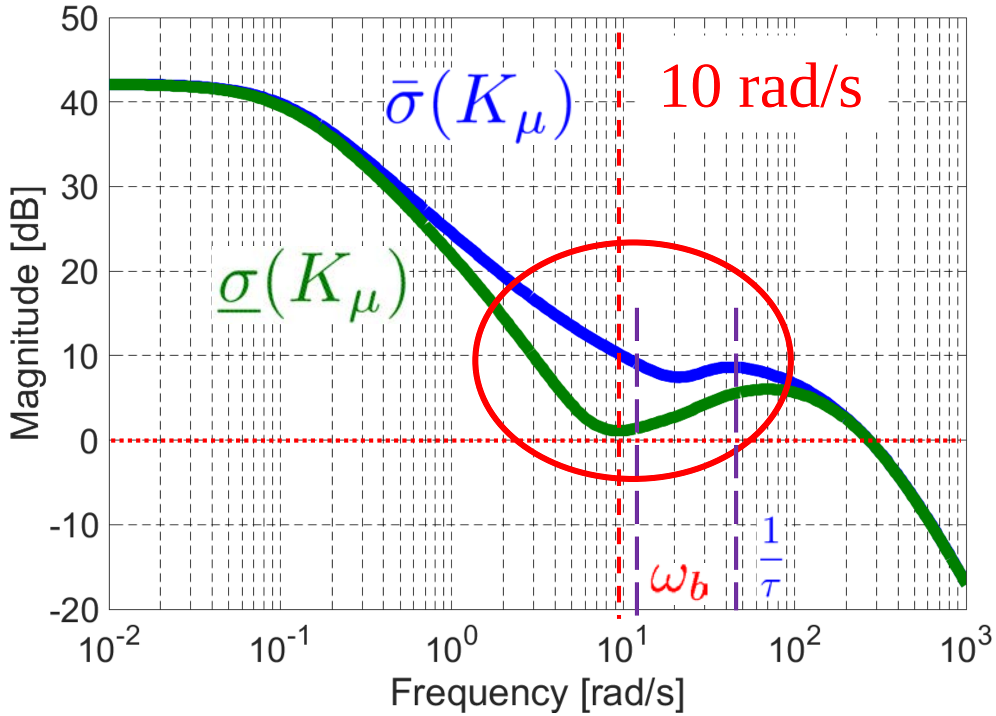
"If the desired specification cannot be achieved, then it probably means that your control problem is not well formulated and you should think again"

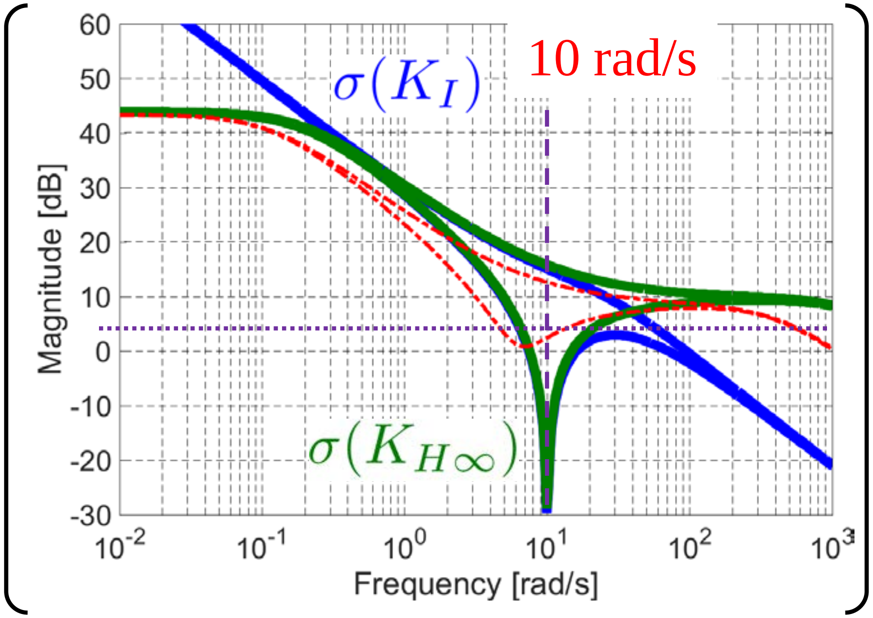
Ex.) Weights, Interconnection Structure

Spinning Satellite: μ-Optimal Controller K_μ

$$K_\mu(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$ Order 22

MATLAB Command
figure; sigma(Kdk)

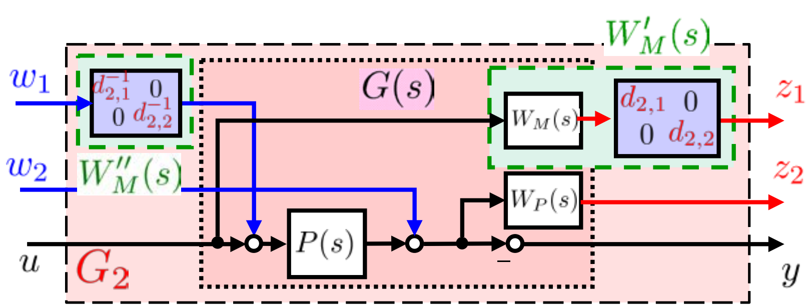




Poles p $\mathrm{Re}[p] < 0, \ ^\forall p$

Zeros z $\mathrm{Re}[z] < 0, \ ^\forall z$

Generalized Plant: Order 22



Numerical problems or inaccuracies may be caused too high order fit or poor fit of the D-scales

Difficult to implement

Spinning Satellite: Model Reduction of K_μ by scale-fitting

$D(s)$: Auto-Prefit (apf) $D(s)$: 1st order

K_μ Order 22 → K'_μ Order 10

(D: order 4 × 4 =16) (D': order 1 × 4 =4)

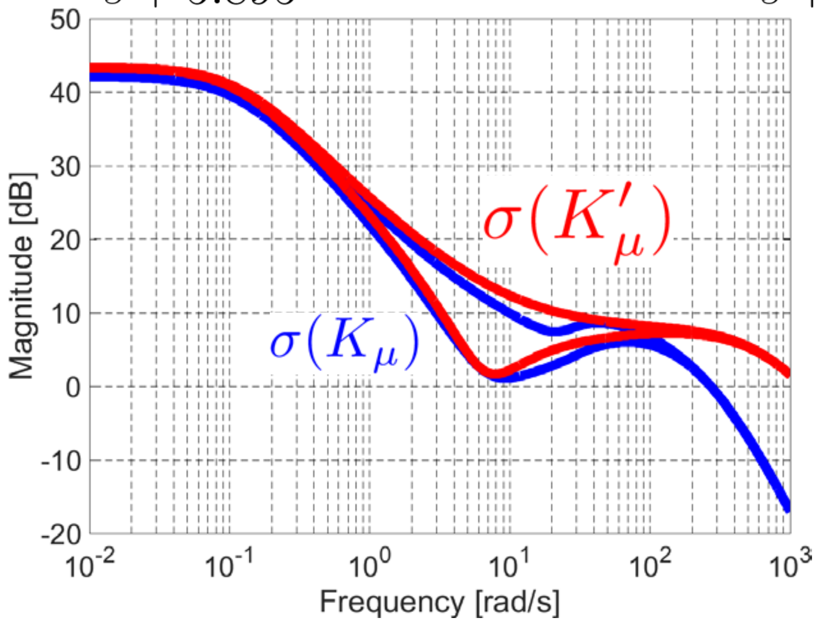
Scale-fitting Options

apf	Auto-PreFit
nd	Move to Next D-scaling
n	Fit with n'th order
e	Exit with Current Fittings

MATLAB Command Window

Iteration #	1	2	3
Controller Order	6	10	10
Total D-Scale Order	0	4	4
Gamma Acheived	1.709	1.064	0.877
Peak mu-Value	1.618	1.068	0.882 < 1

$$d'_{3,1}(s) = \frac{0.028611(s+438.7)}{s+5.895} \quad d'_{3,2}(s) = \frac{0.028401(s+441.6)}{s+5.888}$$

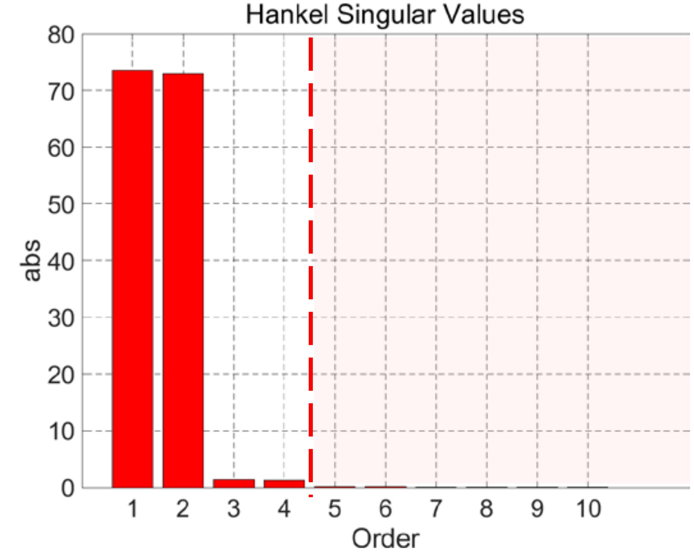


Command Window

Enter Choice (return for list): 1
Fitting with order = 1...
Done
Enter Choice (return for list): nd
Moving to next scaling entry...
Done
Enter Choice (return for list): 1
Fitting with order = 1...
Done
Enter Choice (return for list): nd
Moving to next scaling entry...
Done
Enter Choice (return for list): 0
Fitting with order = 0...
Done
Enter Choice (return for list): e

Spinning Satellite: Model Reduction of K'_μ

$$K''_\mu(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$



Balanced Truncation/Residualization

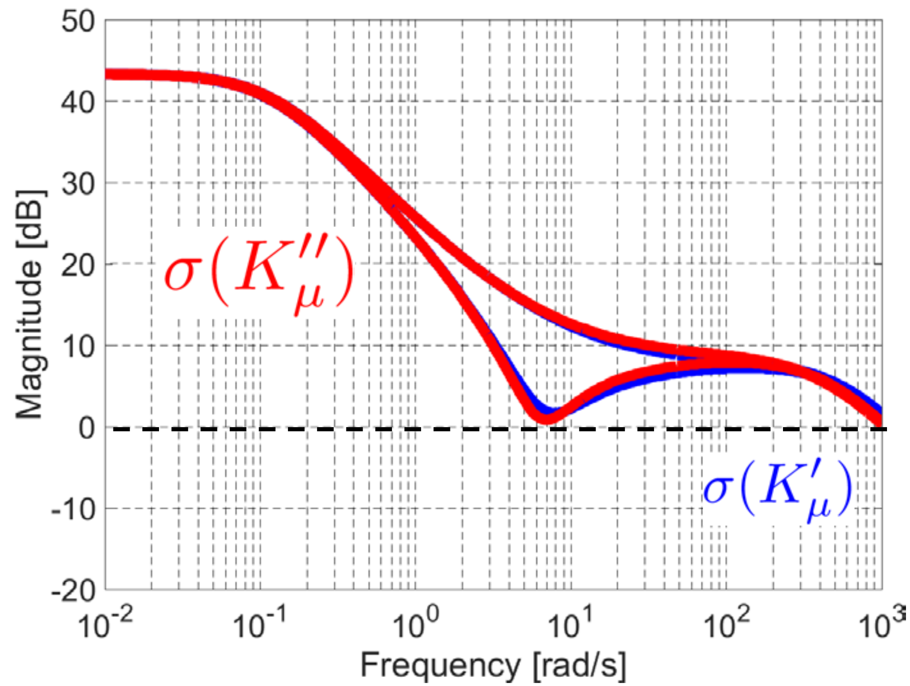
MATLAB Command
[Kred,info] = balancmr(Kdk);

Order: 10 to 4

Poles $-424.99 \pm 13.041j$

$-0.1158, -0.1151$

Zeros $-2.6037 \pm 5.7839j$



Discretization

Remark [SP05, p. 317]

"Note that NS is not guaranteed by $\mu_{\hat{\Delta}}(N) < 1$ and must be tested separately. (Beware! It is a common mistake to get a design with apparently great RP, but which is not NS and thus is not actually RS)"

Spinning Satellite: Beyond SISO Loop Shaping

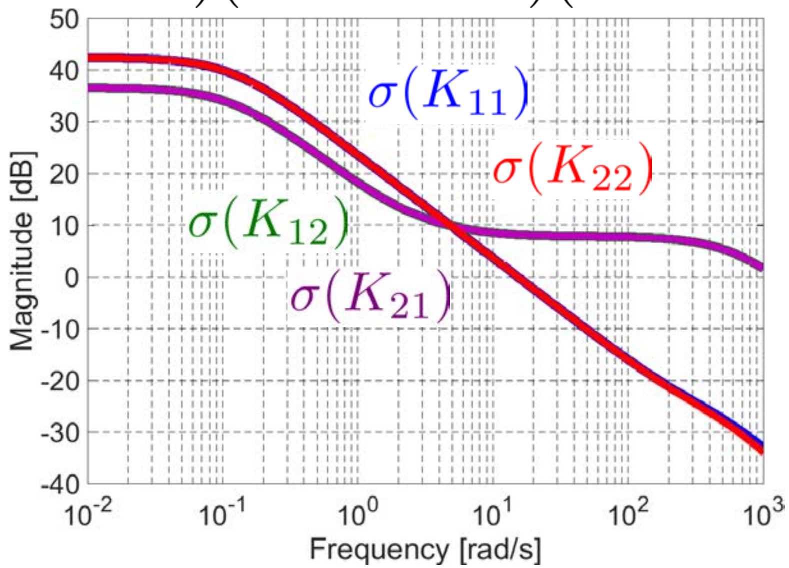
$$K''_\mu(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$

$$K_{11}(s) = \frac{-19.772(s-1140)(s-121.5)(s+0.1149)}{(s+0.1158)(s+0.1151)(s^2+850s+1.81\times10^5)}$$

$$K_{12}(s) = \frac{-1147.6(s+419.3)(s+2.904)(s+0.1152)}{(s+0.1158)(s+0.1151)(s^2+850s+1.81\times10^5)}$$

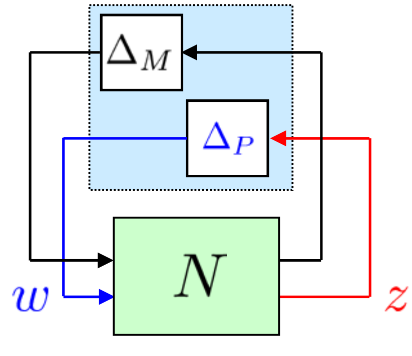
$$K_{21}(s) = \frac{1127.6(s+429.8)(s+2.831)(s+0.1191)}{(s+0.1158)(s+0.1151)(s^2+850s+1.81\times10^5)}$$

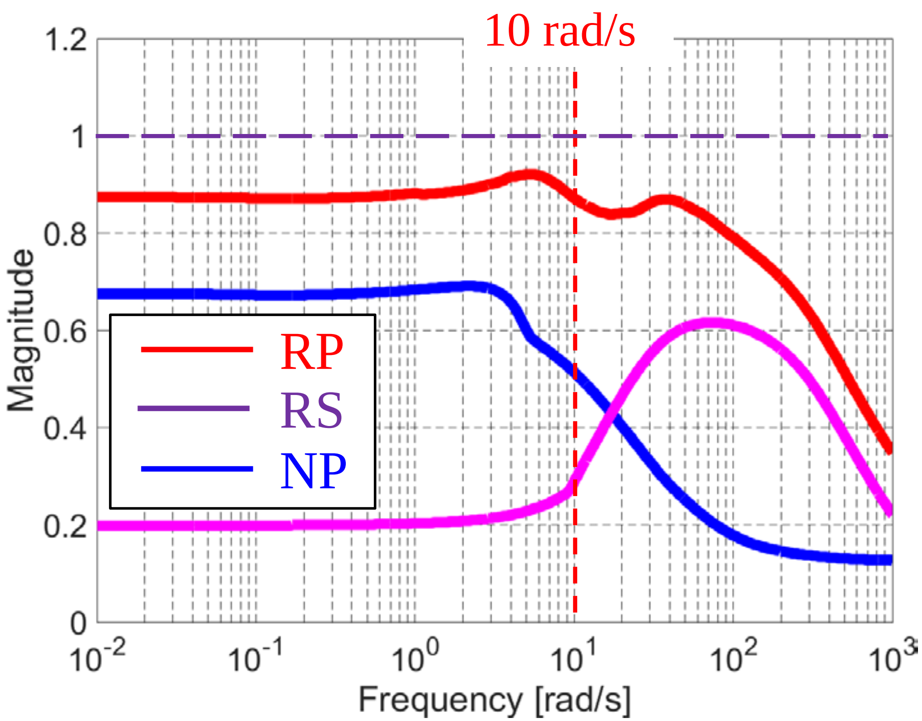
$$K_{22}(s) = \frac{14.602(s+690.6)(s-271.1)(s+0.1152)}{(s+0.1158)(s+0.1151)(s^2+850s+1.81\times10^5)}$$



Spinning Satellite: μ -Analysis for K''_μ mussv

$$N = F_l(G, K_\mu) = \begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} = \begin{bmatrix} -W_M T_i & -W_M K S_o \\ W_P S_o P & W_P S_o \end{bmatrix}$$





$\mu_{\Delta_P}(N_{22})$ = 0.6909 NP ○

$\mu_{\Delta_M}(N_{11})$ = 0.6160 RS ○

$\mu_{\hat{\Delta}}(N)$ = 0.9214 RP ○

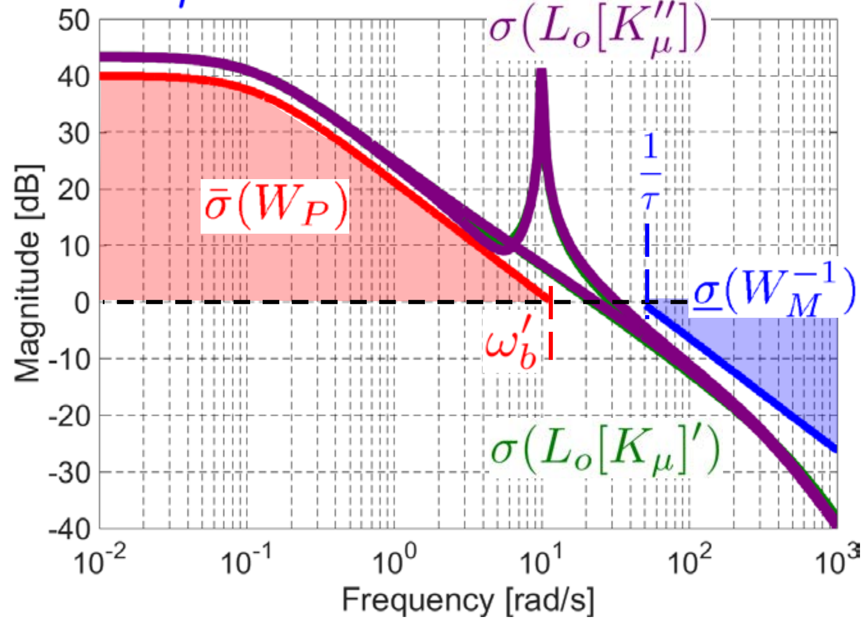
MATLAB Command

```
Blk_unc = [1 1;1 1];
Blk_per = [2 2];
Blk = [Blk_unc; Blk_per];
%%%
w = logspace(-2,2,200);
Nf = frd(N,w);
%%% mu for NP %%%
Nnp = Nf(3:4,3:4);
[MuBnds,MuInfo] = mussv(Nnp,Blk_per,'c');
muNP = MuBnds(:,1);
[muNPinf,muNPw] = norm(muNP,inf);
muNPinf
 %%% mu for RS %%%
Nrs = Nf(1:2,1:2);
[MuBnds,MuInfo] = mussv(Nrs,Blk_unc,'c');
muRS = MuBnds(:,1);
[muRSinf,muRSw] = norm(muRS,inf);
muRSinf
%%% mu for RP %%%
[MuBnds,MuInfoRP] = mussv(Nf,Blk,'c');
muRP = MuBnds(:,1);
[muRPinf,muRPw] = norm(muRP,inf);
muRPinf
%%%
figure; sigma(muNP,muRS,muRP)
```

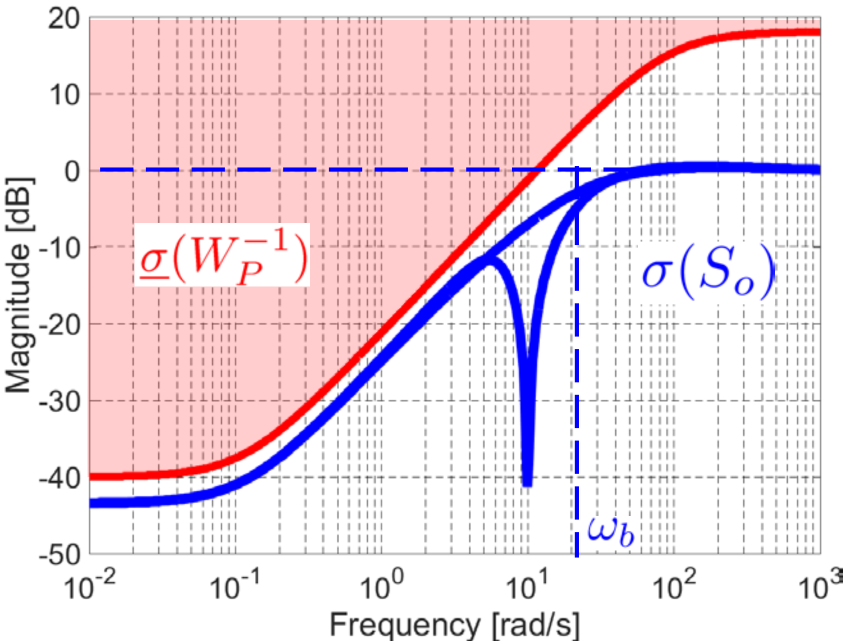
Spinning Satellite: Performance for K_μ''

Open-loop Transfer Function

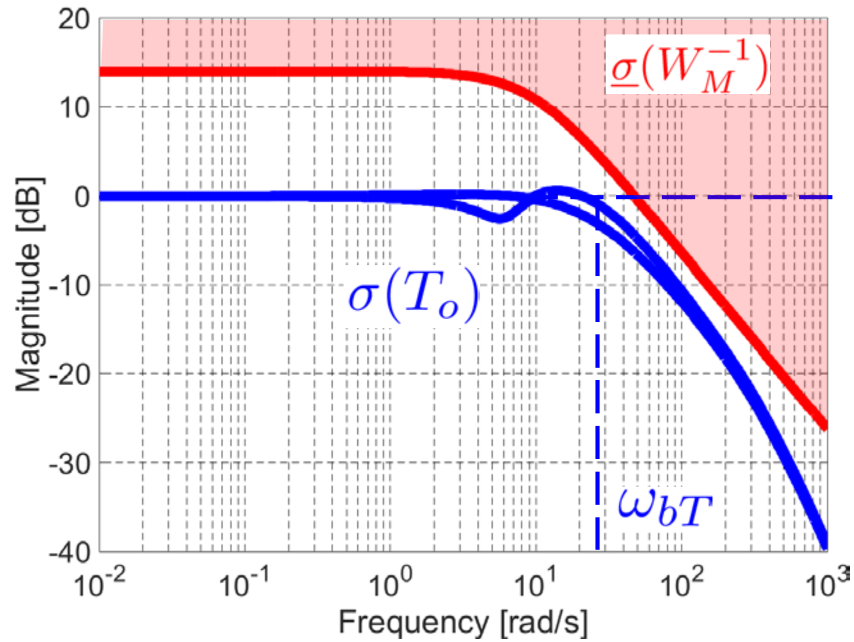
DK-iteration implies auto tuning of W_M and W_P



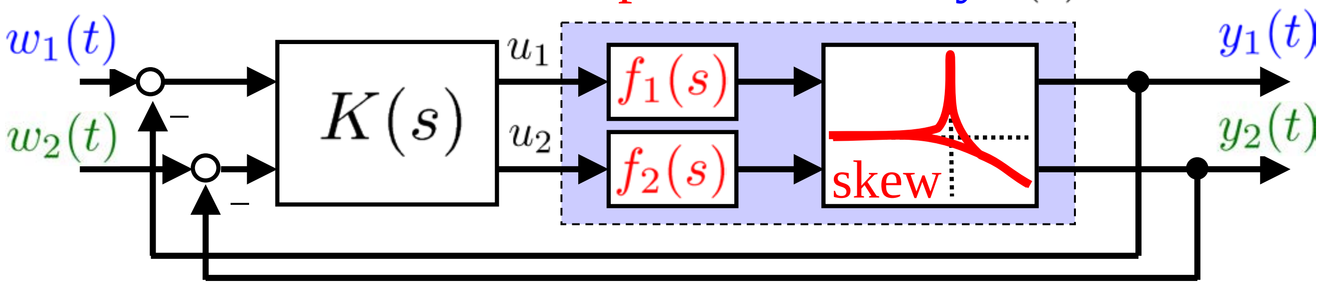
Nominal Performance (NP)

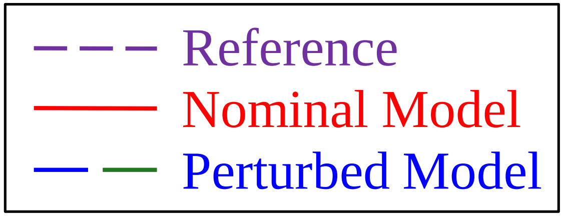


Robust Stability (RS)

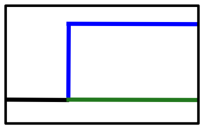


Spinning Satellite: Time Responses for K_μ

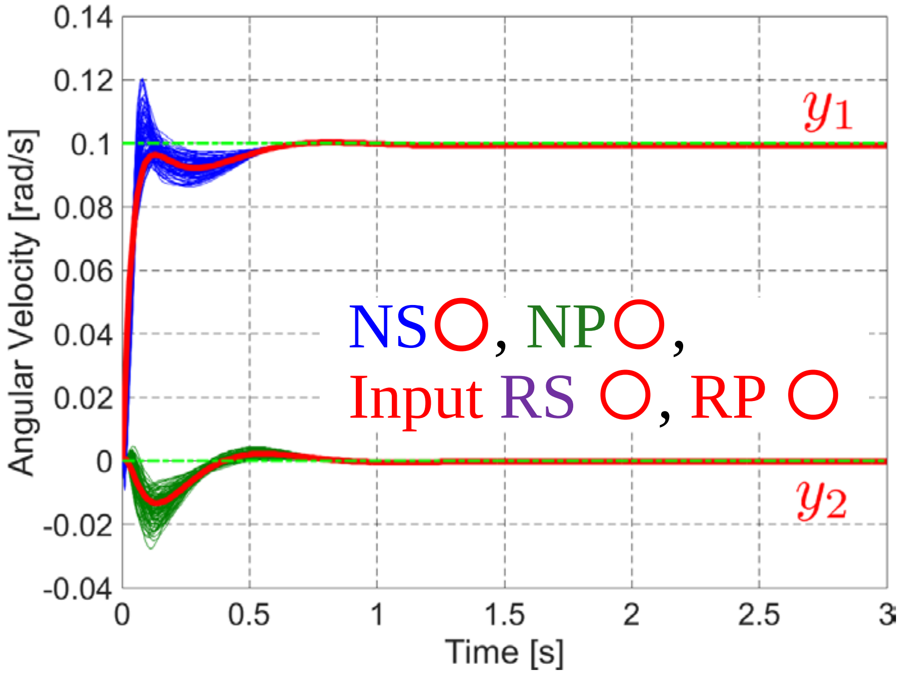


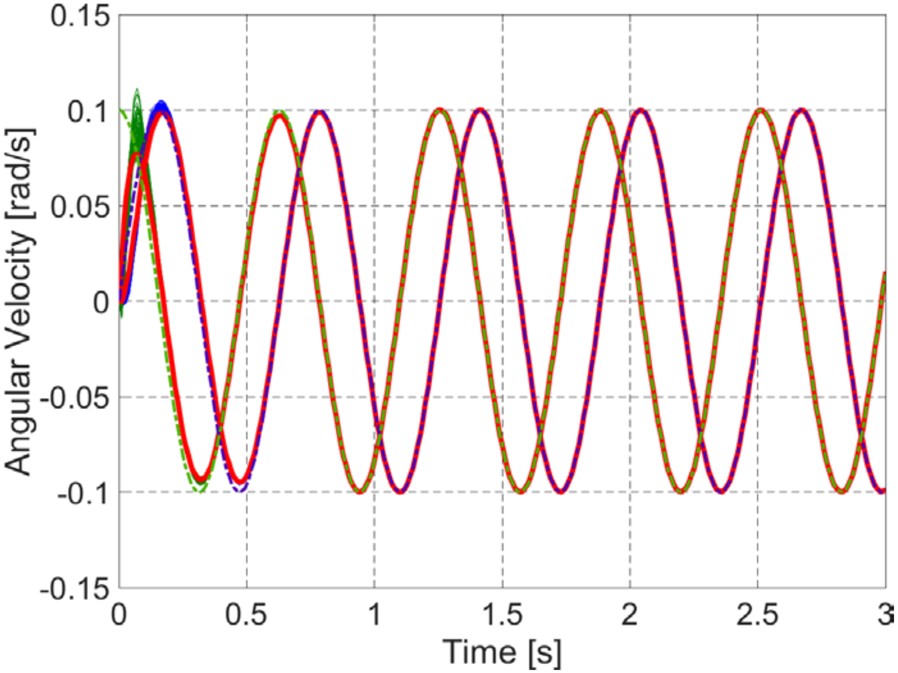


$$w(t) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

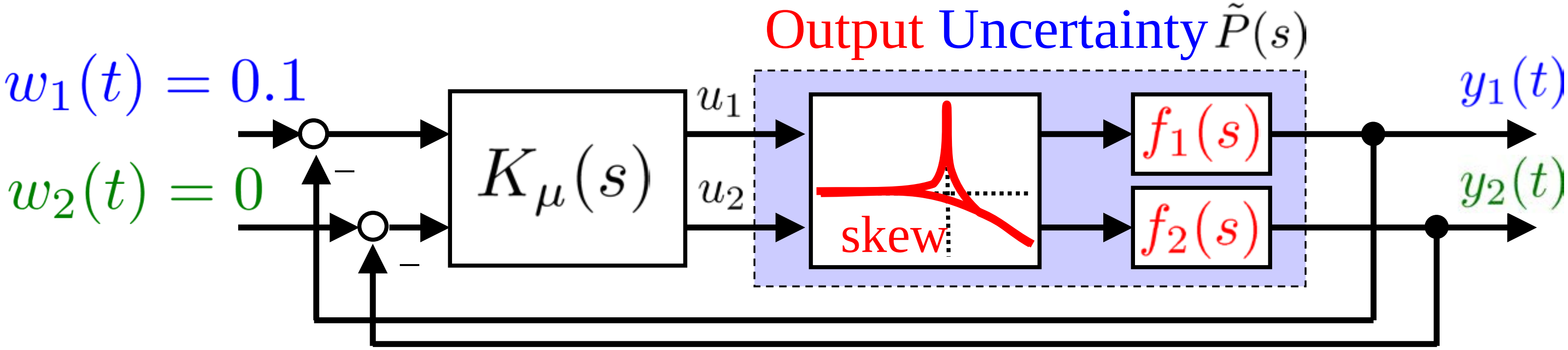


$$w(t) = \begin{bmatrix} 0.1\sin(\omega t) \\ 0.1\cos(\omega t) \end{bmatrix} \quad \omega = 10 \text{ rad/s}$$

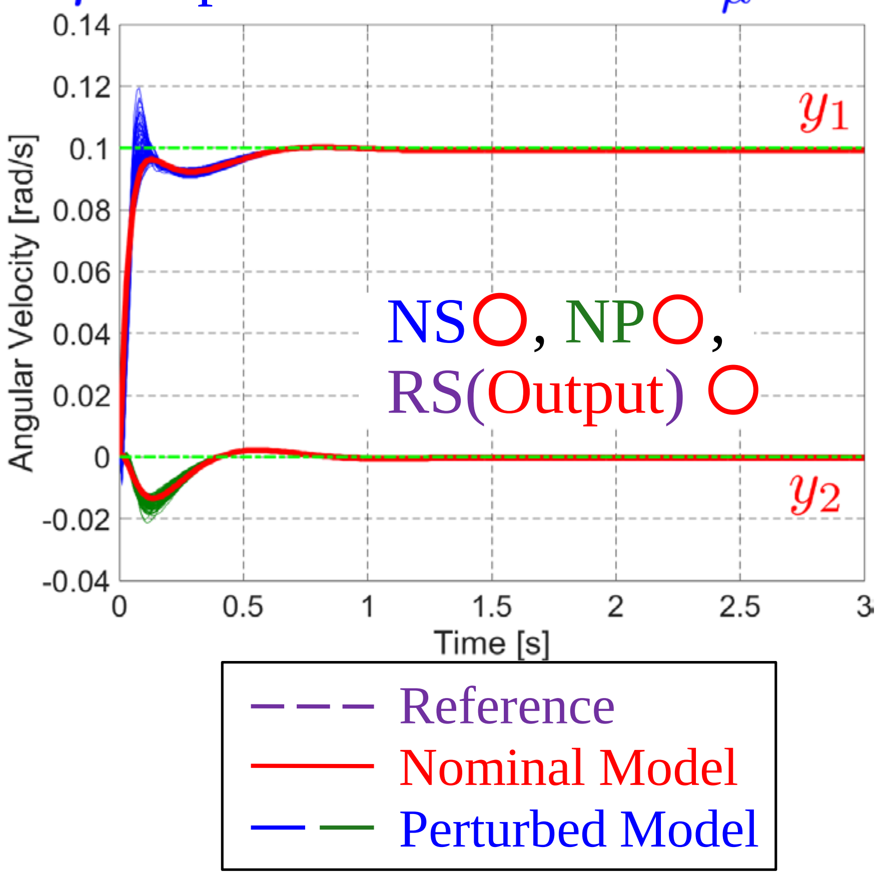




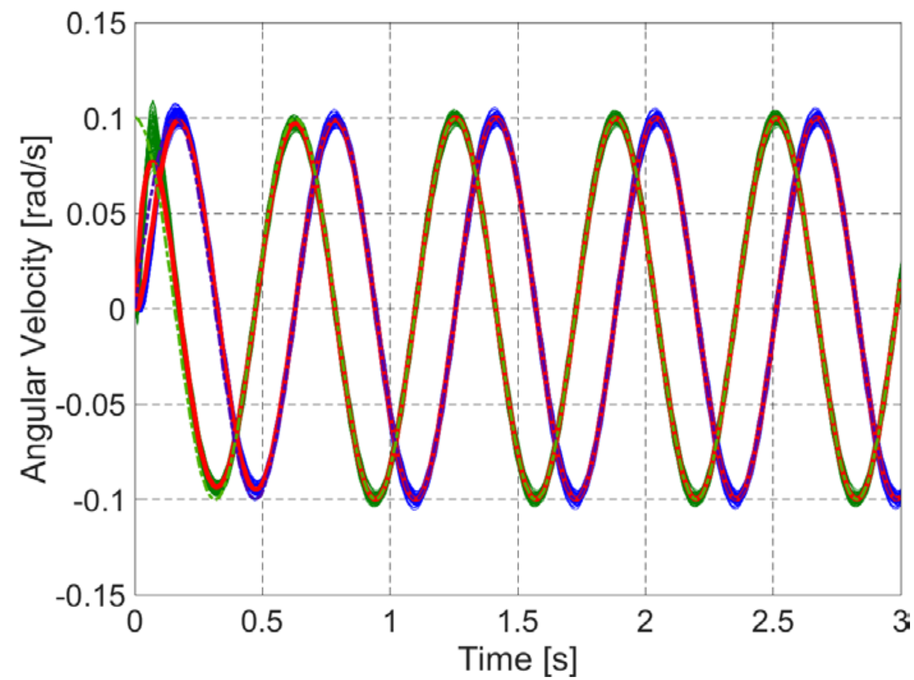
Spinning Satellite: Time Responses for Output Uncertainty



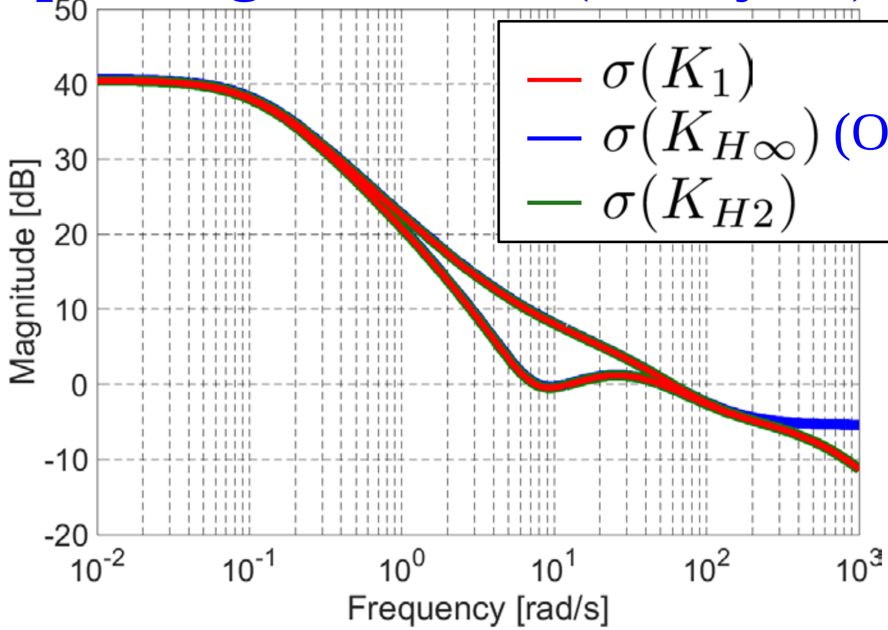
μ-Optimal Controller K_μ



$$w(t) = \begin{bmatrix} 0.1\sin(\omega t) \\ 0.1\cos(\omega t) \end{bmatrix} \quad \omega = 10 \text{ rad/s}$$

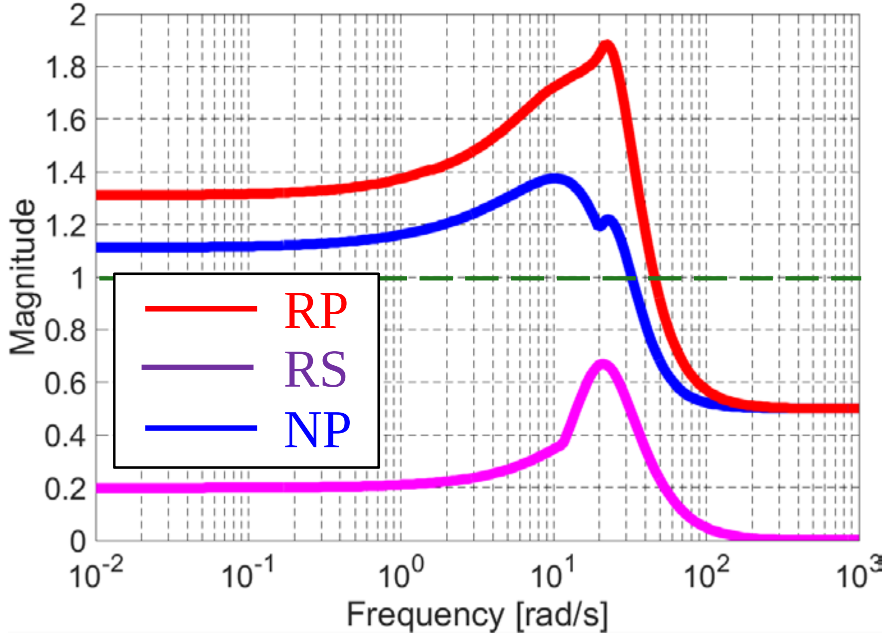


Spinning Satellite: (Analysis) H_∞ Controller K_1

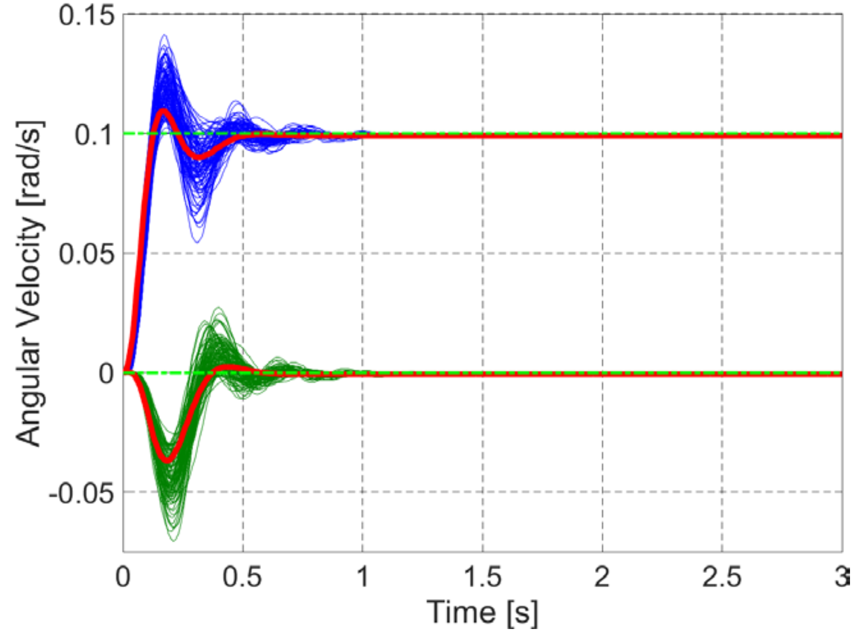


The controller K_1 yields a blend of H_∞ and H_2 optimality with a controller which usually has a steeper high-frequency roll-off than the H_∞ optimal controller.

Close-loop and NP/RS Test



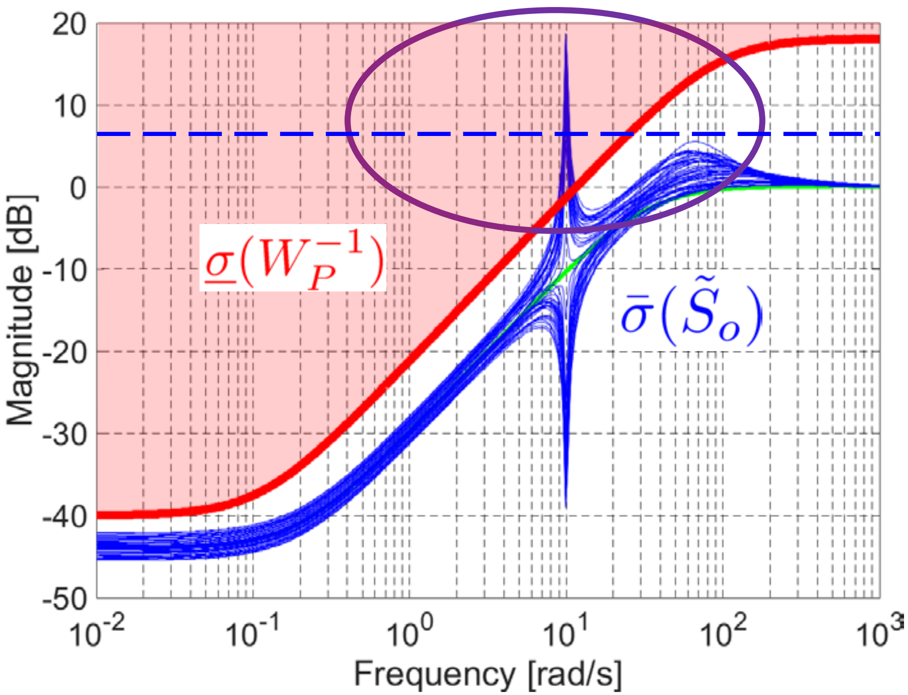
Time responses



Spinning Satellite: Robust Performance

RP $\|W_P \tilde{S}\|_\infty < 1$

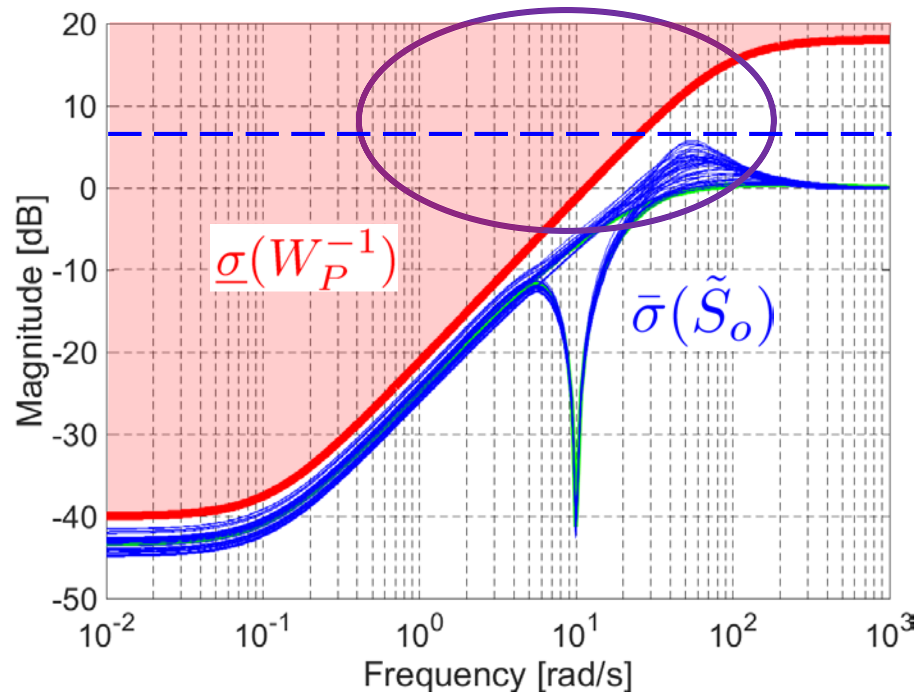
H_∞ Controller $K_{H\infty}$



MATLAB Command

```
Farray = loopsens(Parray,Khi) ;
figure
sigma(Farray.So,inv(WP));
```

μ -Optimal Controller K''_μ

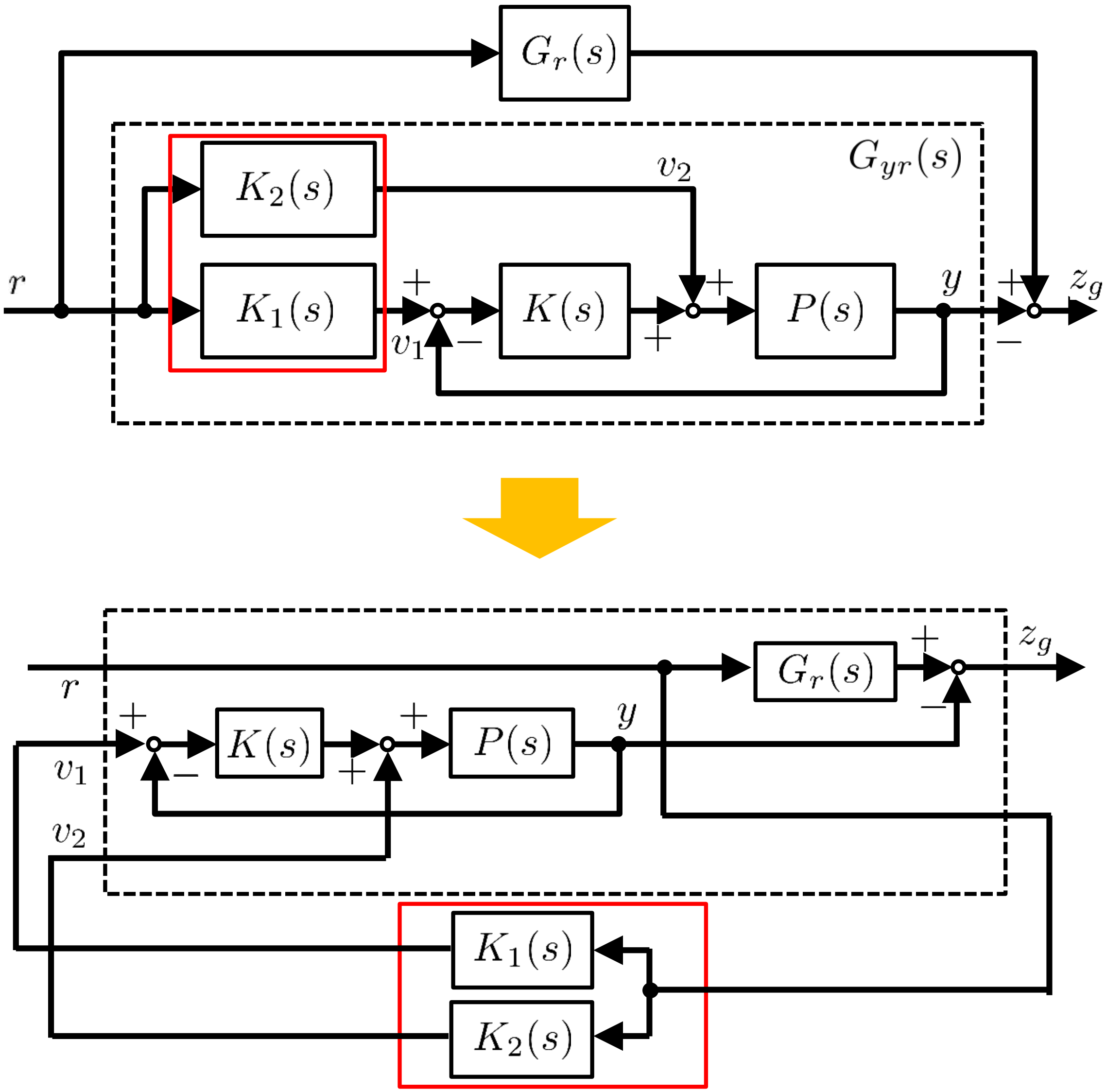


MATLAB Command

```
Farray = loopsens(Parray,Kred) ;
figure
sigma(Farray.So,inv(WP));
```

MIMO 2degrees of freedom controller

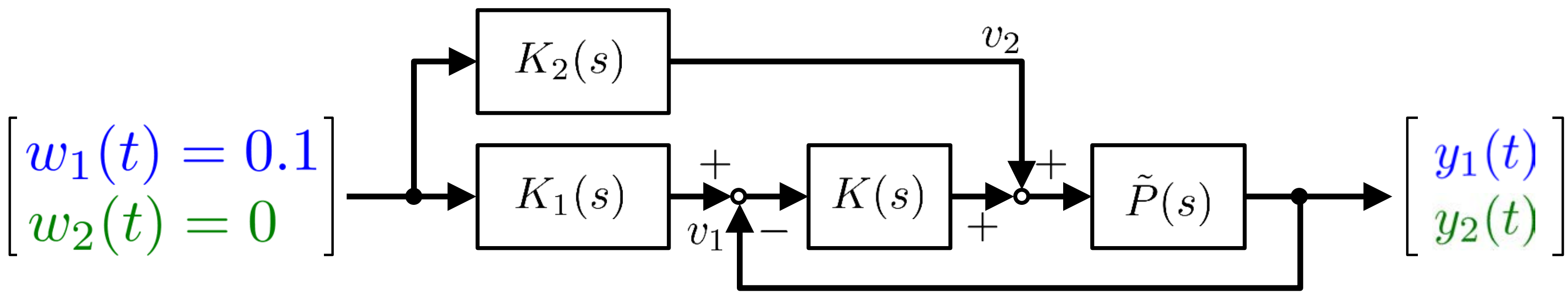
Model matching method: $\|G_r - G_{yr}\|_\infty < \gamma_g$



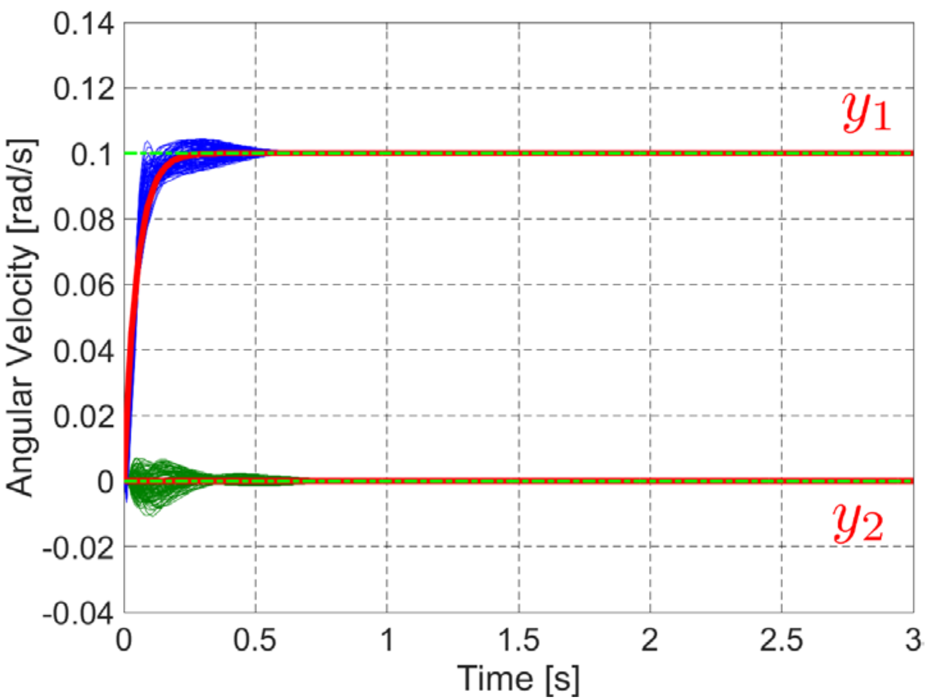
MATLAB Command

```
%% Desired FF Controller%
Gr = tf(30,[1 30])*eye(2) ;

%% Generalized Plant%
systemnames = 'Pnom Kred Gr';
inputvar = '[r(2);u1(2);u2(2)]';
outputvar = '[Gr-Pnom;r]';
input_to_Pnom= '[Kred+u2]';
input_to_Kred = '[u1-Pnom]';
input_to_Gr = ' [r]';
G = sysic;

%% Controller %
nmeas = 2;
ncon = 4;
[Kf,CLf,gf,hfinfo] = …
hinfsyn(G,nmeas,ncon, 'Method', 'lmi' );
gf
```

MIMO 2degrees of freedom controller



$$G_r(s) = \frac{20}{s+20} I_2 \ (\gamma_g = 9.17 \times 10^{-5})$$



$K_1(s), K_2(s)$ (order 8)

MATLAB Command

```
time = 0:0.01:3;
step_ref = ones(1,length(time));
ref = [0.1*step_ref; 0*ones(1,length(time))];
figure; hold on; grid on;
for i = 1 : 100
    FI = loopsens(Parray(:,:,i),Kred );
    T1 = series( Kf(1:2,1:2), FI.To ) ;
    T2 = series( Kf(3:4,1:2), FI.PSi ) ;
    TF = parallel(T1, T2 ) ;
    [yhi,t] = lsim(TF,ref,time);
    plot(t,yhi(:,1),'b-'); plot(t,yhi(:,2),‘g-');
end
FI = loopsens(Pnom,Kred );
T1 = series( Kf(1:2,1:2), FI.To ) ;
T2 = series( Kf(3:4,1:2), FI.PSi ) ;
TF = parallel(T1, T2 ) ;
[yhi,t] = lsim(TF,ref,time);
plot(t,yhi,'r-'); plot(time,ref,'g-.');
```

MIMO 2degrees of freedom controller

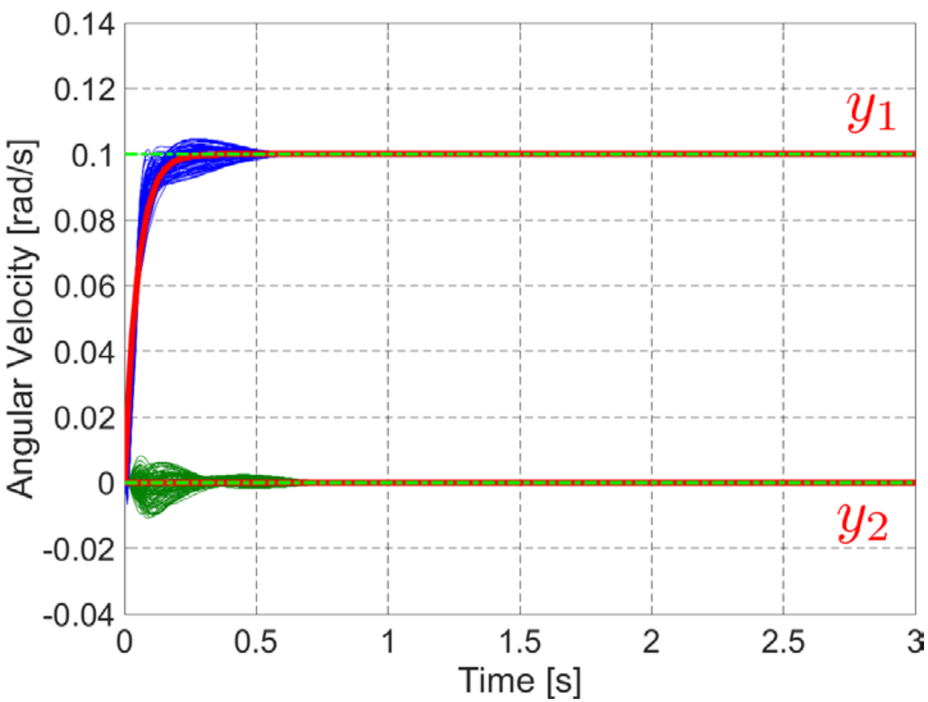
Reduced Feedforward Controllers

$$K_1(s) = \begin{bmatrix} \frac{20.0}{s+20} & \frac{-1.6\times10^{-6}(s+99.7)(s+10.52)}{(s+20)^2} \\ \frac{2.6\times10^{-6}(s^2+9.217s+762.6)}{(s+20)^2} & \frac{20.0}{s+20} \end{bmatrix} \approx G_r(s)$$

$$K_2(s) = \begin{bmatrix} \frac{0.198(s-100)}{s+20} & \frac{-1.98(s+1)}{s+20} \\ \frac{1.98(s+1)}{s+20} & \frac{0.198(s-100)}{s+20} \end{bmatrix} = \frac{0.198}{s+20}\begin{bmatrix} s-100 & -10(s+1) \\ 10(s+1) & s-100 \end{bmatrix}$$

$$\approx P^{-1}(s)G_r(s)$$

$$(\gamma_g = 1.38\times10^{-5})$$



MATLAB Command

```
Kfred1 = hankelmr( Kf(1:2,1:2), 4 ) ;
Kfred2 = hankelmr( Kf(3:4,1:2), 4 ) ;
[Kfred1,] = modreal( Kfred1, 2 ) ;
[Kfred2,] = modreal( Kfred2, 2 ) ;
for i = 1 : 2 ; for j = 1 : 2 ;
Kfred1(i,j) = minreal( Kfred1(i,j), 1e-1 ) ;
Kfred2(i,j) = minreal( Kfred2(i,j), 1e-1 ) ;
end; end;
```