Robust Control

Spring, 2016

Instructor: Prof. Masayuki Fujita (S5-303B)

3rd class

Tue., 19th April, 2016, 10:45～12:15,

S423 Lecture Room

3. SISO Loop Shaping

3.1 Computer Controlled System

3.2 Modeling

[SP05, Sec. 3.7, 1.4, 1.5]

3.3 Example

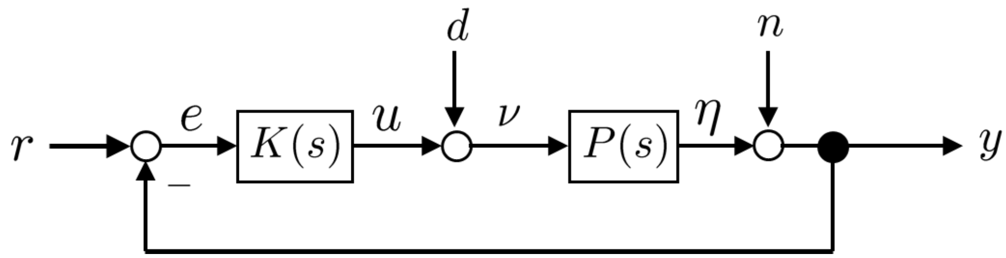
[SP05, Sec. 2.6, 5.6, 5.7, 5.9]

Reference:

[SP05] S. Skogestad and I. Postlethwaite,
Multivariable Feedback Control; Analysis and Design,
Second Edition, Wiley, 2005.

SISO Loop Shaping

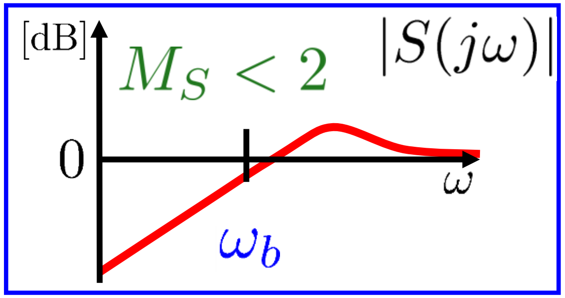
Loop Transfer Function



$$L(s) = P(s)K(s)$$

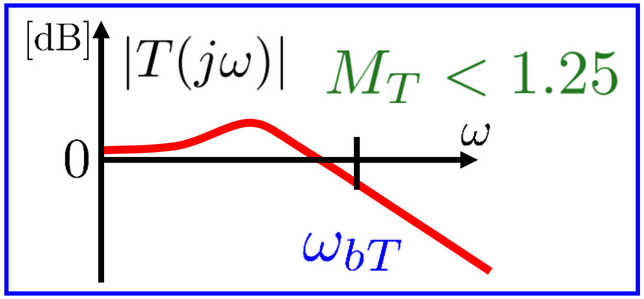
Sensitivity: $S = \frac{1}{1+L}$

Comp. Sensitivity: $T = \frac{L}{1+L}$



$+$

$S + T = 1$



$|L| \gg 1 \rightarrow |S| \ll 1$
large small

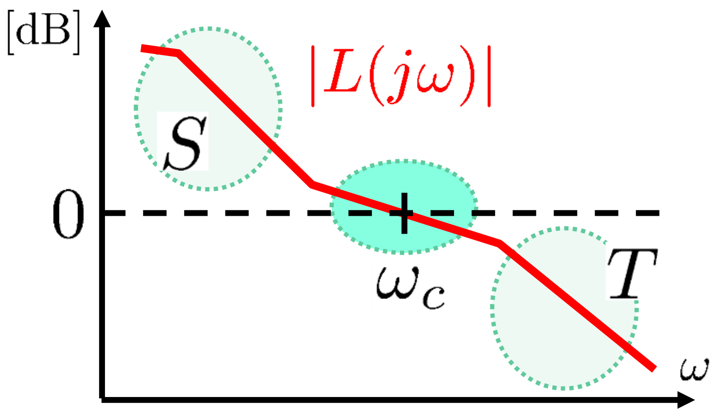
$|L| \ll 1 \rightarrow |T| \ll 1$
small small

Loop Shaping

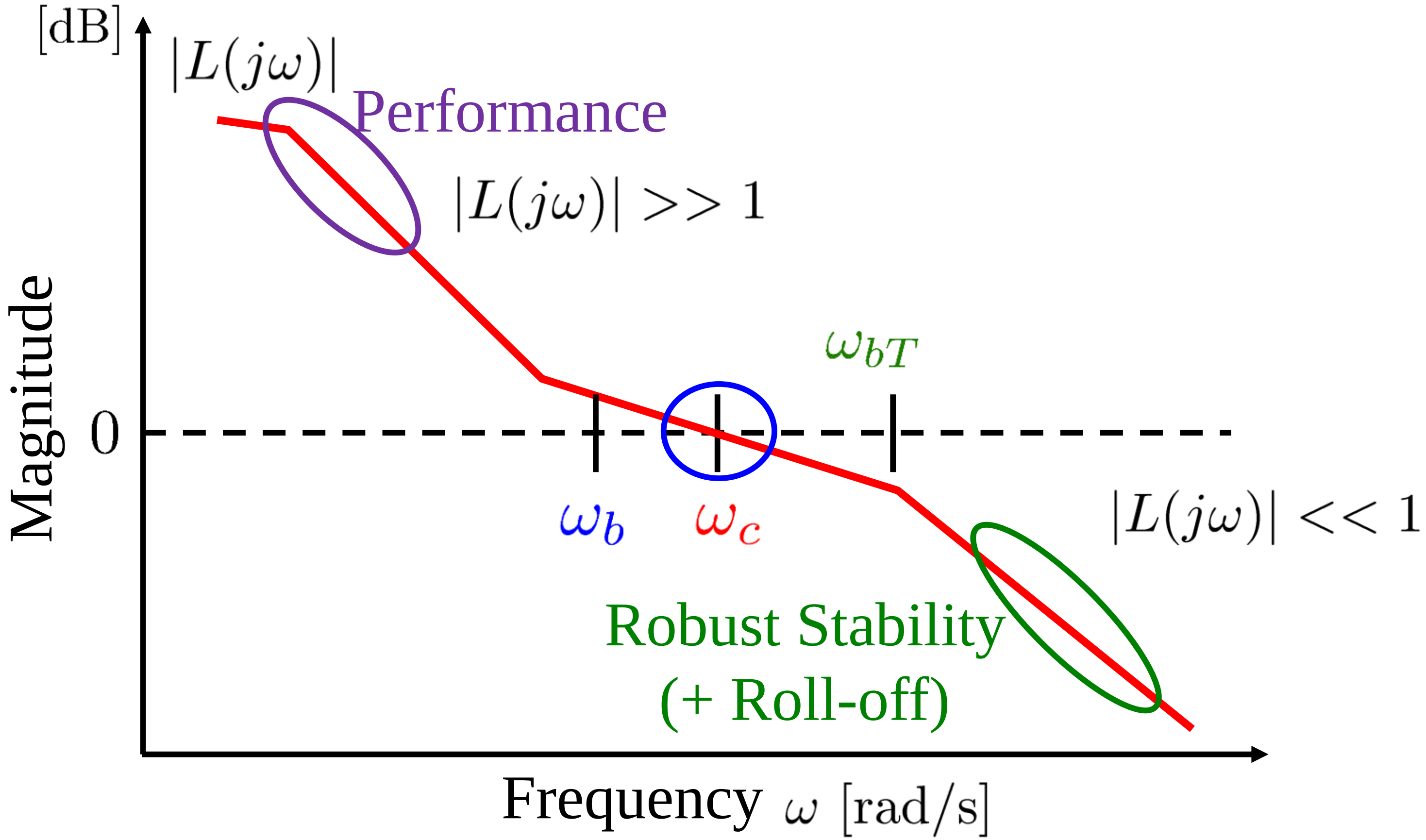
Closed-loop S, T

→ Open Loop L

Stability, Performance



SISO Loop Shaping [SP05, pp. 41, 42, 343]



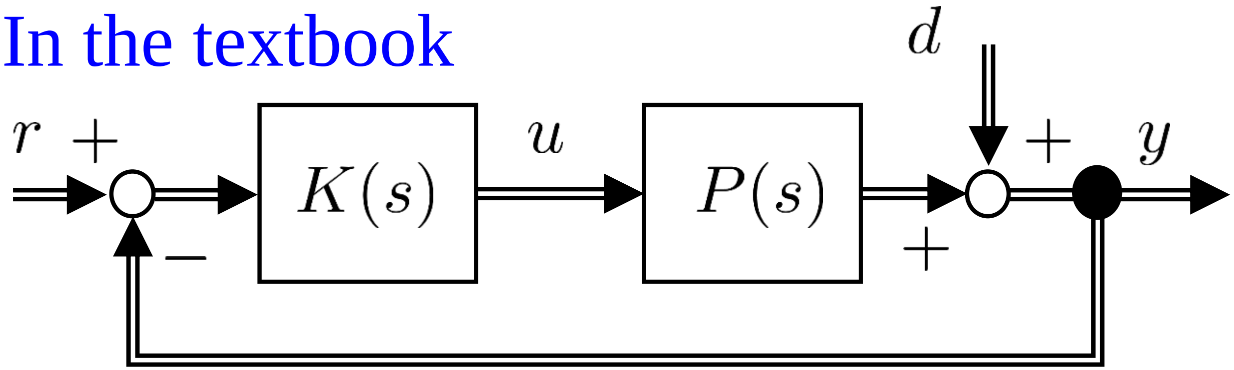
Loop Shaping Specifications

- Gain Crossover Frequency ω_c
- Large Magnitude at Lower Frequencies
- Small Magnitude at Higher Frequencies

Target Loop

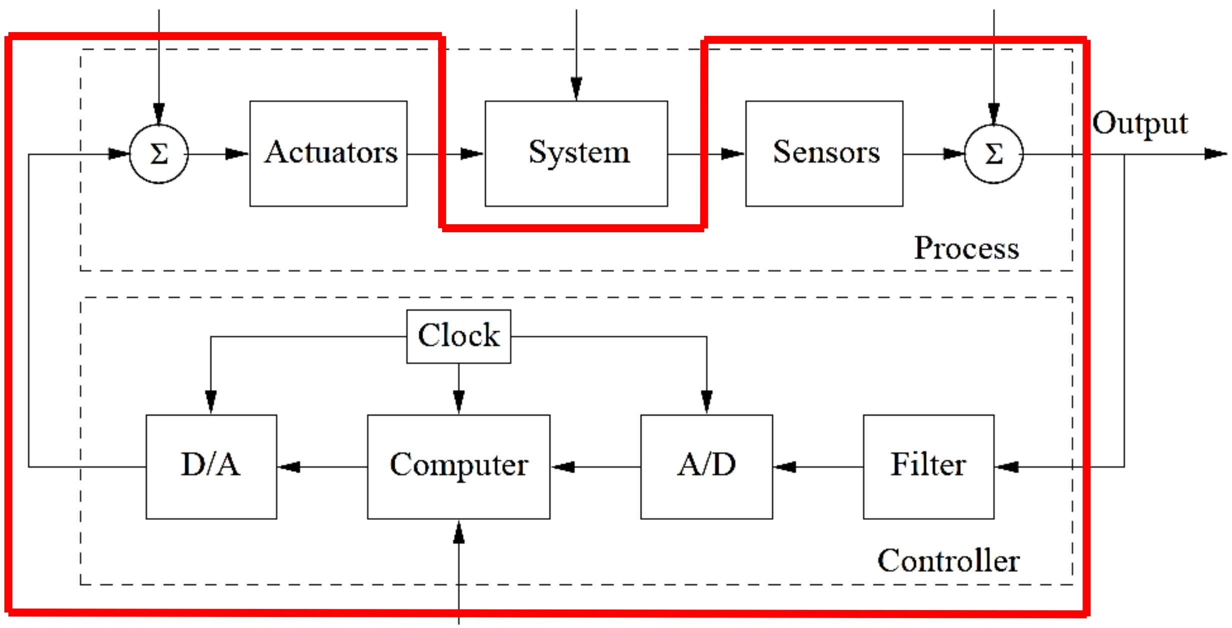
Computer Controlled System

In the textbook



Idealized
Too simplified

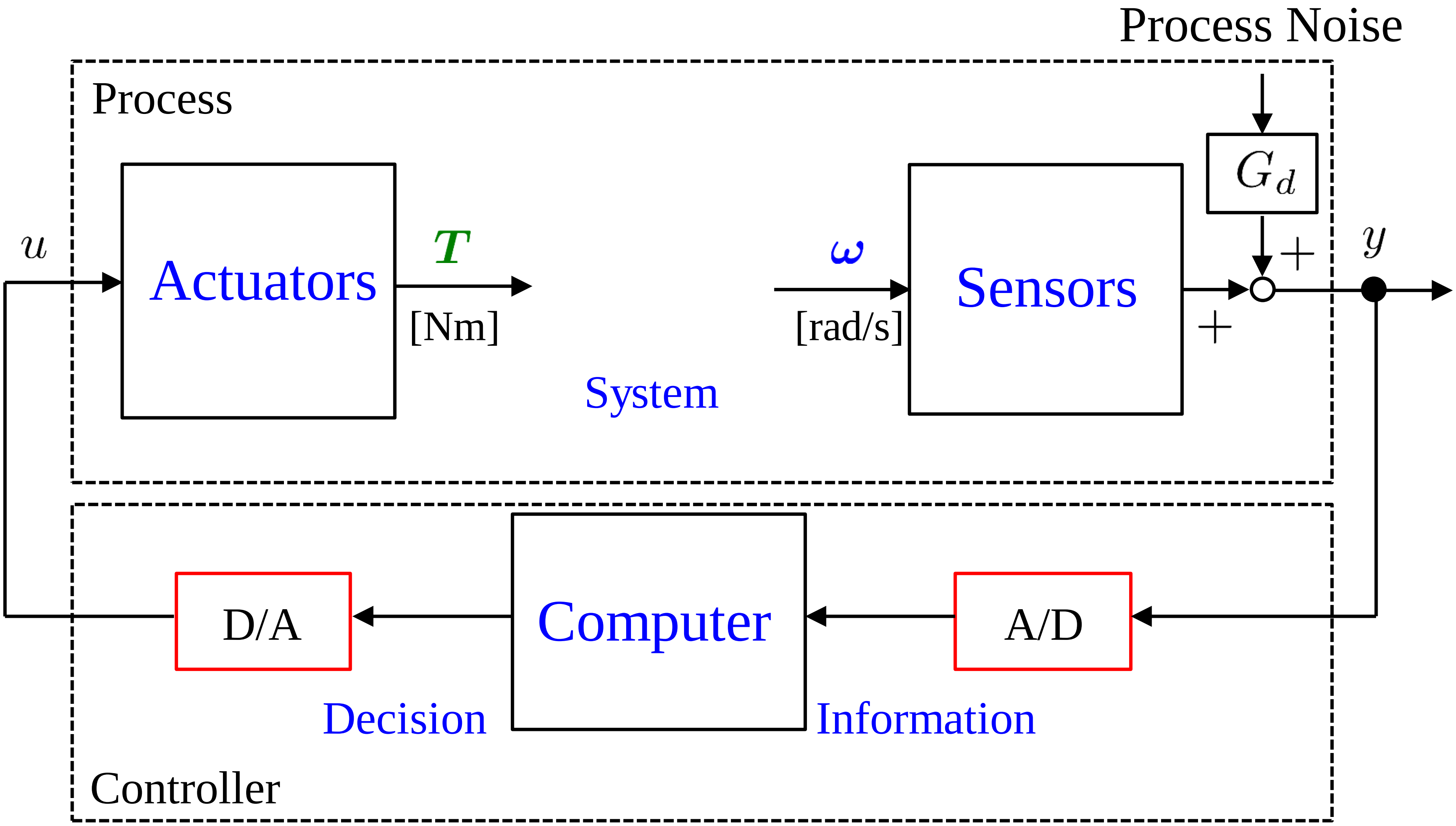
In the real world



Real System
Complicated

[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008.

Computer Controlled System



Sensor: Gyroscope

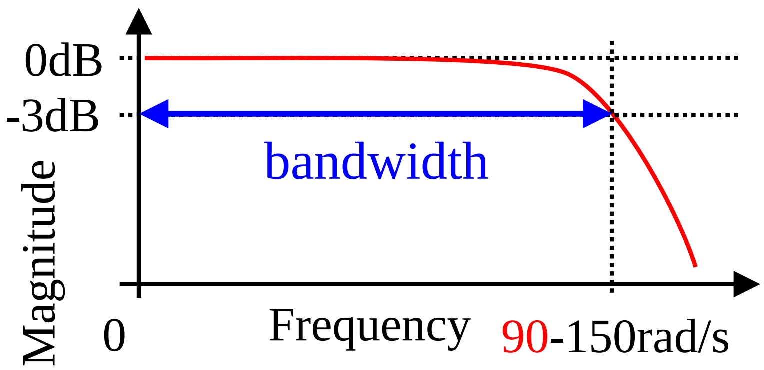
Kinematics

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & a \\ -a & 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

Interaction (Coupling)

(Sensor measurement are poorly aligned.)

Frequency Response



90-150rad/s (15-25Hz)

Resolution

$$\delta y_i = 5.3 \times 10^{-4} \text{ rad/s} = 0.03 \text{ deg/s}$$

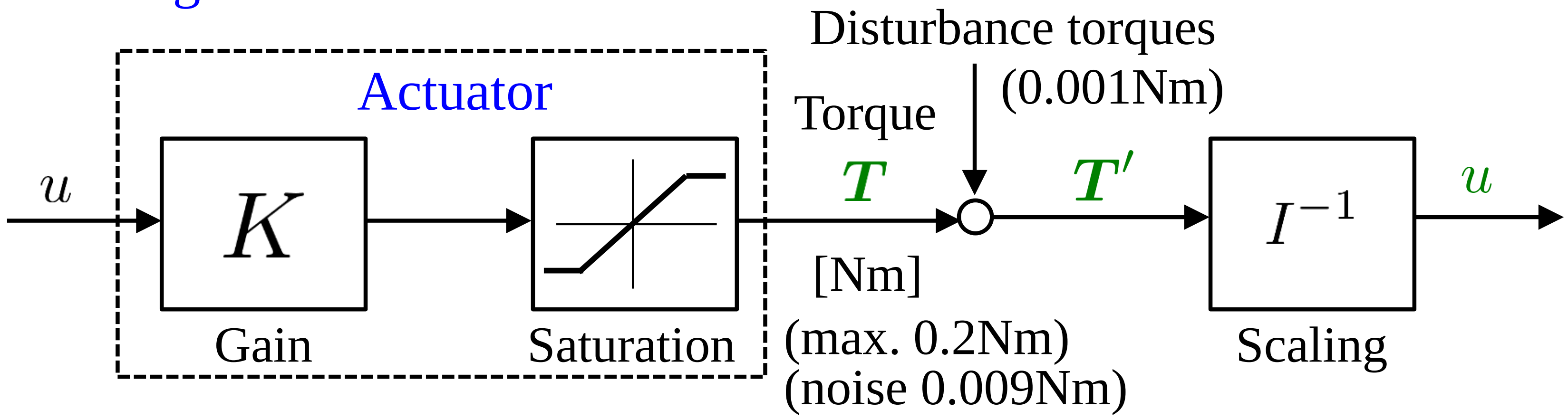
Measurable Range

$$y_i \in [-y_i^{\max}, y_i^{\max}],$$

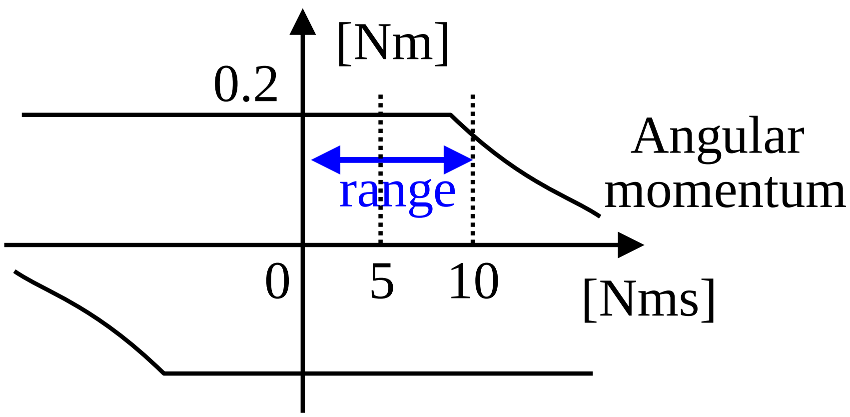
$$y_i^{\max} = 0.26 \text{ rad/s} = 15 \text{ deg/s}$$

Actuator: Reaction Wheel

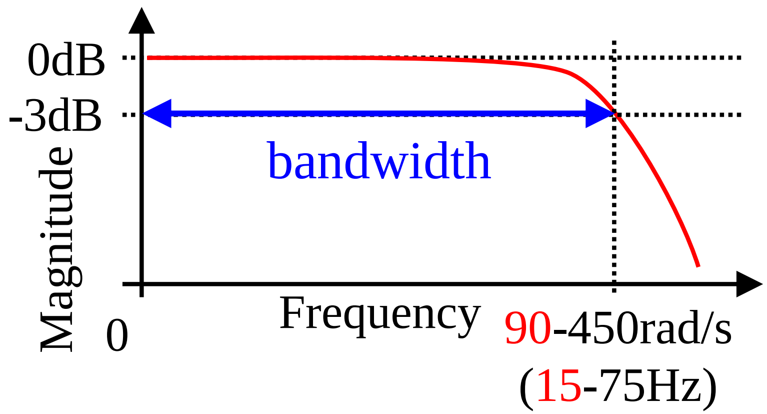
Block diagrams



Torque-momentum limitations



Frequency response



Controller: Computer

Onboard computer loaded in 1989:

80386/80387 RISC processor (Intel, Loral RAD-6000 32bit)

Control law		
LQG	(Processing Time)	94-184 ms (5-10Hz)

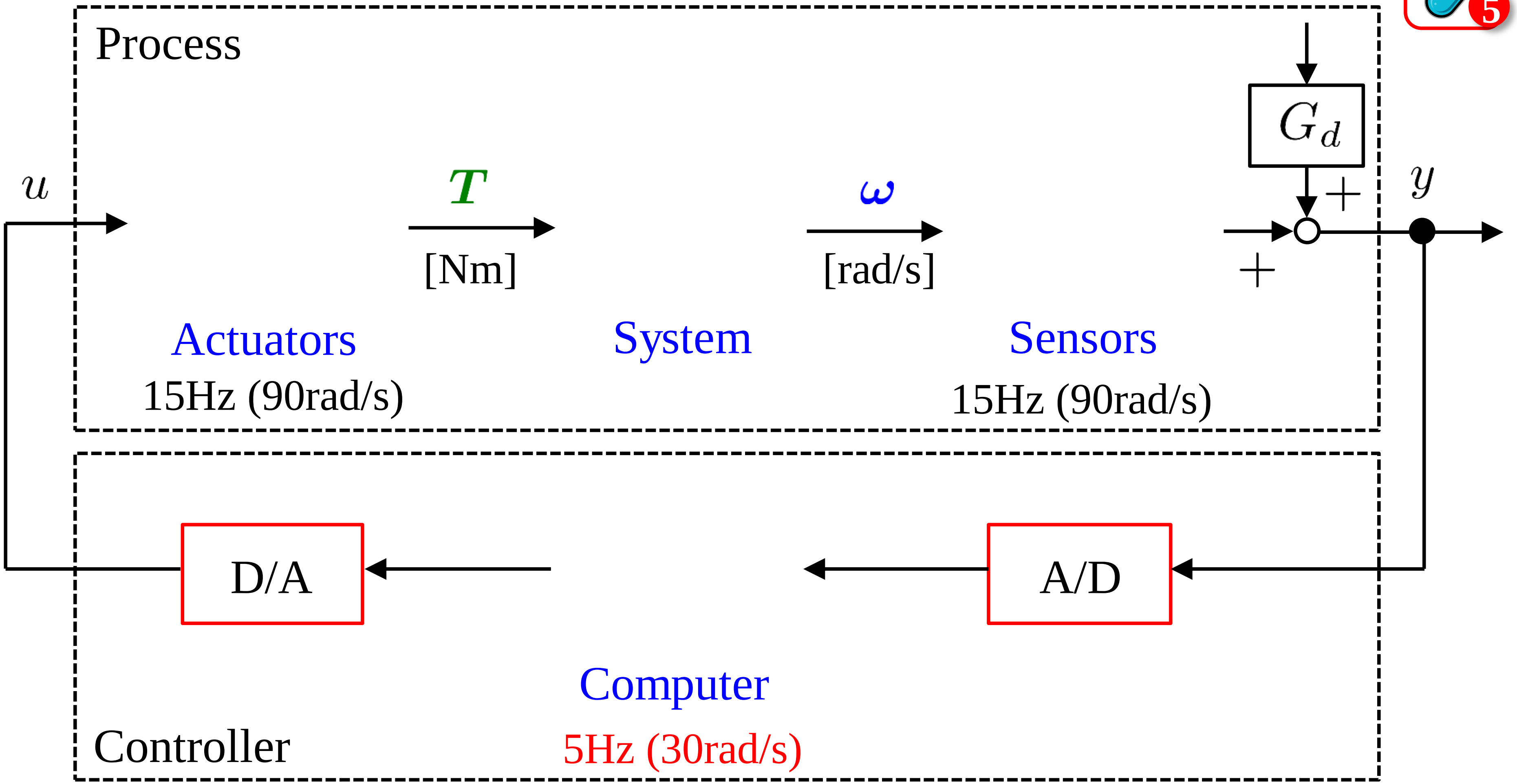
cf. [HOKN11, Table 2] ETS-VIII (launched by 2006)

Processing time and memory requirement of each control law

Control law	Order of Controller	Processing Time	Required memory
Gain Scheduling	14	1.40 ms (714 Hz)	4140 byte
μ -synthesis	27	2.28 ms (438 Hz)	7948 byte
DVDFB	--	3.20 ms (312 Hz)	14780 byte

[HOKN11] Y. Hamada, T. Ohtani, T. Kida and T. Nagashio. Synthesis of a linearly interpolated gain scheduling controller for large flexible spacecraft ETS-VIII, *Control Engineering Practice*, **19**(6) 611-625, 2011.

Computer Controlled System



Real physical systems have a multitude of limitations on available bandwidth ➡ **System Bandwidth**

$$f_c = 5\text{Hz}$$

- Time Delay Margin

$$0 \leq \theta \leq 0.02\text{s} = \frac{1}{10 f_c}$$

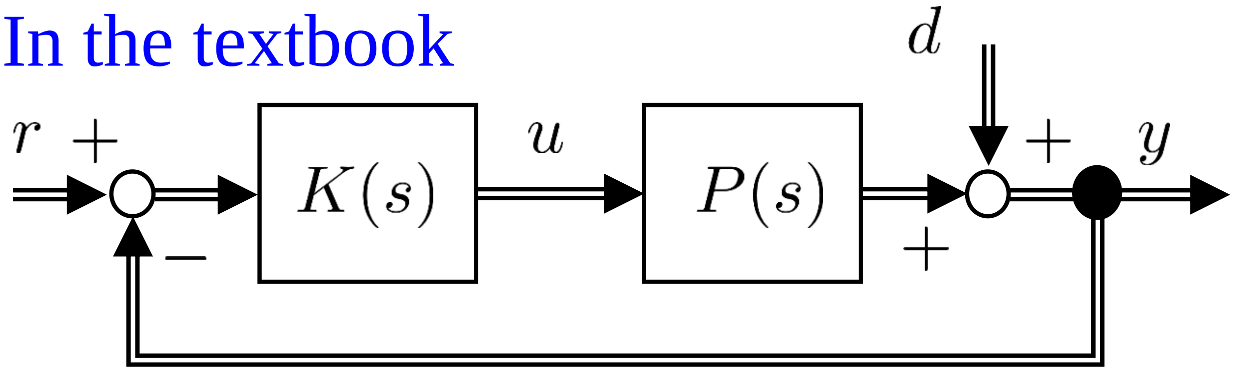
- Gain Margin

$$0.8 \leq k \leq 1.2$$

(20% variation $GM \geq 2\text{dB}$)

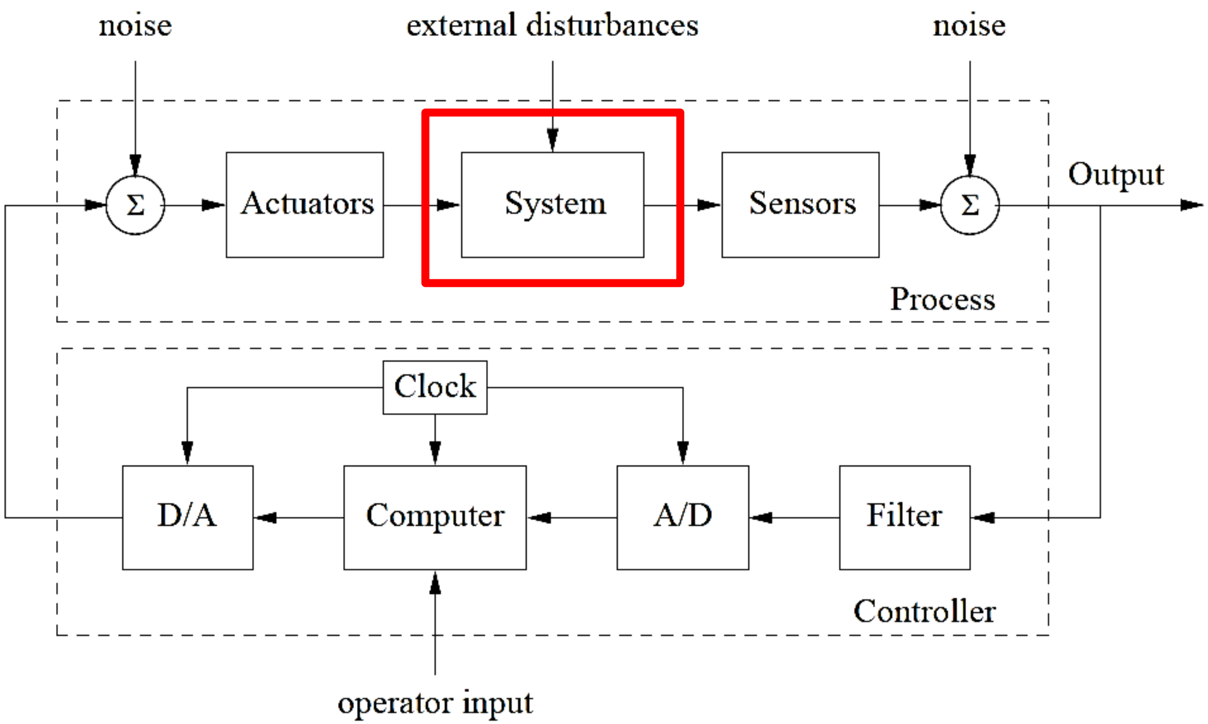
Computer-controlled System

In the textbook



Idealized
Too simplified

In the real world



Real System
Complicated

[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008.

Modeling

STEP 1.

Real Physical System

（実物）

STEP 2.

Ideal Physical Model

Conceptual/Schematic model

（図式化・概念化）

STEP 3.

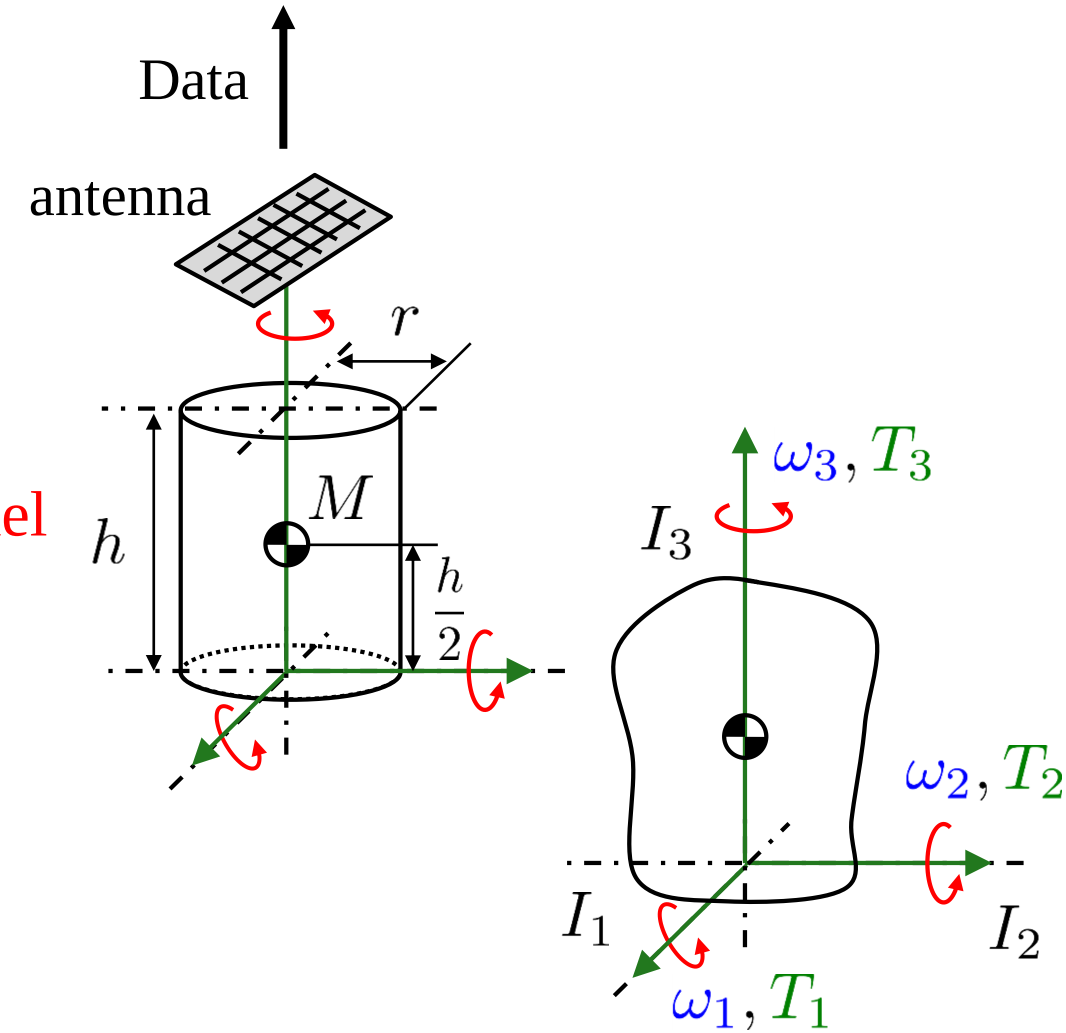
Ideal Mathematical Model

Idealization（理想化）

STEP 4.

Reduced Mathematical Model

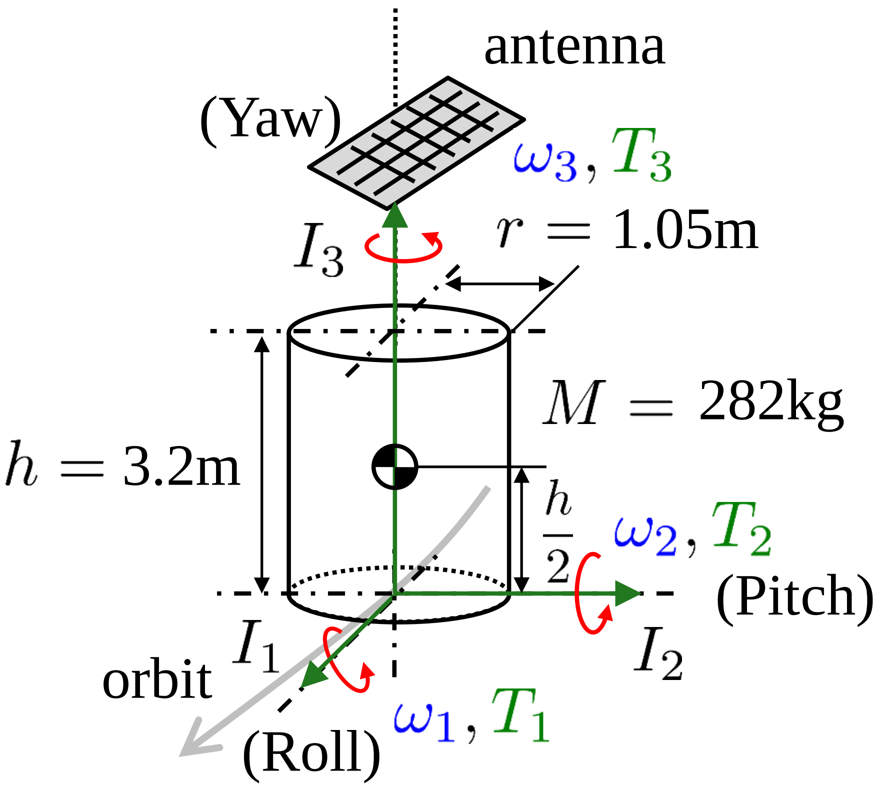
Linearization（線形化）



$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

STEP 2. Ideal Physical Model

Conceptual/Schematic model
（図式化･概念化）



Cylindrical Shape

Objective

The angular velocity (ω_1, ω_2) control of a satellite spinning about principal (yaw) axis

Attitude Control Type:

Spin-stabilization
on the principle (yaw) axis

ω_i : angular velocity
T_i : torque input
I_i : inertia

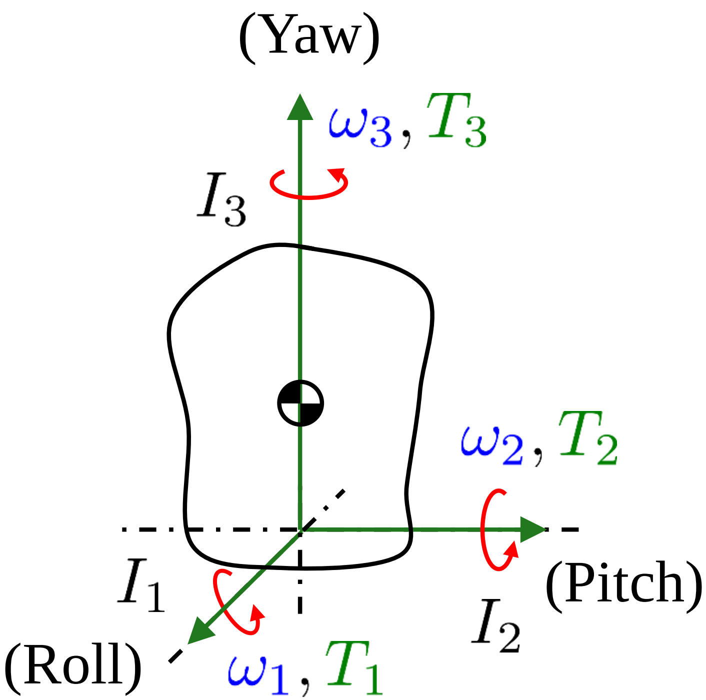
STEP 3. Ideal Mathematical Model

Idealization
（理想化）

Assumptions

- The satellite is regarded as a **rigid body**
- The satellite is **symmetric** about yaw axis

$$I_1 = I_2 = 1.04 \times 10^3 \ \text{kgm}^2$$
$$I_3 = 0.15 \times 10^3 \ \text{kgm}^2$$



Dynamics Euler's Moment Equation

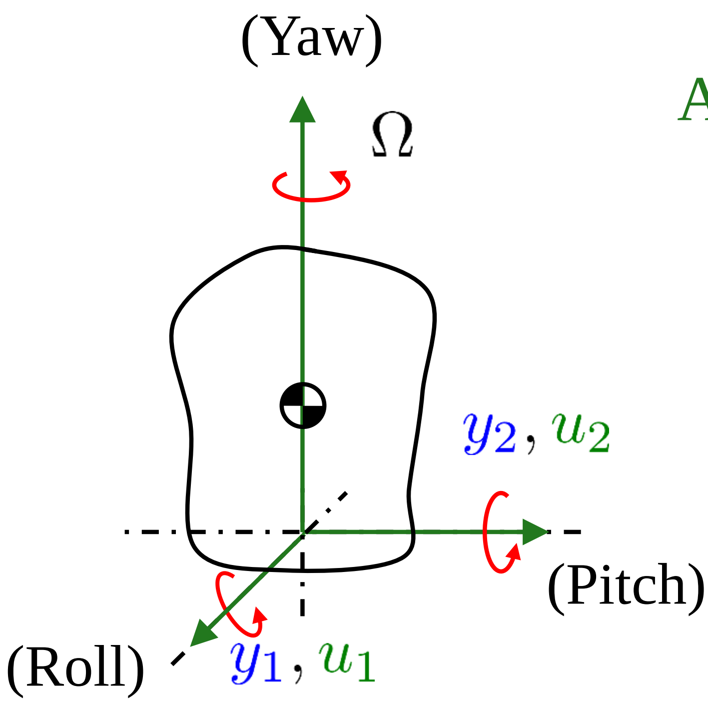
$$\boldsymbol{I} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) = \boldsymbol{T}$$

$$\Rightarrow \begin{cases} I_1\dot{\omega}_1 - \omega_2\omega_3(I_2 - I_3) = T_1 \\ I_2\dot{\omega}_2 - \omega_3\omega_1(I_3 - I_1) = T_2 \\ I_3\dot{\omega}_3 = T_3 \end{cases}$$

Nonlinear System

STEP 4. Reduced Mathematical Model

Linearization
（線形化）



Assumption

- The spin rate about yaw axis is constant

$\omega_3 = 100 \text{ rpm} = 10.46 \text{ rad/s}$ (Constant)

Control Inputs:

Scaled torque

$$u_1 := \frac{T_1}{I_1}, \quad u_2 := \frac{T_2}{I_2}$$

Measured Outputs:

Angular velocity y_1, y_2

State Space Representation

$$\begin{bmatrix} \dot{\omega}_1 \\ \dot{\omega}_2 \end{bmatrix} = \begin{bmatrix} 0 & a \\ -a & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & a \\ -a & 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}$$

Linear System, No State Feedback

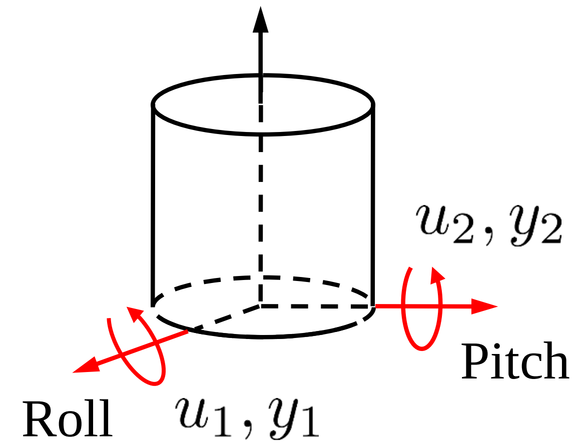
Structural Mode

$$a := \frac{I_2 - I_3}{I_1} \omega_3 \approx 10 \text{ rad/s}$$

Spinning Satellite: Plant Model [SP05, p. 98]

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{s-a^2}{s^2+a^2} & \frac{a(s+1)}{s^2+a^2} \\ \frac{-a(s+1)}{s^2+a^2} & \frac{s-a^2}{s^2+a^2} \end{bmatrix}}_{P(s) \quad (a = 10 \text{ rad/s})} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}$$

$$P(s) = C(sI - A)^{-1}B + D$$



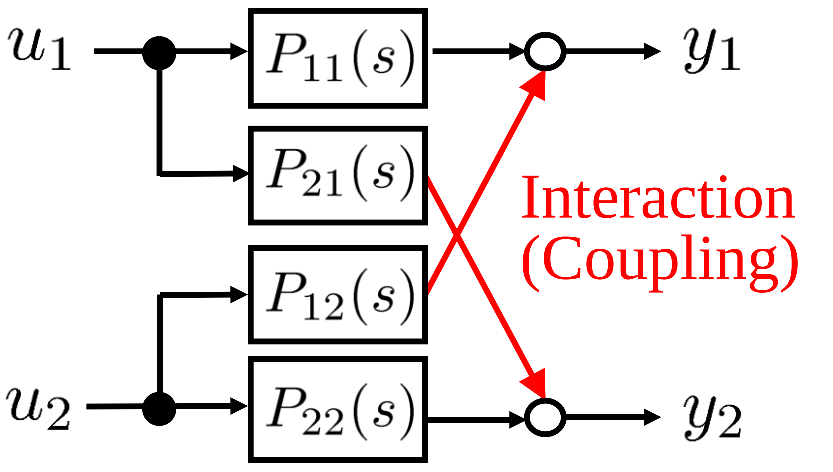
Transfer Function Matrix

$$P(s) = \begin{bmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{bmatrix}$$

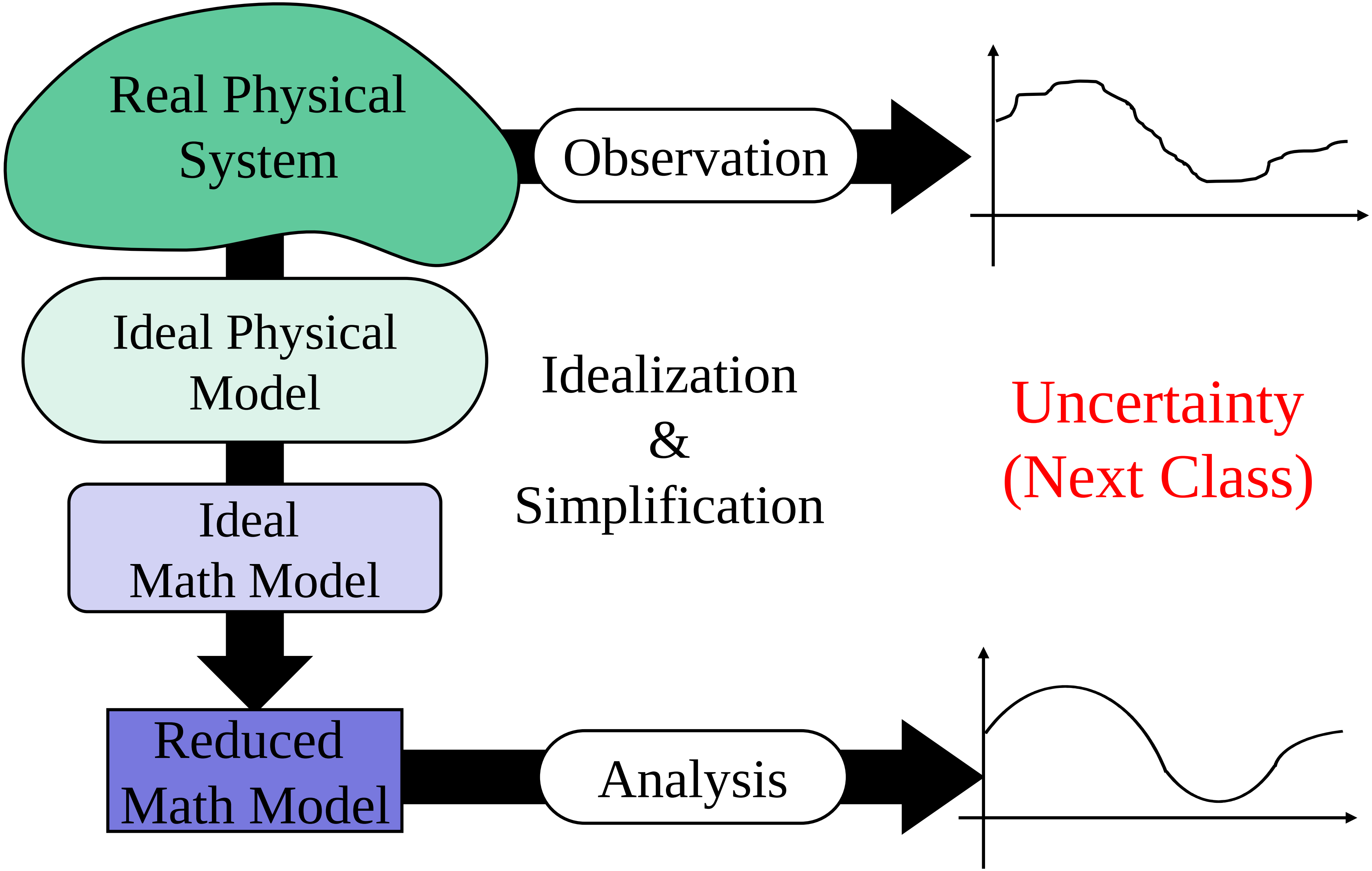
$$= \begin{bmatrix} \frac{s-100}{s^2+100} & \frac{10s+10}{s^2+100} \\ \frac{-10s-10}{s^2+100} & \frac{s-100}{s^2+100} \end{bmatrix}$$

Poles: $s = \pm 10j$

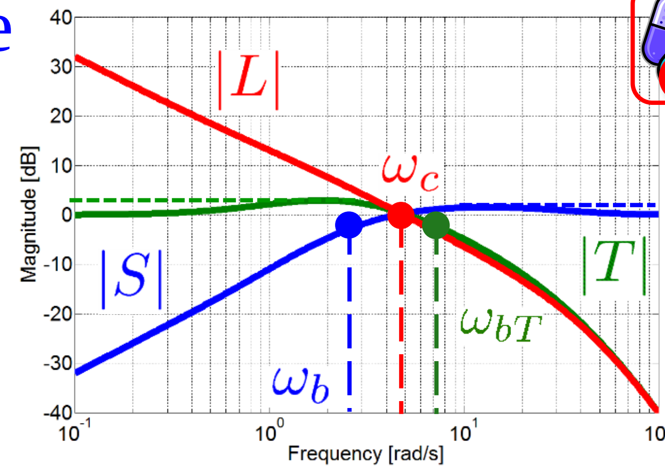
MIMO System



System and Model



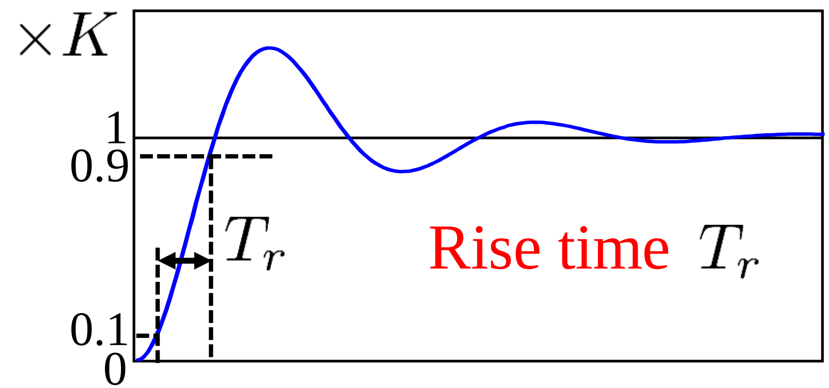
System Stabilization and Performance



Unstable Plant [SP05 Sec 5.9]

- Real RHP Poles: $2p < \omega_c$
- Imaginary Poles: $1.15|p| < \omega_c$
- Complex RHP Poles:

$$0.67(x + \sqrt{4x^2 + 3y^2}) < \omega_c$$



Stable Plant

First-order System

$$G_1(s) = \frac{K}{Ts+1} \quad K > 0 \quad T > 0$$

Rise time $T_r = (\ln 9)T \approx 2.2T$

Second-order System

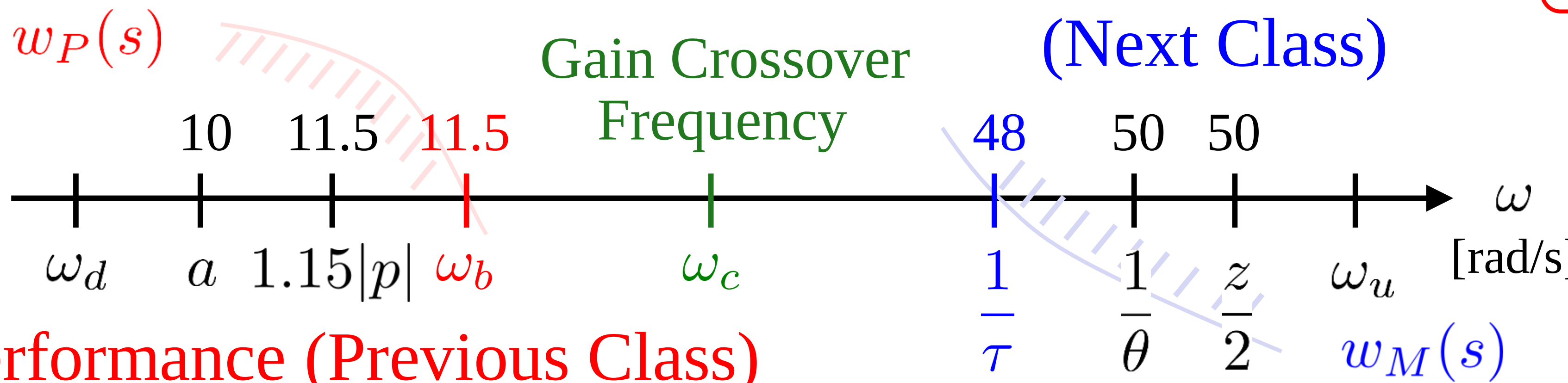
$$G_2(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \omega_n > 0 \quad \zeta \geq 0$$

Rise time $T_r = \frac{\pi/2 + \arcsin\zeta}{\omega_n\sqrt{1-\zeta^2}}$

[QZ07] L. Qiu and K. Zhou (2007) *Introduction to Feedback Control*, Prentice Hall.

Performance Specification

Uncertainty (Next Class)



Performance (Previous Class)

- Structural Mode: $a = 10$ rad/s
- Imaginary Poles:

  $p = \pm aj = \pm 10j$

  ➡ $\omega_c > 1.15|p| = 11.5$ rad/s
- Disturbance Noise: $\omega_d < \omega_c$

  Phase Stabilization

- **Performance Weight** (See 2nd lecture)

  $\omega_c > \omega_b \geq 11.5$ rad/s $\quad w_P(s)$

- Time Delay: $\theta \leq 0.02$s

  ➡ $\omega_c < 1/\theta = 50$ rad/s
- Unstable Zero: $z = a^2 = 100$

  ➡ $\omega_c < z/2 = 50$ rad/s
- Phase Lag of Plant: $\omega_u > \omega_c$

- **Uncertainty Weight** (See 4th lecture)

  $\omega_c < 1/\tau = 48$ rad/s $\quad w_M(s)$

Spinning Satellite: SISO Plant Model

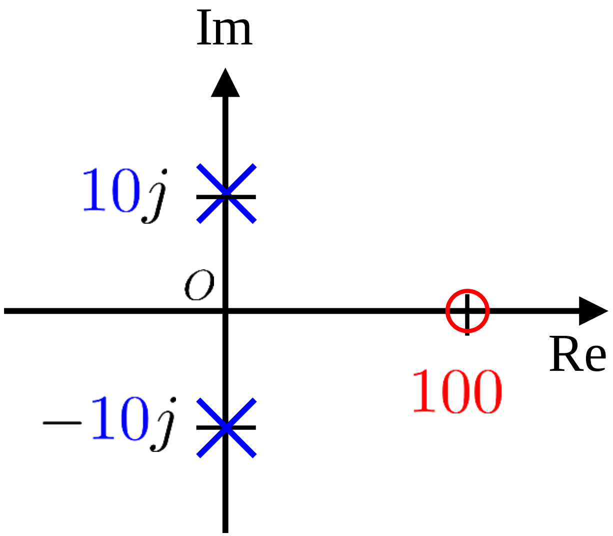
Plant $P_s(s) = \dfrac{s-100}{s^2+100}$

Poles: $s = \pm 10j$ (on the imaginary axis)

Vibratory System

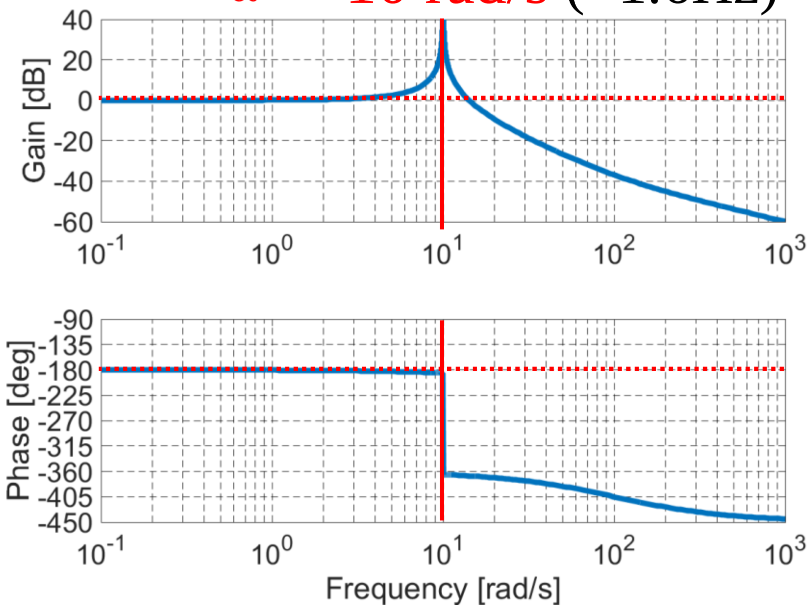
Zero: $s = 100$ (unstable zero)

Non-minimum Phase System

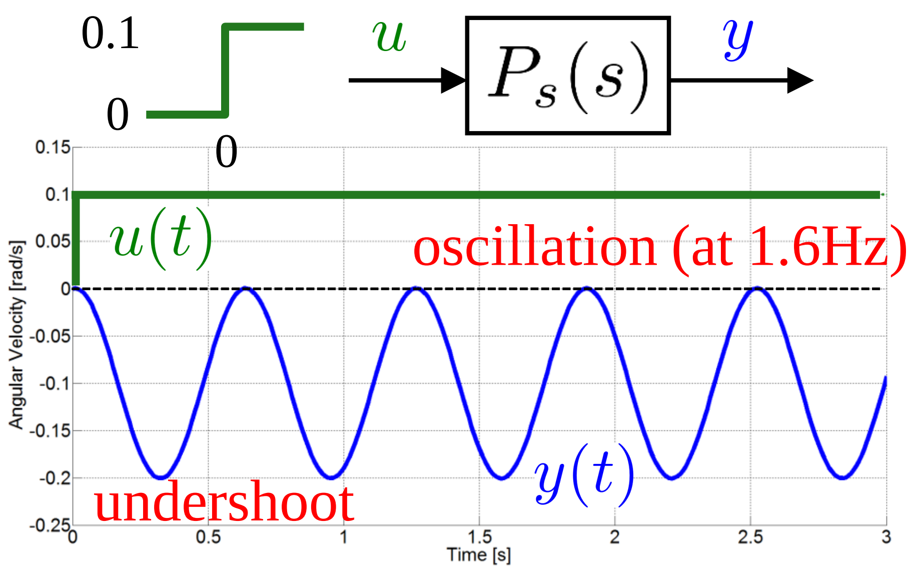


Frequency Response (Bode Plot)

$a = 10$ rad/s (=1.6Hz)



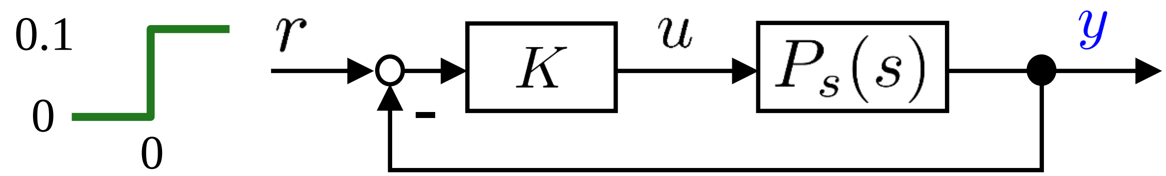
Step Response

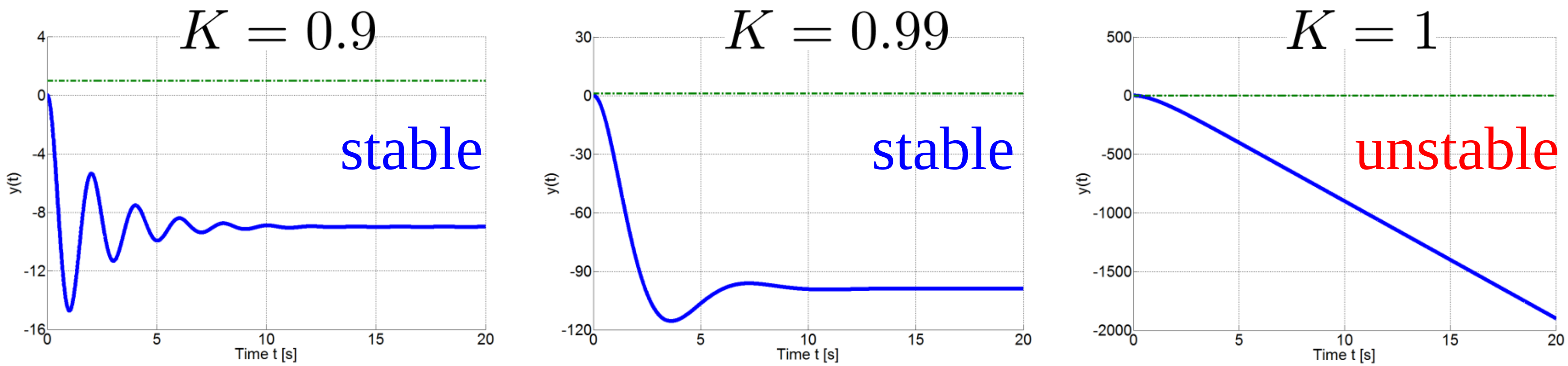


Marginally Stable

Spinning Satellite: SISO Controller Design (Z-N tuning)

Inverse response process





Ziegler-Nichols tuning [SP05, p. 29]

$K_u = 1$ K_u : Maximum (ultimate) P controller gain

$P_u =$? P_u : corresponding period of oscillations

This system has one right half-plane zero and two undamped complex poles. The process is difficult to control. … None of the standard methods for tuning PID controllers work well for this system.

[AH06] Karl J. Astrom and Tore Hagglund (2006) *Advanced PID Control*, ISA.

考えてみれば古典制御論も，合理的な筋道を通した方法を確立したとは言えない

[木村83] 木村：ロバスト制御，計測と制御，22(1) 50/52, 1983

Spinning Satellite: SISO Controller Design

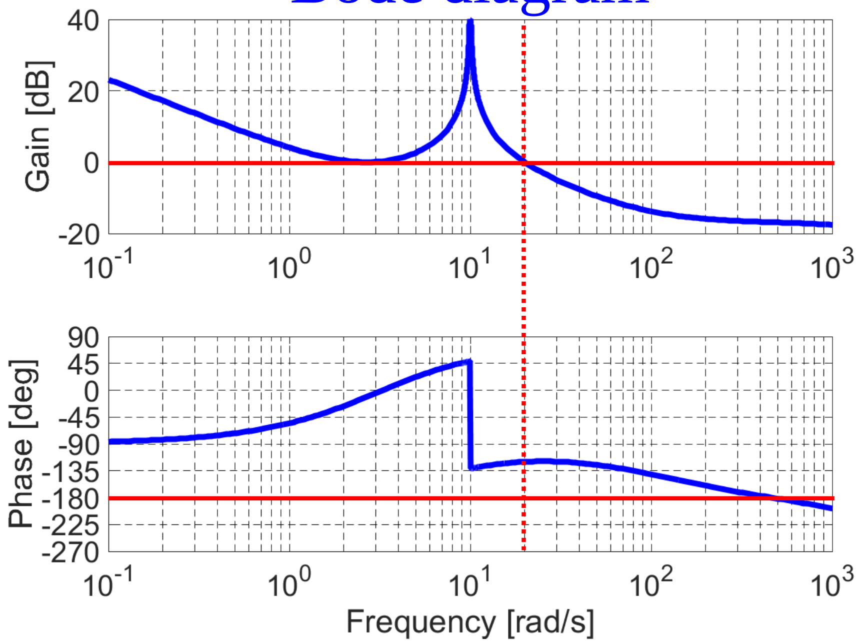
古典制御は周波数領域におけるループ整形の手法と現場調整に基づくPID制御を2本の柱にしていた[木村89]

PID tuning command: pidtune

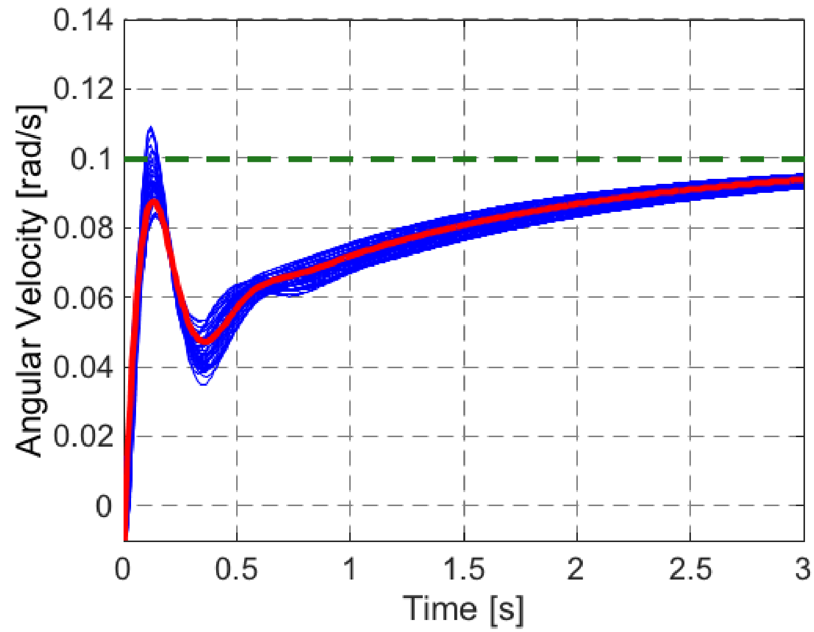
11 ~ 17

$$K_s^{\mathrm{pid}}(s) = -0.86 - \frac{1.34}{s} - 0.135s$$

Bode diagram



Step responses



$\mathrm{GM} = 16.8\mathrm{dB}$ $\omega_{pc} = 462\mathrm{rad/s}$

$\mathrm{PM} = 60\mathrm{deg}$ $\omega_c = 20.0\mathrm{rad/s}$

Nominal Plant Model

Perturbed Plant Model

この補償法は決して体系的なものではなく，今から思えば使いやすいものでもない

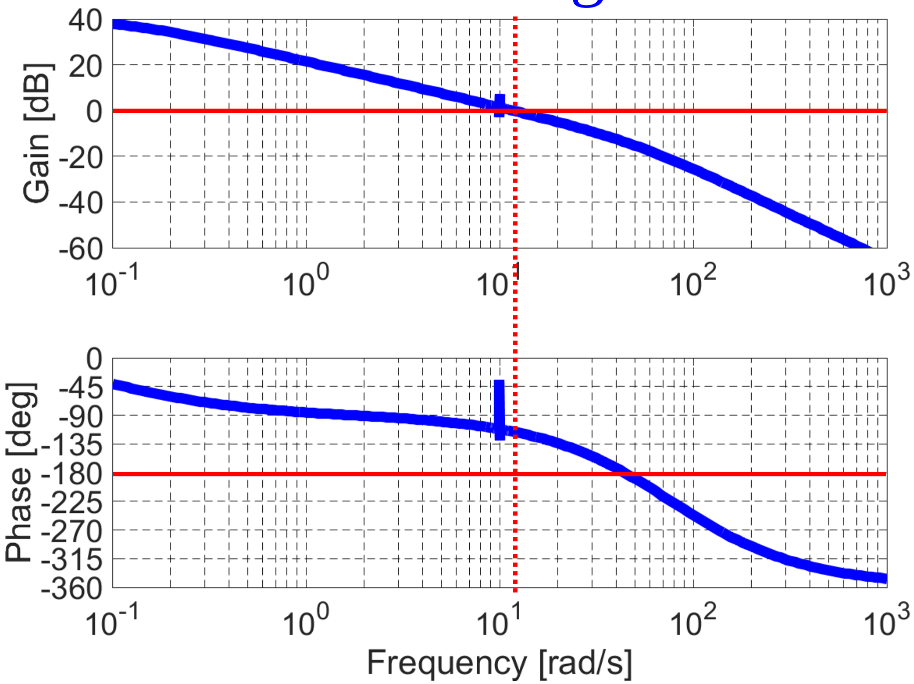
Spinning Satellite: SISO Controller Design

(See 6th lecture) 11~17

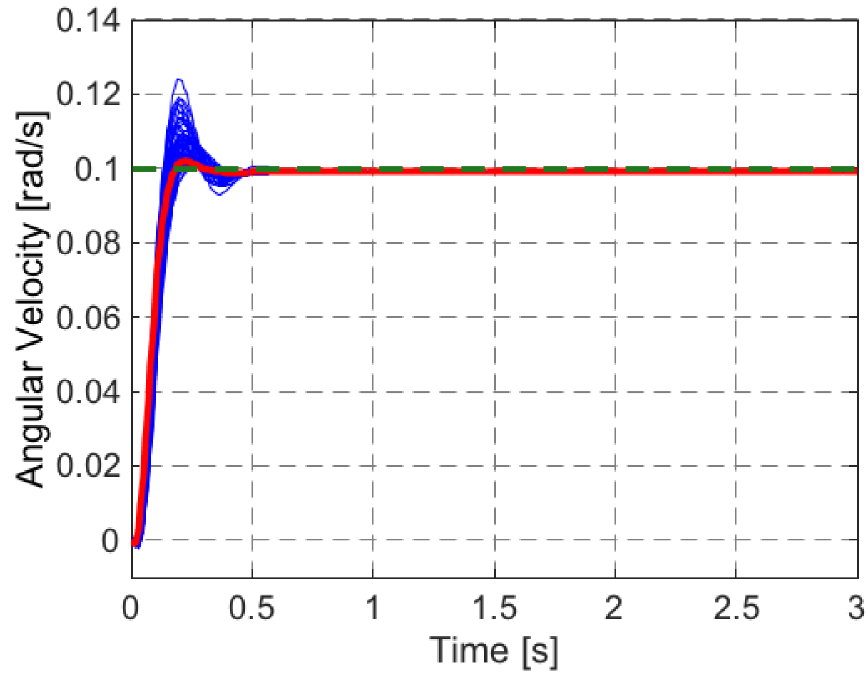
H_2 Controller

$$K_s^2(s) = \frac{-542.44(s^2 + 0.0006864s + 99.99)}{(s+0.115)(s^2 + 134.9s + 4558)}$$ (Reduced Order 3)

Bode diagram



Step responses



$\mathrm{GM} = 13.7\mathrm{dB}$ $\quad \omega_{pc} = 44.1\mathrm{rad/s}$

$\mathrm{PM} = 64.4\mathrm{deg}$ $\quad \omega_c = 11.6\mathrm{rad/s}$

— Nominal Plant Model

— Perturbed Plant Model

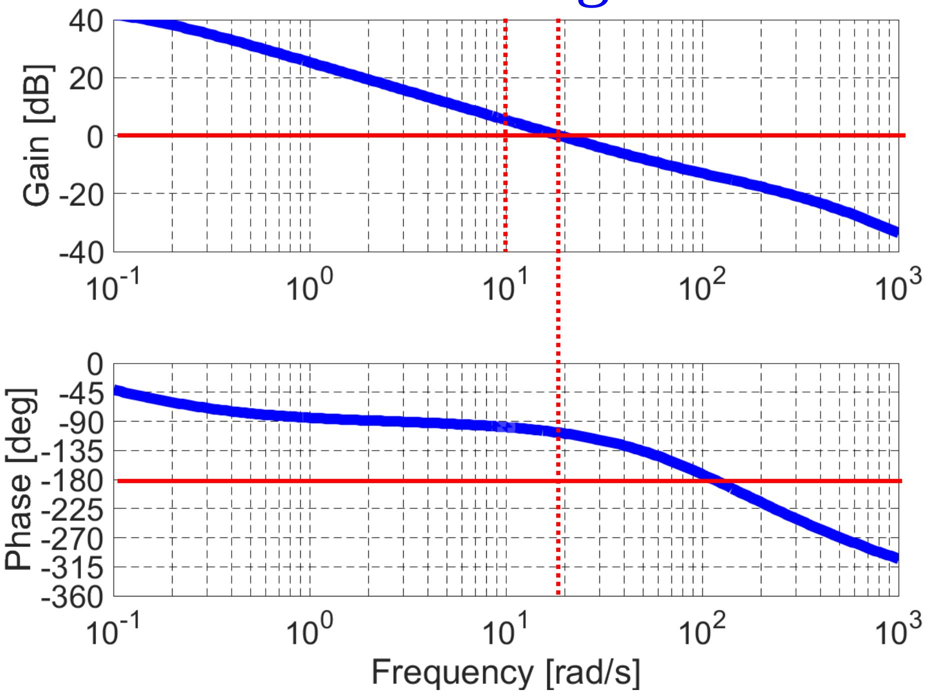
Spinning Satellite: SISO Controller Design

H_∞ Controller

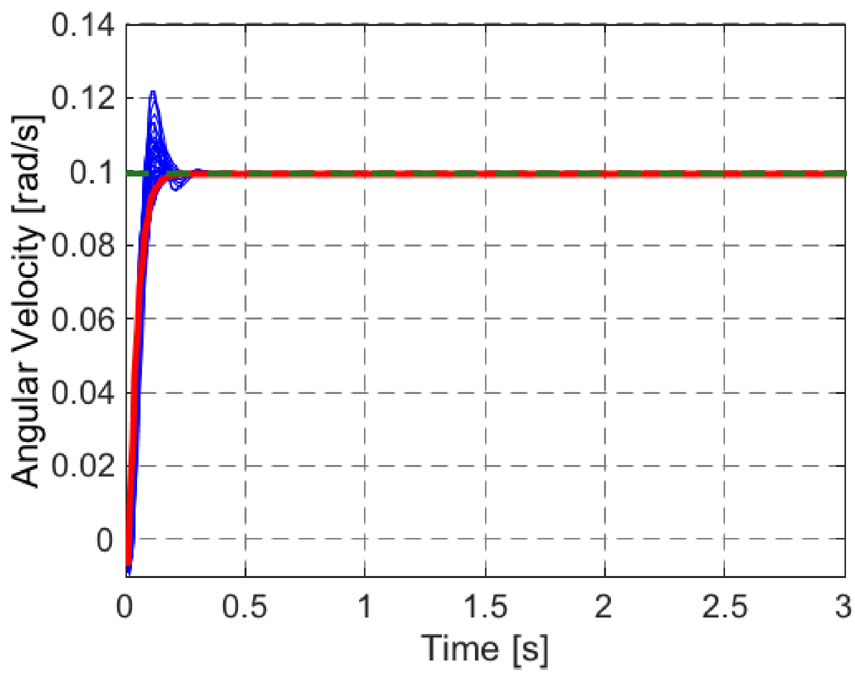
(See 6th lecture)

$$K_s^\infty(s) = \frac{-28210(s^2+100)}{(s+897.3)(s+169.7)(s+0.1136)}$$ (Reduced Order 3)

Bode diagram



Step responses



$\mathrm{GM} = 13.9\mathrm{dB}$ $\omega_{pc} = 114\mathrm{rad/s}$

$\mathrm{PM} = 72.2\mathrm{deg}$ $\omega_c = 18.7\mathrm{rad/s}$

Nominal Plant Model

Perturbed Plant Model

3. SISO Loop Shaping

3.1 Computer Controlled System

3.2 Modeling

[SP05, Sec. 3.7, 1.4, 1.5]

3.3 Example

[SP05, Sec. 2.6, 5.6, 5.7, 5.9]

Reference:

[SP05] S. Skogestad and I. Postlethwaite, ***Multivariable Feedback Control; Analysis and Design,*** Second Edition, Wiley, 2005.

4. Robustness and Uncertainty

4.1 Why Robustness? [SP05, Sec. 4.1.1, 7.1, 9.2]

4.2 Representing Uncertainty [SP05, Sec. 7.2, 7.3, 7.4]

4.3 Uncertain Systems [SP05, Sec. 8.1, 8.2, 8.3]

4.4 Systems with Structured Uncertainty

[SP05, Sec. 8.2]

Reference:

[SP05] S. Skogestad and I. Postlethwaite, ***Multivariable Feedback Control; Analysis and Design,*** Second Edition, Wiley, 2005.

Scaling [SP05, pp. 5-7]

Amplitude Scaling $x' = S_x x \; , \quad x'_i \in [-1, 1]$ Normalization

A method for accomplishing the best scaling for a complex system is first to estimate the maximum values for each system variable and then to scale the system so that each variable varies between -1 and 1

Ex. $P = D_y^{-1} \hat{P} D_u$

$$D_u := \mathrm{diag}(\hat{u}_1^{\max}, \hat{u}_2^{\max}) \; , \quad D_y := \mathrm{diag}(\hat{y}_1^{\max}, \hat{y}_2^{\max})$$

$$u_i = \hat{u}_i / \hat{u}_i^{\max} \; , \; u_i \in [-1, 1] \; , \; i = 1, 2$$

$$y_i = \hat{y}_i / \hat{y}_i^{\max} \; , \; y_i \in [-1, 1] \; , \; i = 1, 2$$

Spinning Satellite $\hat{u}_i^{\max} = 1 \times 10^{-4} \;\mathrm{N/kgm} \quad \hat{y}_i^{\max} = 0.26 \;\mathrm{rad/s}$

Time Scaling $\tau = \omega_0 t$

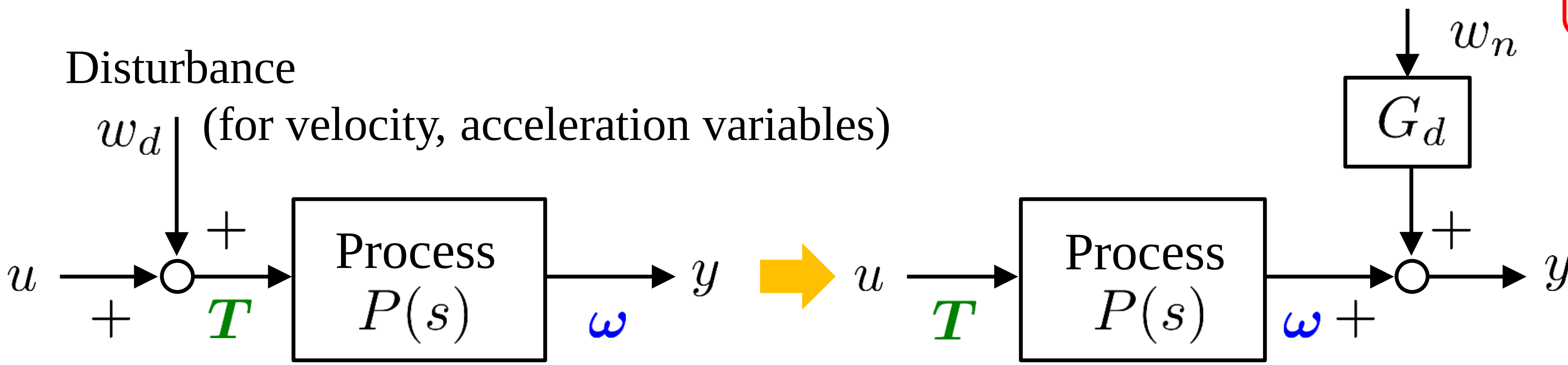
Ex. t : measured in seconds τ : measured in milliseconds

$\omega_0 = 1000$: scaling coefficient

Ex. $$\dot{x} = \frac{dx}{dt} = \frac{dx}{d(\tau/\omega_0)} = \omega_0 \frac{dx}{d\tau} \; , \quad \ddot{x} = \frac{d^2 x}{dt^2} = \omega_0^2 \frac{dx}{d\tau^2}$$

[FPN09] G.F. Franklin, J.D. Powell and A. E.-Naeini (2009) *Feedback Control of Dynamic Systems*, Sixth Edition, Prentice Hall.

Process Noise Model



$$\dot{x} = A(x + w_d) + Bu \qquad \dot{x} = Ax + Bu + A_d w_d$$
$$y = Cx + Du + w_n \qquad y = Cx + Du + w_n$$

- Process Noise Model $G_d(s) = C(sI - A)^{-1} A_d$
- Process Noise

It is assumed that the measurement noise inputs w_n and disturbance signals (process noise) w_d are stochastic with known statistical properties. These noises are usually assumed to be uncorrelated zero-mean Gaussian stochastic processes with constant power spectral density matrices V and W respectively.

$$E\{w_d(t)w_d(t)^T\} = W\delta(t - \tau) \qquad E\{w_d(t)w_n(t)^T\} = 0$$
$$E\{w_n(t)w_n(t)^T\} = V\delta(t - \tau) \qquad E\{w_n(t)w_d(t)^T\} = 0$$

Sensors

Primal-sensor: Gyroscope

These sensors are based on the gyroscopic stiffness of revolving moments of inertia

Single-Axis Gyroscope

Assumption: "Synchro", "Torque": small

$$\begin{bmatrix} \Delta\dot{\theta}_y \\ \Delta\dot{\omega}_y \end{bmatrix} = \begin{bmatrix} \Delta\omega_y \\ (T_x + T_y)/I_y \end{bmatrix}$$

Torques $T_x = H_r \Delta\omega_x$

$$T_y = k_{\theta_y}\Delta\theta_y + k_{\omega_y}\Delta\omega_y$$

H_r : Spin-rotor Angular momentum

Medium-accuracy RIG

Assumption

The sensor measurements are poorly aligned with the axis of rotation being measured

Rate gyro(RG)

Analogous to a mechanical **spring** restraint

$$T_y = k_{\theta_y}\Delta\theta_y$$

$$\Delta\dot{\omega}_y = 0 \Rightarrow \Delta\theta_y = -\frac{H_r}{k_{\theta_y}}\Delta\omega_x$$

Rate-integrating gyro(RIG)

Analogous to a mechanical **damper** restraint

$$T_y = k_{\omega_y}\Delta\omega_y$$

$$\Delta\dot{\omega}_y = 0$$

$$\Rightarrow \Delta\omega_y = -\frac{H_r}{k_{\omega_y}}\Delta\omega_x$$

$$\Rightarrow \Delta\theta_y = -k_{xy}\Delta\theta_x$$

Sub-sensors

Earth sensor, sun sensor

[Si97] M.J. Sidi (1997) *Spacecraft Dynamics and Control: A Practical Engineering Approach*, Cambridge University Press.

Actuators

Primal Actuator: Reaction Wheel

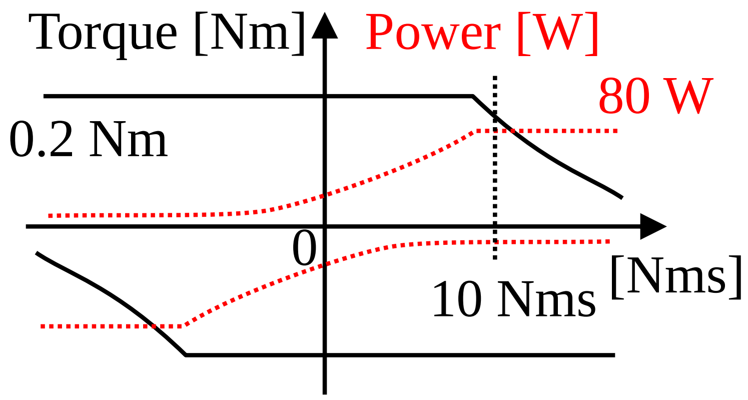
Four reaction wheels [Si97, pp. 167-172]

A Fourth RWA(reaction wheel assembly) is installed in order to increase the reliability the entire control system.

Kinematics

$$\begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} T_{i1} \\ T_{i2} \\ T_{i3} \\ T_{i4} \end{bmatrix}$$

Torque-momentum limitations



Angular momentum commands

$$\Delta T_{ij} = \Delta h_w K_j, \ \ j = 1, 2, 3, 4$$

$$\Delta h_w(s) = \frac{\omega_{w1}(0)}{s + 4K/I_w} \quad \text{[Nms]}$$

Disturbance torques Solar pressure torque, Gravity-gradient torque
Aerodynamic torque, Magnetic-field torque

Sub-actuator: Thruster

Switching time: 20-40ms (25-50Hz)

[FPN09] G.F. Franklin *et al.*(2009) *Feedback Control of Dynamic Systems*, 6th Ed., Prentice Hall.
[Si97] M.J. Sidi (1997) *Spacecraft Dynamics and Control*, Cambridge University Press.

Performance Specification

- System Bandwidth: $f_c = 5\text{Hz}$
- Sampling Time θ [s]

$$\frac{1}{40f_c} < \theta < \frac{1}{10f_c} \quad \text{[Le10]}$$

$$0.005 < \theta < 0.02$$

➡ Time Delay Variation

$$0 \leq \theta \leq 0.02\text{s}$$

- Uncertain Gain

  caused by process noise and sensor measurements

$$0.8 \leq k \leq 1.2$$

(20% variation, $GM \geq 2\text{dB}$)

- Other requirements:

  Phase margin $PM \geq$ 30deg

  (A direct safeguard against time delay uncertainty)

  Gain margin $GM \geq 2$ (=6dB)

  (A direct safeguard against steady-state gain uncertainty/error)

  Maximum peak gain of T :

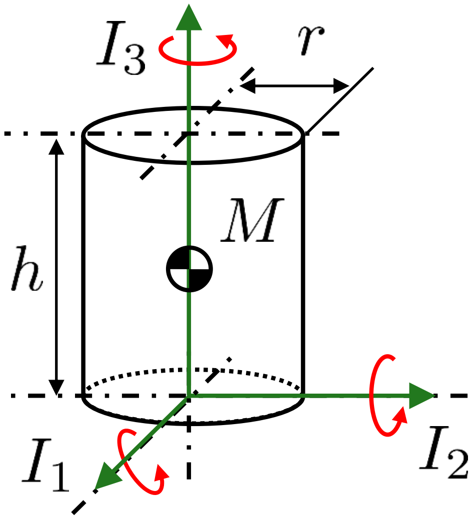
$$M_T \lessapprox 1.25(=2\text{dB})$$

  Maximum peak gain of S :

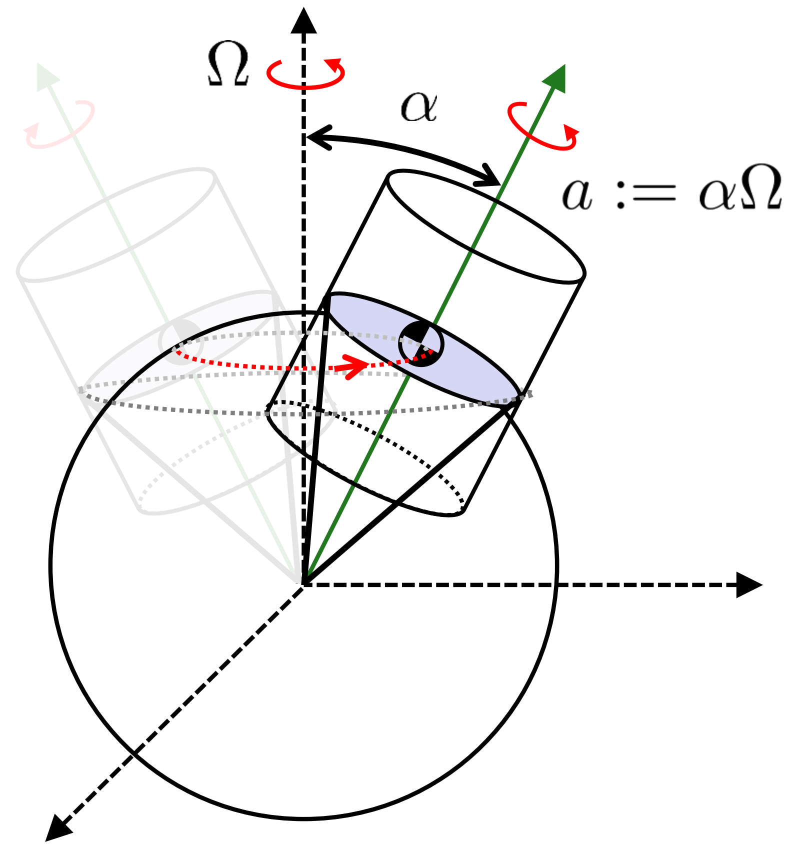
$$M_S \lessapprox 2(=6\text{dB})$$

[Le10] W.S. Levine (Eds.) (2010) *The Control Handbook, Second Edition: Control System Fundamentals, Second Edition*, CRC Press.

Characteristics of Rotational Motion of a Spinning Body



Principal Axis of Inertia



$$M = 130\text{kg} \sim 1100\text{kg}$$

$$r = 0.7\text{m} \sim 1.1\text{m}$$

$$h = 1.5\text{m} \sim 3.2\text{m}$$

$$\Omega = 90\text{rpm} \quad \text{or} \quad 100\text{rpm}$$

$$I_1 = I_2 = \frac{M(3r^2 + h^2)}{12} + \frac{Mh^2}{4}$$

$$I_3 = \frac{1}{2}Mr^2$$

MOIR: moment of inertia ratio $\sigma = \dfrac{I_3}{I_1}$

Nutation angle $\alpha = 1 - \sigma$

Precession angle β

Spin angle $\gamma \quad (\Omega = \dot{\gamma})$

Rigid Body Attitude Configurations

Attitude Representations	Global?	Unique?
Euler angles	×	×
Rodrigues parameters	×	×
Modified Rodrigues parameters	×	×
Quaternions,	○	×
Axis-angle	○	×
Rotation matrix	○	○

Rotation matrix $R \in SO(3)$ The set of all rotation matrices

$SO(3)$ Special orthogonal group of rigid rotations in $\mathbb{R}^3$

Kinematics $\dot{R} = R\,\mathrm{sk}(\omega)$ $\quad \mathrm{sk}(\omega) = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}$

$\omega \in \mathbb{R}^3$ The angular velocity of the body relative to the reference frame

Dynamics $\boldsymbol{I} \cdot \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\boldsymbol{I} \cdot \boldsymbol{\omega}) = \boldsymbol{T}$ (Euler's moment equations)

[CSC11] N.A. Chaturvedi, A.K. Sanyal and N.H. McClamroch. Rigid-body Attitude Control, *IEEE Control Systems Magazine*, **31**(3) 30-51, 2011.

Design Relations

Maximum Peak Magnitude of T

$$M_T \simeq \frac{1}{2\sin(PM/2)}$$

Phase Margin

$$PM = \tan^{-1}\left[\frac{2\zeta}{\sqrt{\sqrt{1+4\zeta^4}-2\zeta^2}}\right]$$

$$\approx 100\zeta \quad (< 70^\circ)$$

Bandwidth

$$\omega_c \le \omega_{bT} \le 2\omega_c$$

$$\omega_{bT} = \omega_c \quad \text{if} \quad PM = 90^\circ$$

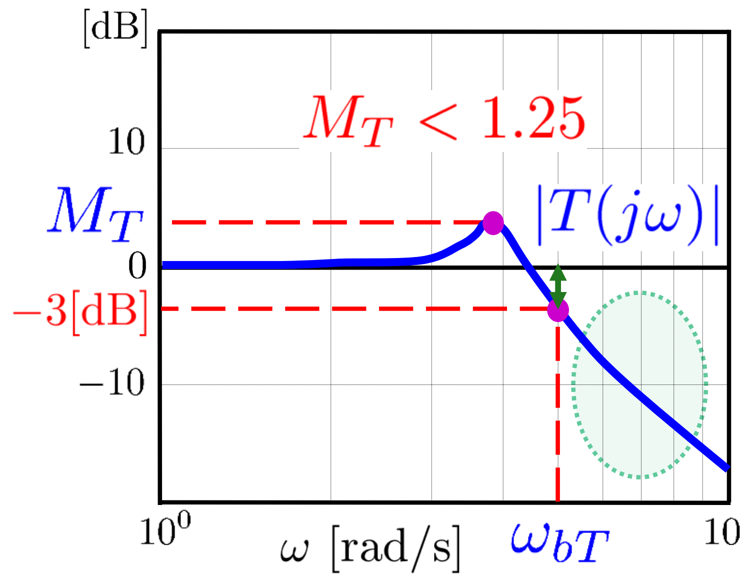
$$\omega_{bT} \simeq 2\omega_c \quad \text{if} \quad PM \le 45^\circ$$

$$0 < \omega_b \le \omega_c$$

$$\omega_b = \omega_c \quad \text{if} \quad PM = 90^\circ$$

Complementary Sensitivity

$$T = \frac{PK}{1+PK}$$



M_T : Maximum Peak Magnitude of T

$$M_T = \max_{\omega} |T(j\omega)| < 1.25 \ (2 \text{ dB})$$

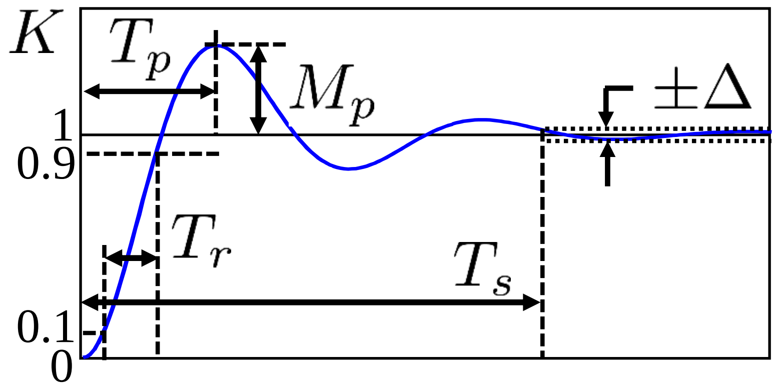
ω_{bT} : Bandwidth Frequency of T

$$|T(j\omega_{bT})| = \frac{1}{\sqrt{2}} \quad (-3 \text{ dB})$$

[FPN09] G.F. Franklin, J.D. Powell and A. E.-Naeini (2009) *Feedback Control of Dynamic Systems*, Sixth Edition, Prentice Hall.

Step response analysis/Performance criteria

- Rise time T_r
- Settling time T_s
- Peak time T_p
- Overshoot M_p
- Error tolerance Δ



$K > 0$

First-order System

$$G_1(s) = \frac{K}{Ts+1} \quad T > 0$$

Rise time

$$T_r = (\ln 9)T \approx 2.2T$$

Settling time

$$T_s \approx \begin{cases} 3T & \text{if } \Delta = 5\% \\ 4T & \text{if } \Delta = 2\% \end{cases}$$

Overshoot

$$M_p = 0$$

Second-order System

$$G_2(s) = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \omega_n > 0, \ \zeta \geq 0$$

Rise time $T_r = \frac{\pi/2+\arcsin\zeta}{\omega_n\sqrt{1-\zeta^2}}$

Settling time

$$T_s \approx \begin{cases} 3/\zeta\omega_n & \text{if } \Delta = 5\% \\ 4/\zeta\omega_n & \text{if } \Delta = 2\% \end{cases}$$

Overshoot $M_p = Ke^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Peak Time $T_p = \pi/(\omega_n\sqrt{1-\zeta^2})$

[QZ07] L. Qiu and K. Zhou (2007) *Introduction to Feedback Control*, Prentice Hall.

Controllability analysis with SISO feedback control

[SP05, pp. 206-209]

M_1 Margin to stay within constraints $|u| < 1$

M_2 Margin for performance $|e| < 1$

M_3 Margin because of RHP-pole p

➡ $2p < \omega_c$

M_4 Margin because of RHP-zero z

➡ $\omega_c < z/2$

M_5 Margin because of frequency ω_u where plant has -180° phase lag

➡ $\omega_c < \omega_u$

M_6 Margin because of delay θ

➡ $\omega_c < 1/\theta$

Typically, the closed-loop bandwidth of the spacecraft is an order of magnitude less than the lowest mode frequency, and as long as the controller does not excite any of the flexible modes, the sampling period may be selected solely based on the closed-loop bandwidth.

[Le10] W.S. Levine (Eds.) (2010) *The Control Handbook, Second Edition: Control System Fundamentals, Second Edition*, CRC Press.

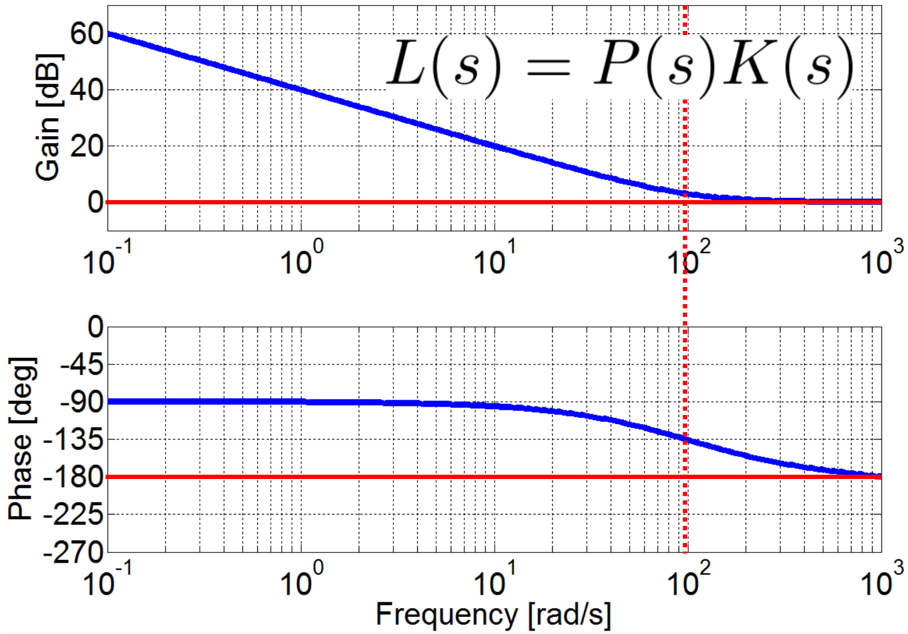
Spinning Satellite: Try SISO Controller Design

Internal Model Controller(IMC) design [SP05, p. 55, Ex. 2.13]

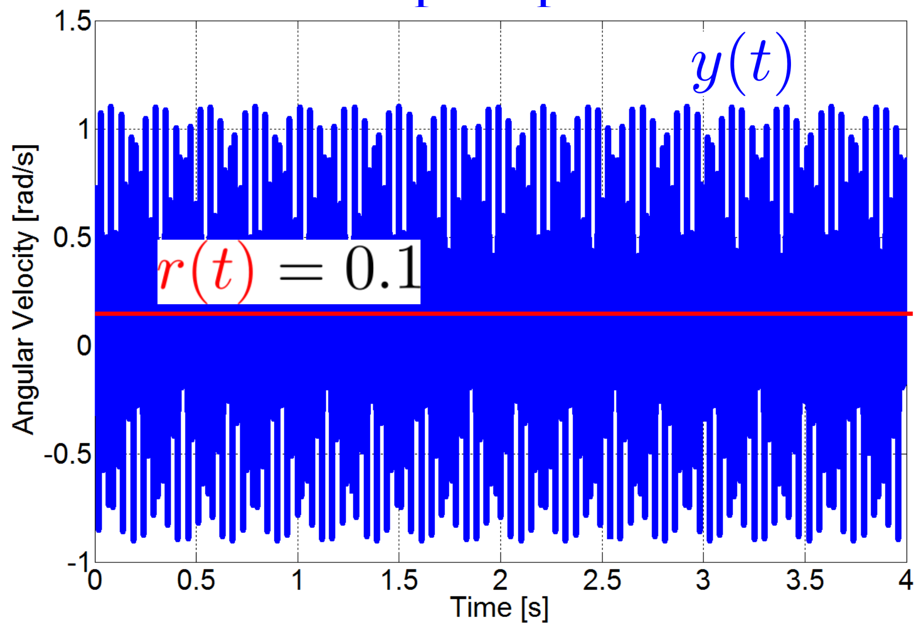
Plant: $P(s) = k\dfrac{1-\theta s}{\tau_0^2 s^2 + 2\tau_0 \zeta s + 1}$ $\quad k = -1 \quad \theta = 0.01 \quad \tau_0 = 0.1 \quad \zeta = 0$

PID: $K(s) = \dfrac{\tau_0^2 s^2 + 2\tau_0 \zeta s + 1}{k\theta s} \simeq -\dfrac{100}{s} - \dfrac{s}{0.001s+1}$

Bode diagram



Step responses



$\mathrm{GM} = 0\mathrm{dB} \quad \omega_{pc} = \infty \quad \mathrm{PM} = 0\mathrm{deg} \quad \omega_{gc} = \infty$

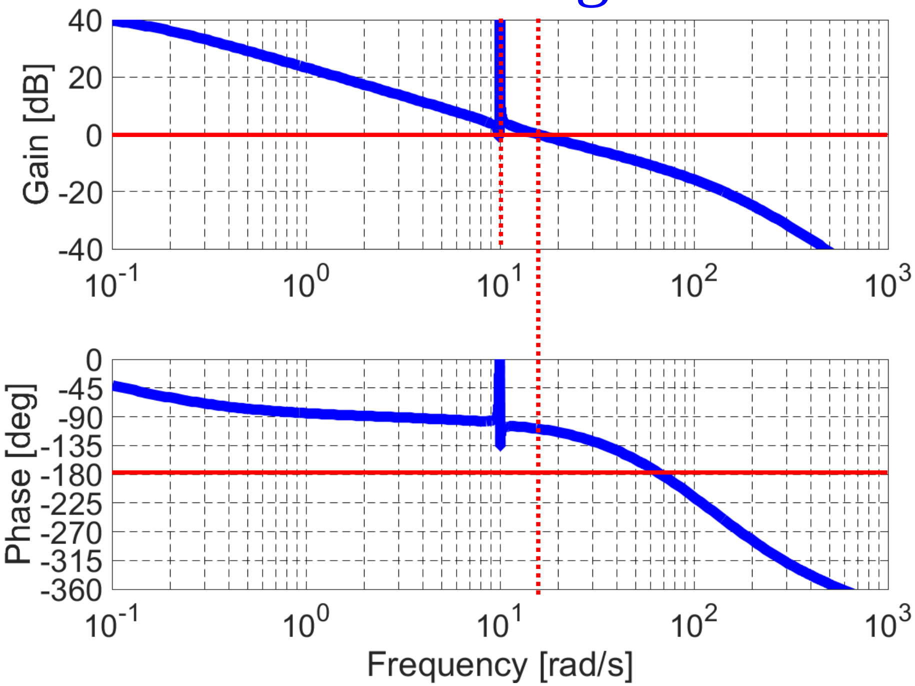
cf. Skogestad's Internal Model Controller(SIMC) [SP05, p. 57]

Spinning Satellite: Try SISO Controller Design

μ Controller (Reduced Order 6)

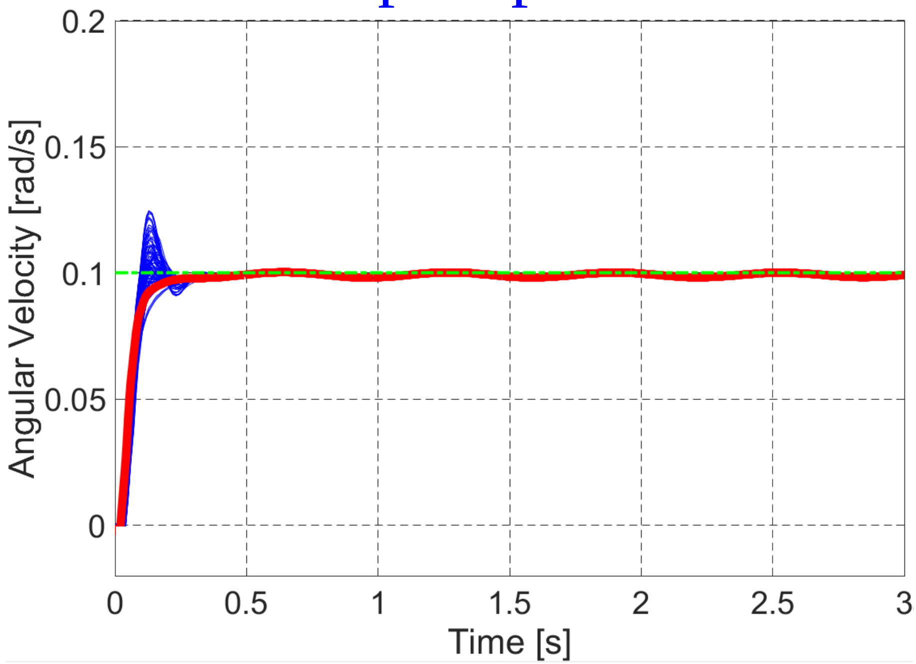
$$K_s^{\mu}(s) = \frac{-377.83(s+5160)(s^2+0.1154s+98.39)(s^2+73.21s+1702)}{(s+702.9)(s+0.115)(s^2+70.57s+1855)(s^2+235.2s+17040)}$$

Bode diagram



$$\mathrm{GM} = 11.9\mathrm{dB}$$
$$\mathrm{PM} = 71.1\mathrm{deg}$$
$$\omega_{pc} = 67.4\mathrm{rad/s}$$
$$\omega_{gc} = 15.7\mathrm{rad/s}$$

Step responses

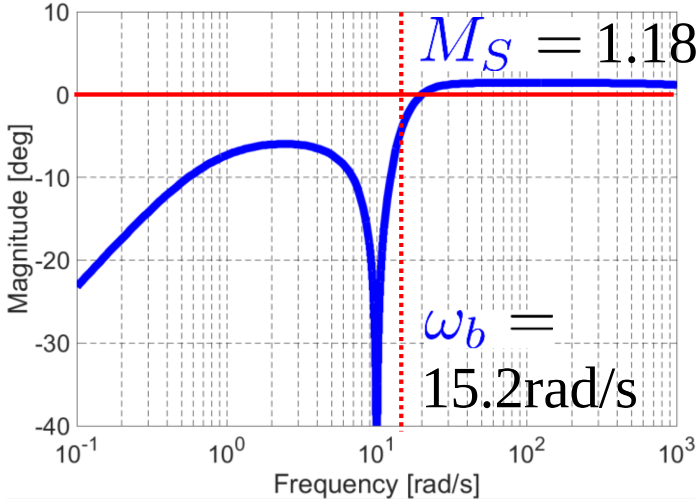


Nominal Plant Model
Perturbed Plant Model

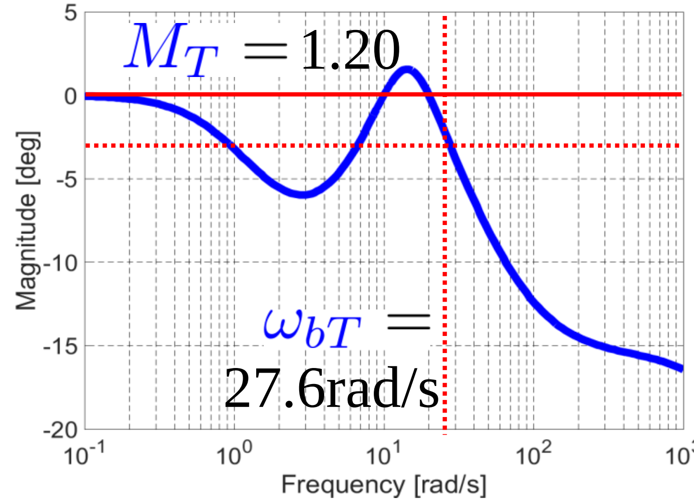
Spinning Satellite: Evaluate SISO Controller Design

PID compensator Gang of Four [AM08, p. 317]

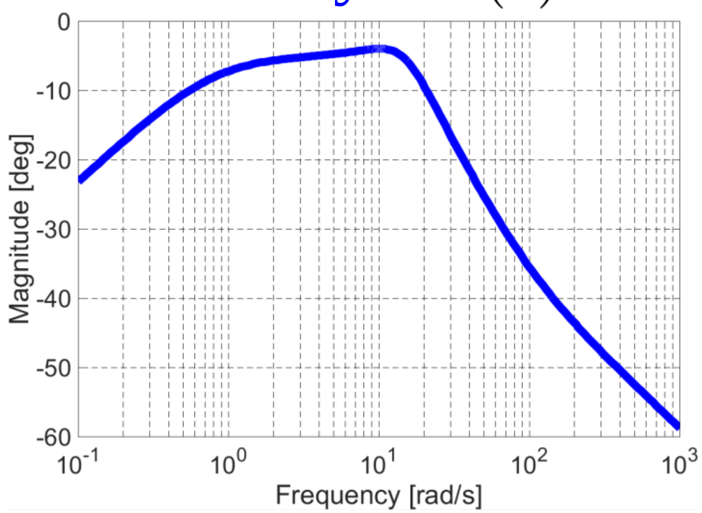
Sensitivity $S(s)$



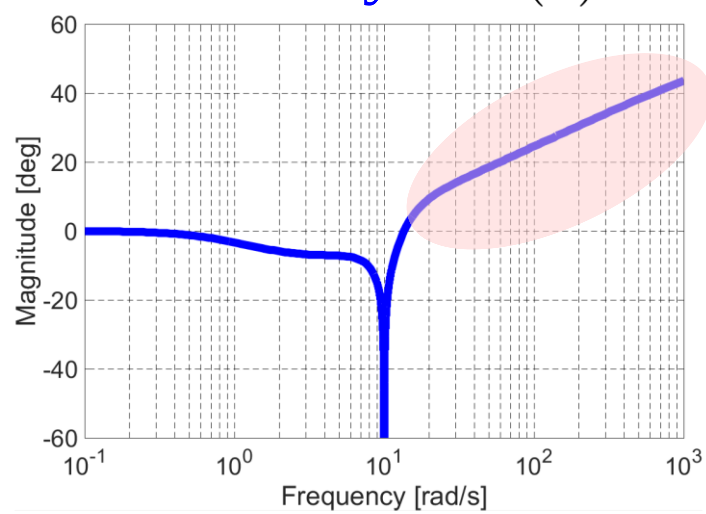
Complementary Sensitivity $T(s)$



Load Sensitivity $PS(s)$



Noise Sensitivity $KS(s)$

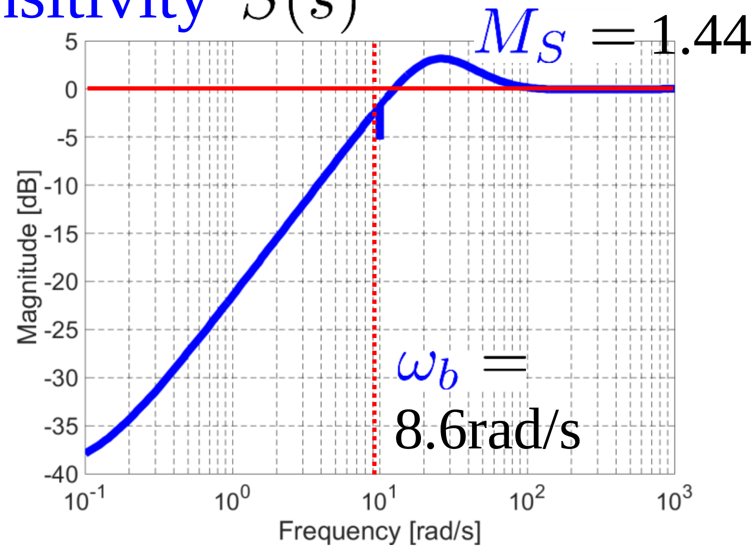


[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008

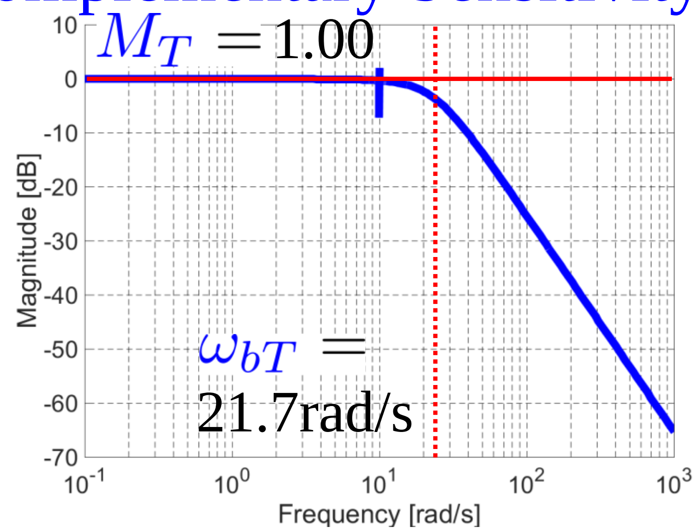
Spinning Satellite: Evaluate SISO Controller Design

H_2 -controller Gang of Four [AM08, p. 317]

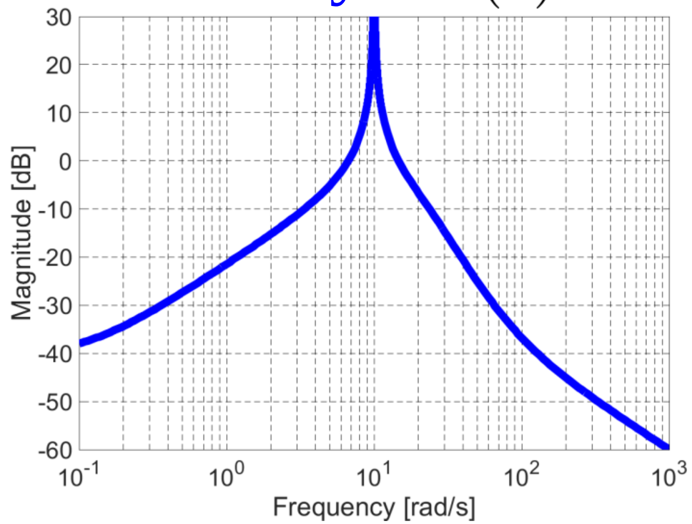
Sensitivity $S(s)$



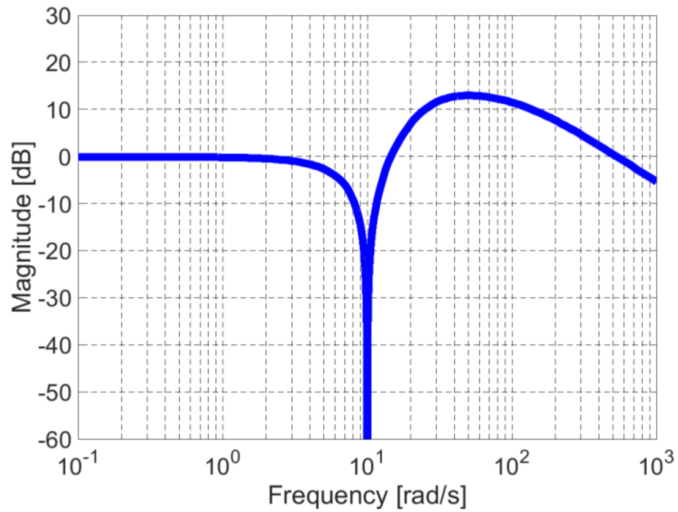
Complementary Sensitivity $T(s)$



Load Sensitivity $PS(s)$



Noise Sensitivity $KS(s)$

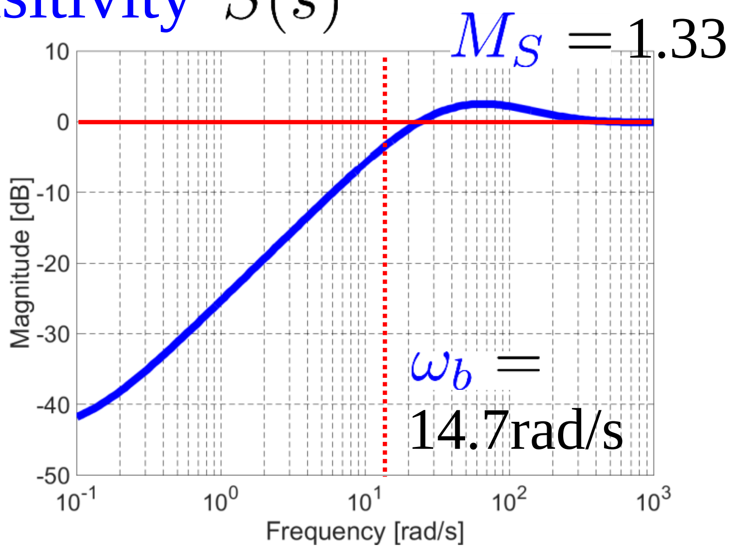


[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008

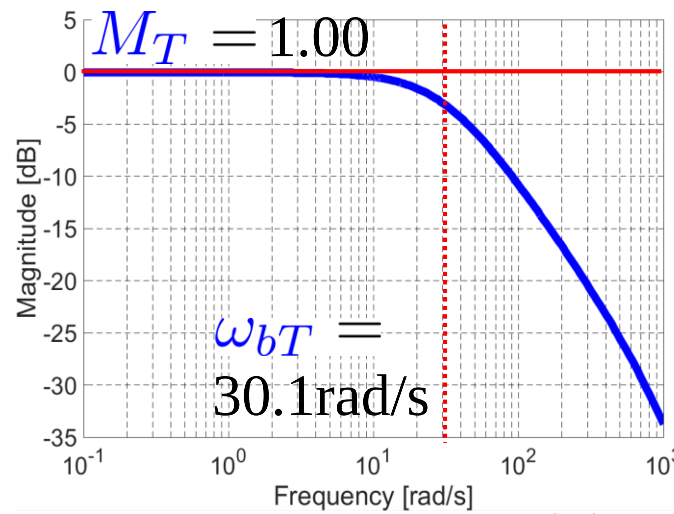
Spinning Satellite: Evaluate SISO Controller Design

H_∞ -controller **Gang of Four** [AM08, p. 317]

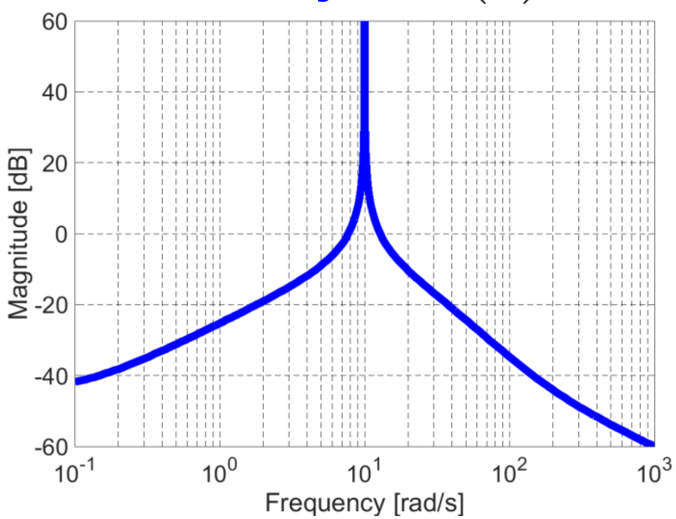
Sensitivity $S(s)$



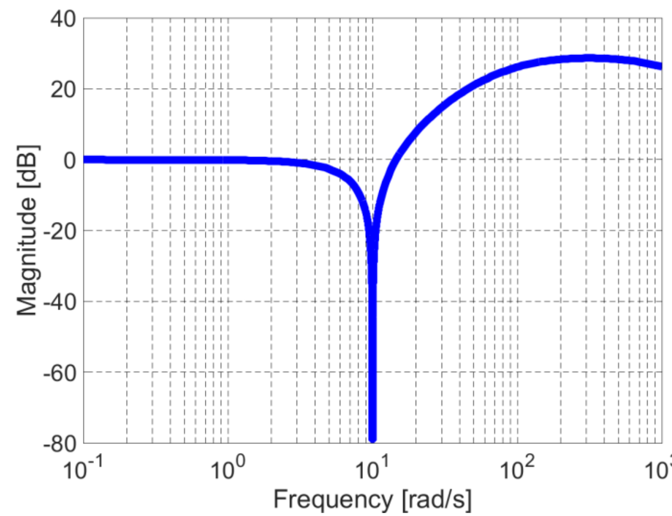
Complementary Sensitivity $T(s)$



Load Sensitivity $PS(s)$



Noise Sensitivity $KS(s)$



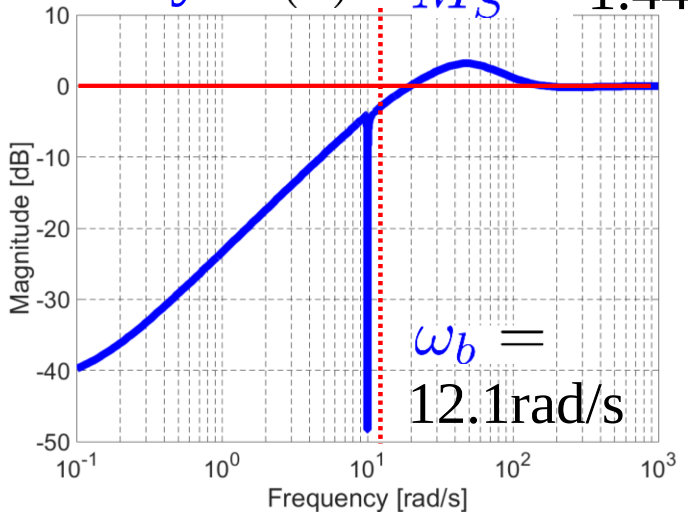
[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008

Spinning Satellite: Evaluate SISO Controller Design

μ -synthesis **Gang of Four** [AM08, p. 317]

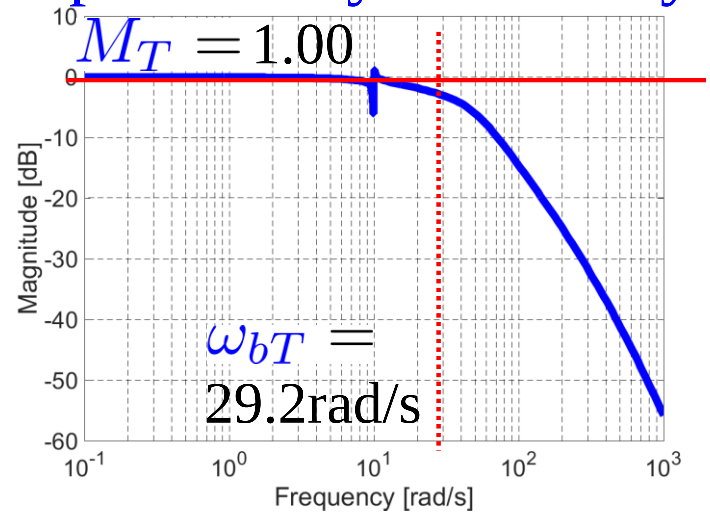
Sensitivity $S(s)$

$M_S = 1.44$

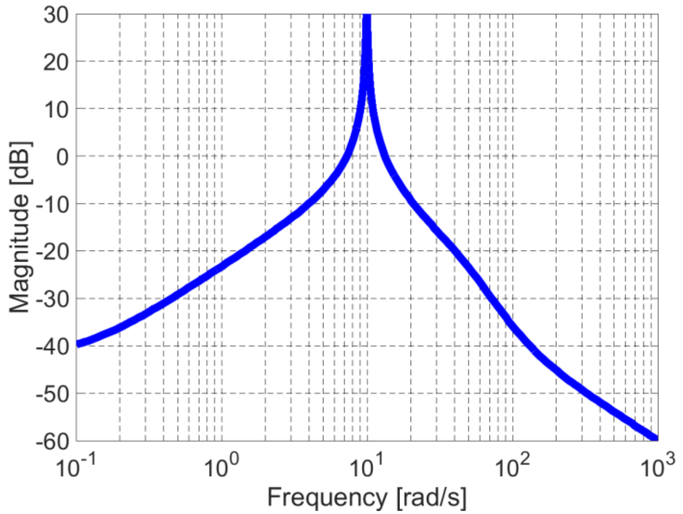


Complementary Sensitivity $T(s)$

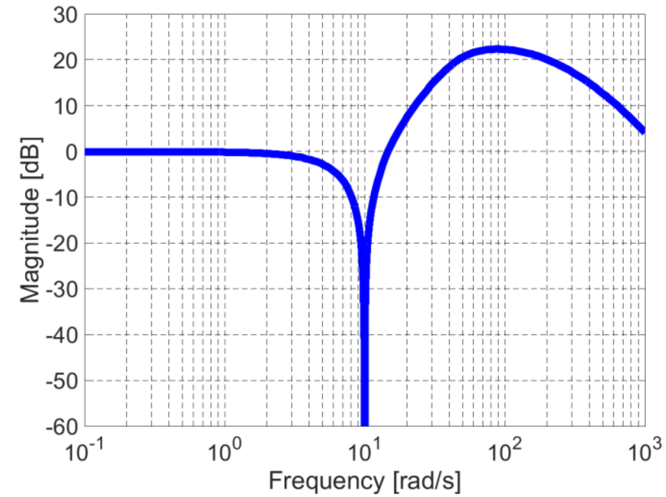
$M_T = 1.00$



Load Sensitivity $PS(s)$



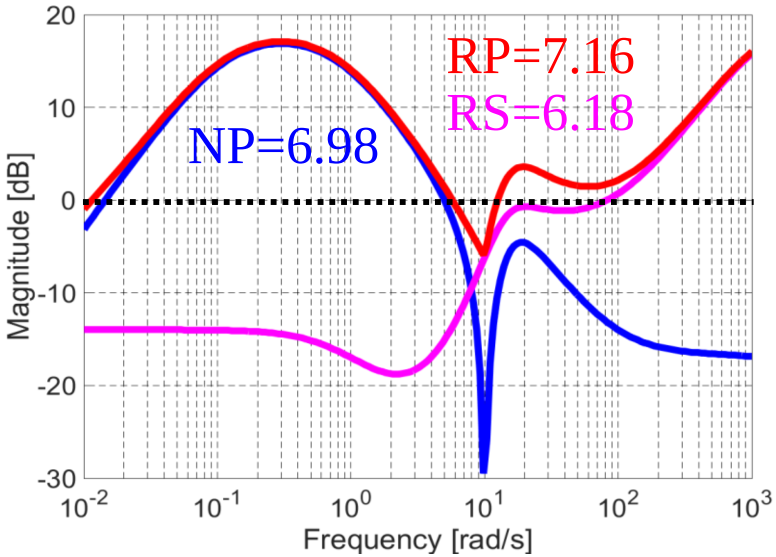
Noise Sensitivity $KS(s)$



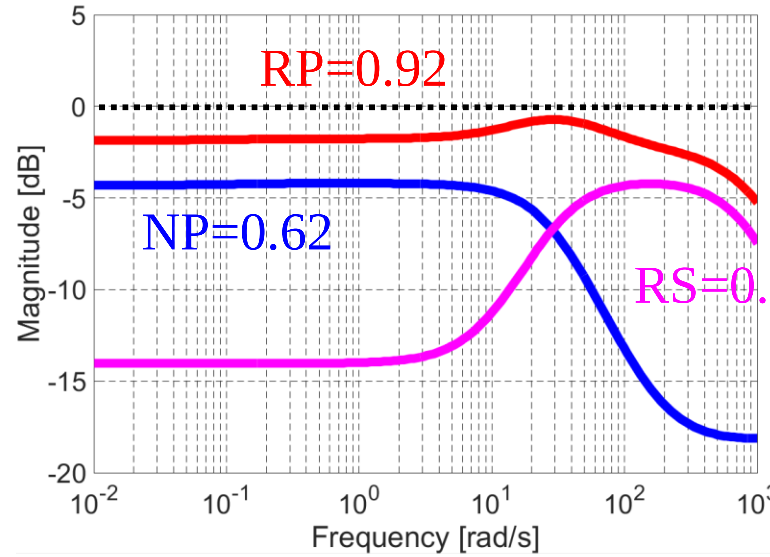
[AM08] K.J. Astrom and R.M. Murray, *Feedback Systems*, Princeton University Press, 2008

Spinning Satellite: SISO Controller Design μ -analysis

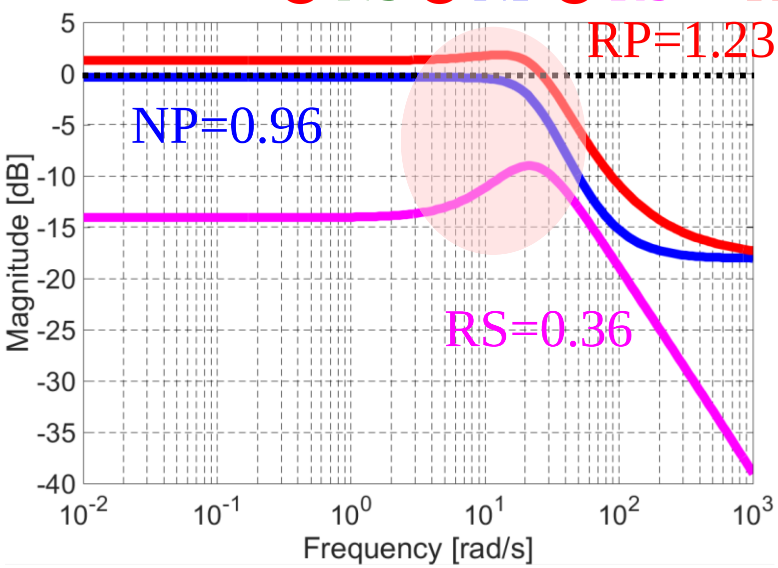
pidtune ○ NS × NP × RS × RP



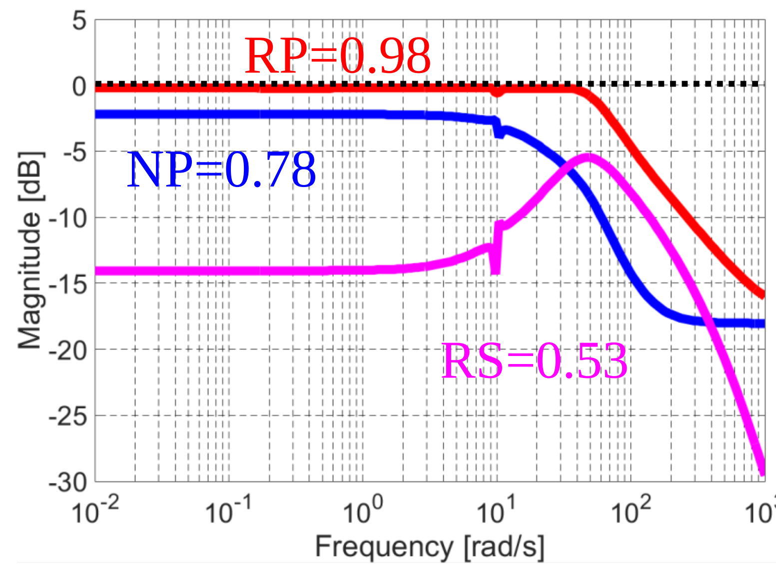
H_∞-controller ○ NS ○ NP ○ RS ○ RP



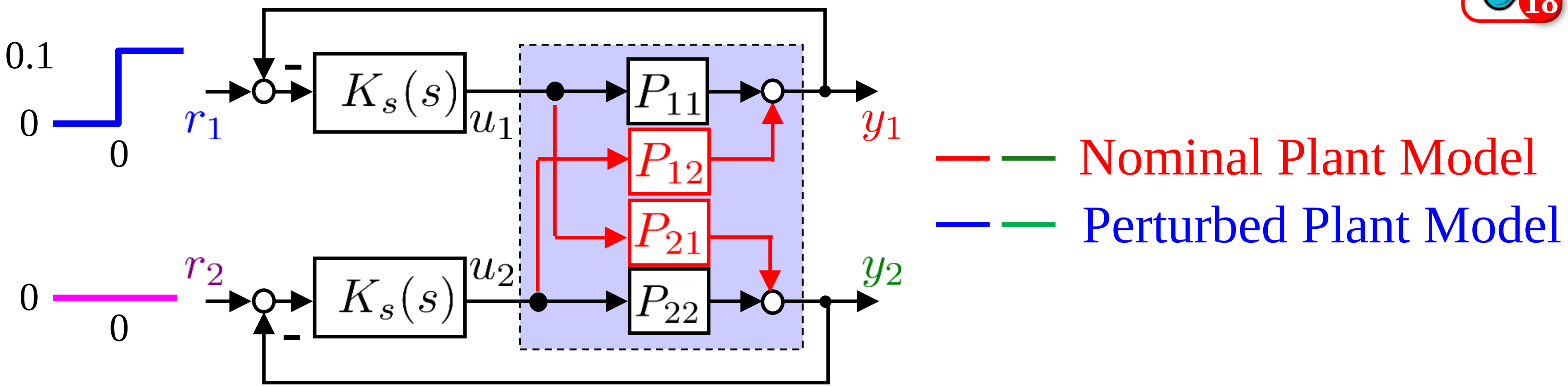
H_2-controller ○ NS ○ NP ○ RS × RP

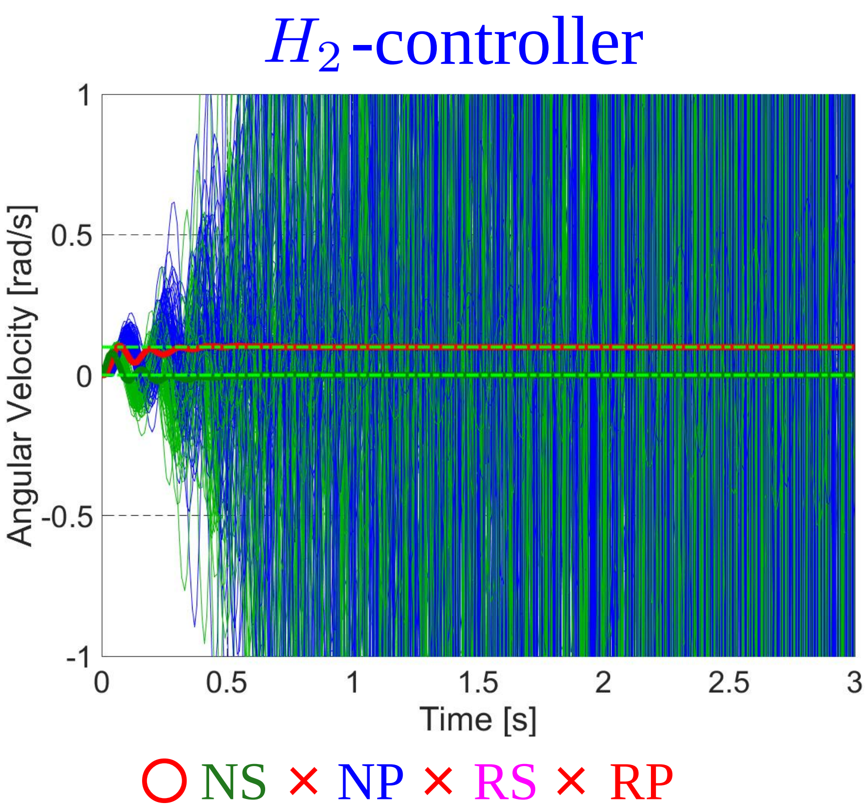


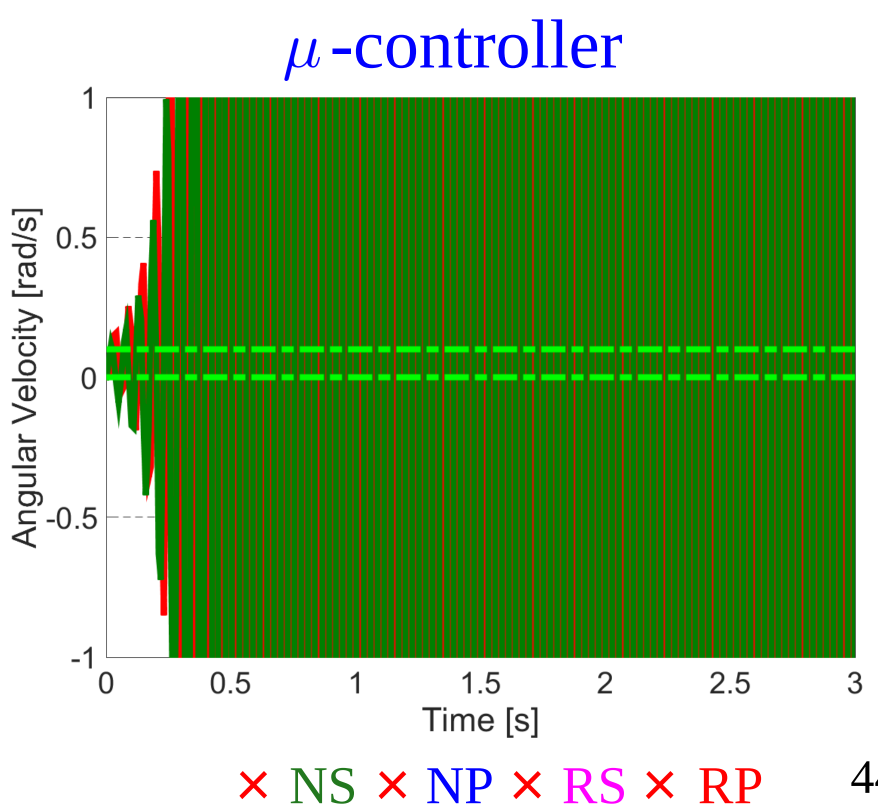
μ -controller ○ NS ○ NP ○ RS ○ RP



1. Diagonal Controller (decentralized control)

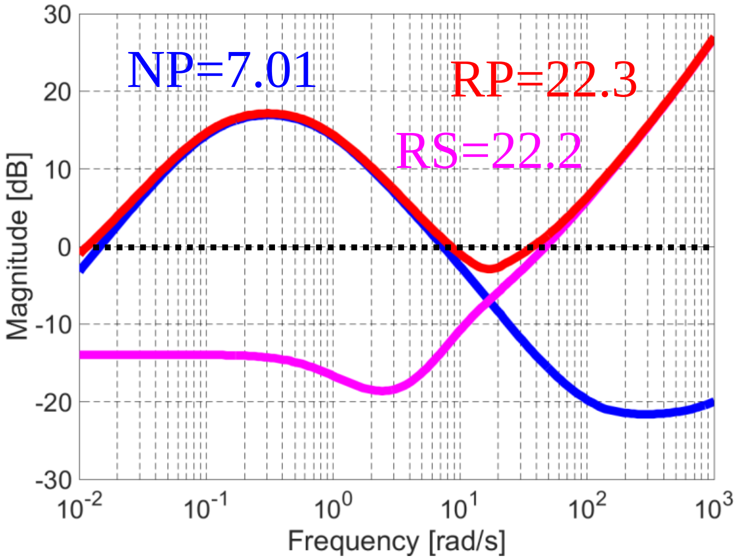




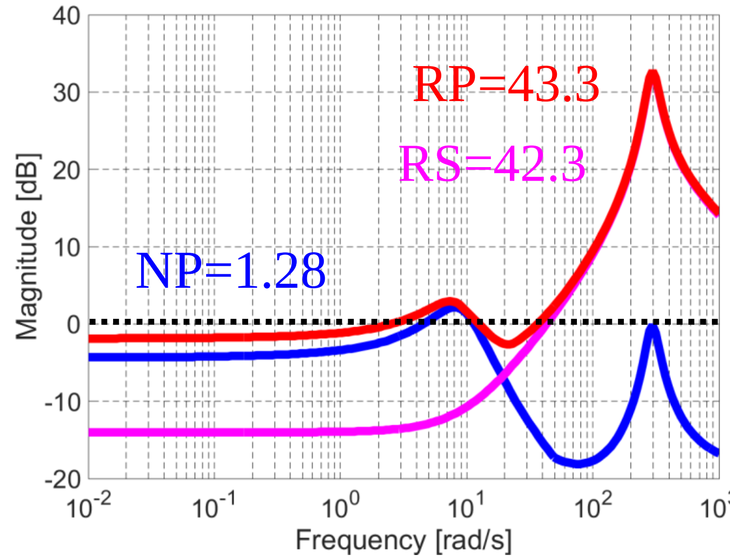


1. Diagonal Controller (decentralized control)

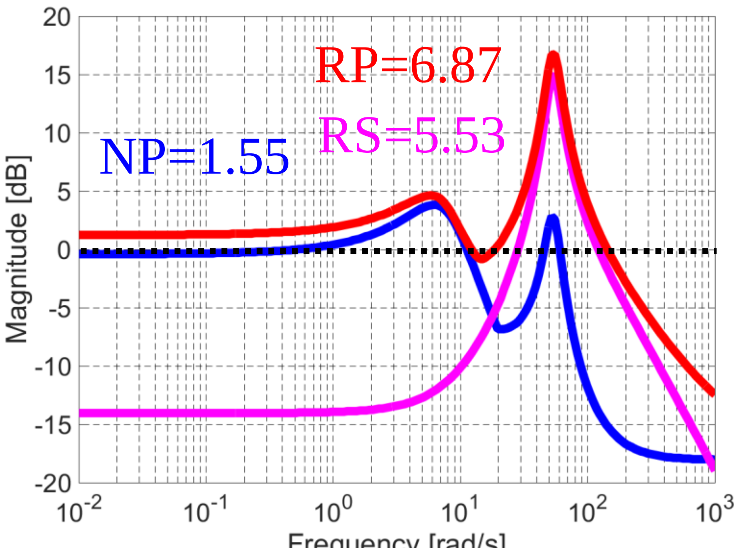
pidtune ○ NS × NP × RS × RP



H_∞-controller ○ NS × NP × RS × RP



H_2-controller ○ NS × NP × RS × RP



μ-controller × NS × NP × RS × RP