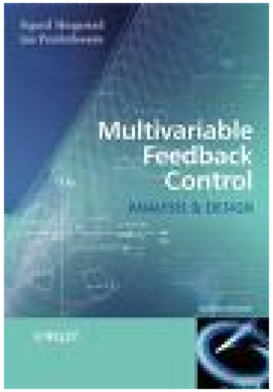
Robust Control, Spring 2019

Instructor: Prof. Masayuki Fujita (S5-303B)

C. Design Example

C.4 HiMAT: H_∞ Control (Highly Maneuverable Aircraft Technology)

Reference:



[SP05] S. Skogestad and I. Postlethwaite, ***Multivariable Feedback Control; Analysis and Design,*** Second Edition, Wiley, 2005.

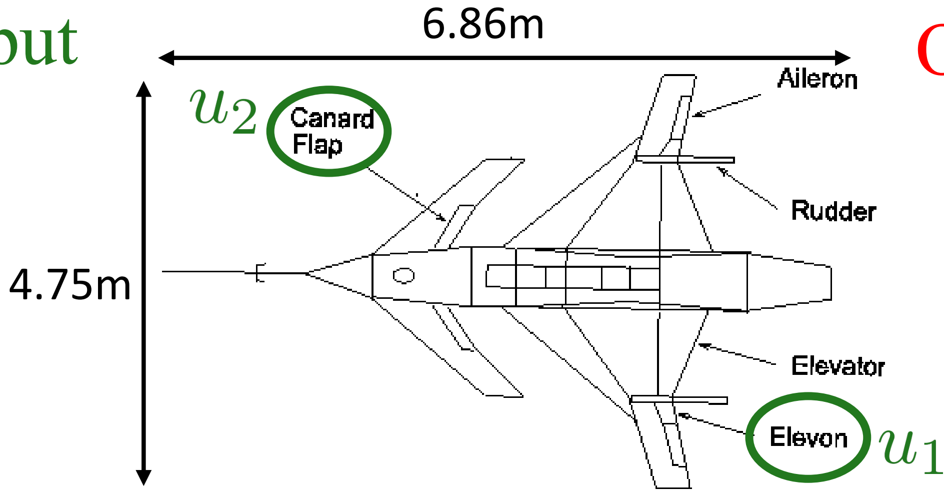
[SLH81] M. G. Safonov, A. J. Laub and G. L. Hartmann, ***Feedback Properties of Multivariable Systems: The Role and Use of the Return Difference Matrix,*** IEEE Transactions on Automatic Control, Vol. 26, No. 1, pp. 47-65, 1981.

[ES84] M. B. Evans and L. J. Schilling, ***The Role of Simulation in the Development and Flight Test of the HiMAT Vehicle,*** NASA Technical Memorandum 84912, 1984.

[CS98] R. Y. Chiang and M. G. Safonov, ***Robust Control Toolbox User's Guide,*** The MathWorks, 1998.

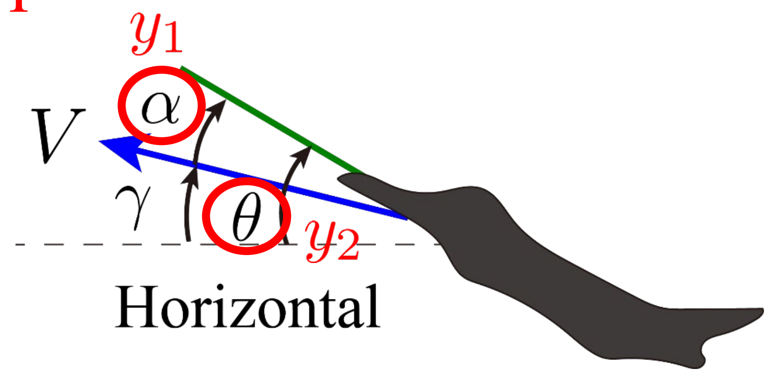
HiMAT: Attitude and flight path control

Input



Max takeoff weight: 1,542kg

Output



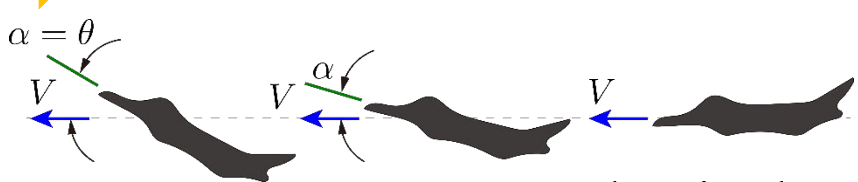
y_1 (Angle of attack)
y_2 (Pitch angle)

Objective

Control attitude (θ) and flight path ($\gamma = \theta - \alpha$) separately

⇒ MIMO System

[Ex.]



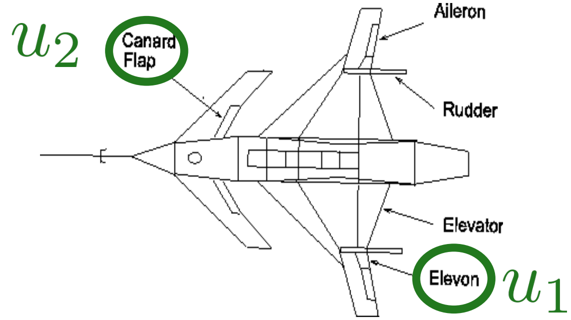
Control attitude without changing flight path

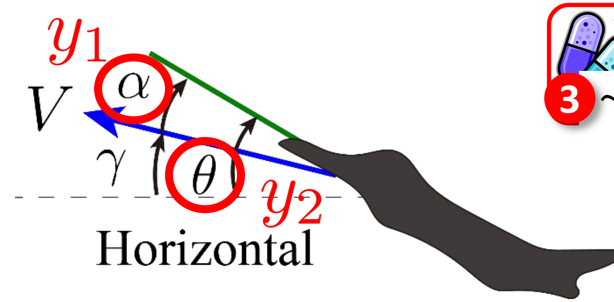
Specification

Robustness Spec.: -40dB/dec roll-off and -20dB at 100rad/s
(Because of the unmodeled low-damped structual mode)

Performance Spec.: Minimize the sens. function as much as possible

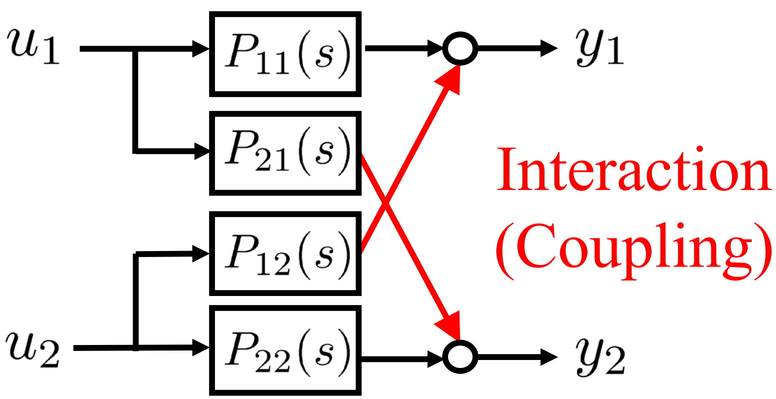
Modeling HiMAT



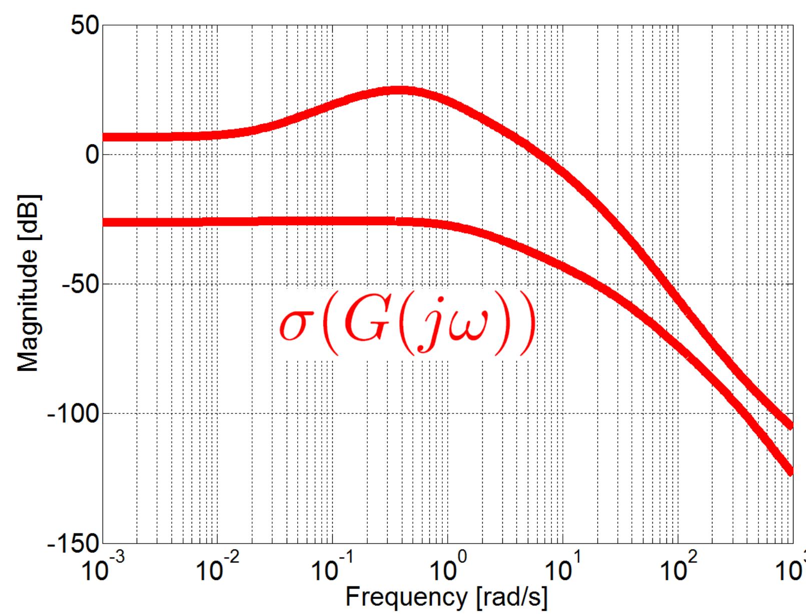


$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-5.124(s+184.5)(s+0.04085)(s-0.02078)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} & \frac{-948.12(s+0.02177)(s+1.963)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} \\ \frac{-0.14896(s-4432)(s+0.06657)(s-0.047)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} & \frac{671.88(s+0.02399)(s+1.895)}{(s+5.676)(s+30)(s+0.2578)(s^2-1.38s+0.5377)} \end{bmatrix}$$



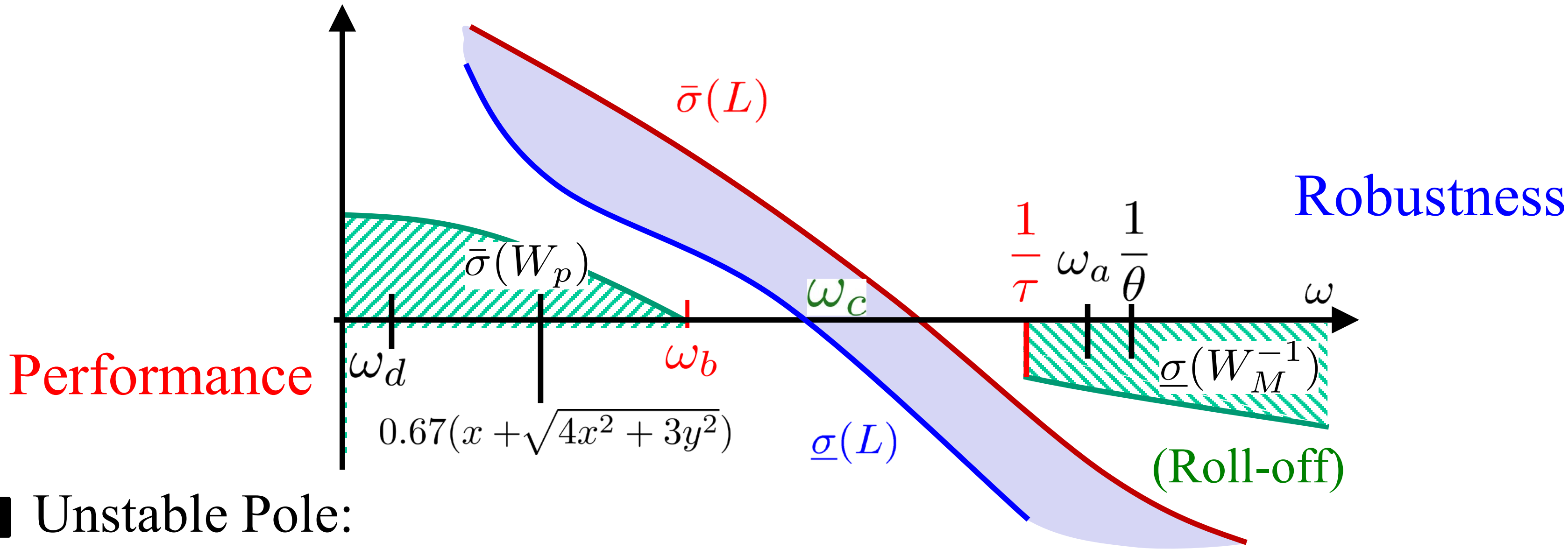
Frequency Response



Poles $-5.6757,\ -0.2578,$
$-30,\ -30,$
$0.6898 \pm 0.2488j$ **Unstable**

Zeros -0.0210

MIMO Loop Shaping [SP05, p. 343]



- Unstable Pole:

$$p = x \pm yj$$
$$= 0.6898 \pm 0.2488j$$

$$\omega_c > 0.67(x+\sqrt{4x^2+3y^2})$$
$$= 1.43\text{rad/s}$$

- Disturbance (Gust):

$$\omega_c > \omega_d = 1\text{rad/s}$$

- Performance Weight

$$\omega_c > \omega_b \approx 1.43 \text{ rad/s} \quad W_P(s)$$

- Time Delay: $\theta \leq 0.025$s

$$\omega_c < 1/\theta = 40 \text{ rad/s}$$

- Actuator dynamics $\omega_a = 30\text{rad/s}$

$$\omega_c < \omega_a = 30\text{rad/s}$$

- Uncertainty Weight

$$\omega_c < \frac{1}{\tau} = 30 \text{ rad/s} \quad W_M(s)$$

HiMAT: Uncertainty Weight

Uncertain Factors:
Actuator Dynamics +
Unmodeled dynamics (High freq.)

Frequency: $1/\tau \leq 30\text{rad/s}$

2 Oscillation mode (2nd order system)

$$95 < \omega_1 < 105[\text{rad/s}],\ 0.05 < \zeta_1 < 0.07$$

$$280 < \omega_2 < 320[\text{rad/s}],\ 0.012 < \zeta_2 < 0.018$$

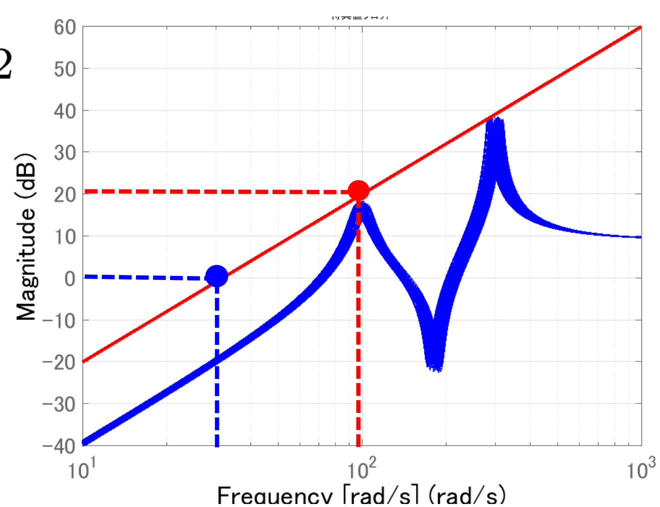
ω_1, ω_2 : Natural angular frequency

ζ_1, ζ_2 : Damping factor

$$W_M(s) = w_M(s) I_2$$

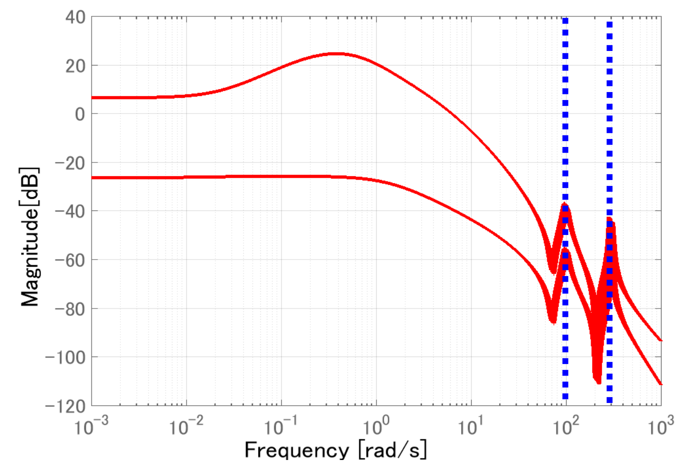
$$w_M(s) = \frac{s^2}{1000}$$

$$\left(\begin{array}{l} \tau = 0.033, \\ 1/\tau = 30 \text{ rad/s} \\ r_0 = 0, \\ r_\infty \to \infty \end{array} \right)$$

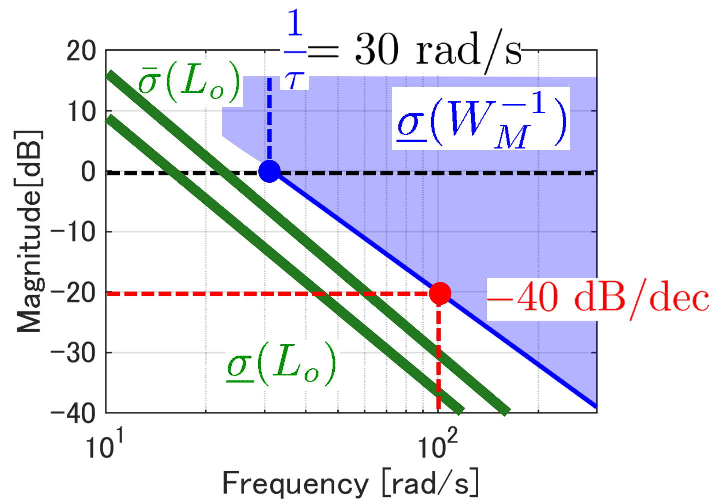


MATLAB Command

```
wM = tf([1 0 0], [1000]);
WM = eye(2)*wM;
```



Target Loop



Roll-off: $-20\text{dB at } 100\text{rad/s}$

-40dB/dec

HiMAT: Performance Weight

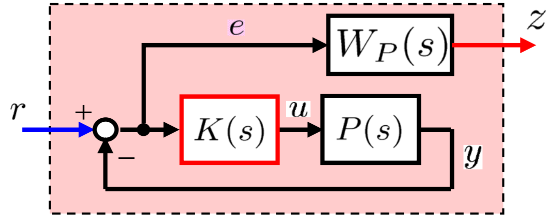
$$W_P(s) = w_p(s) I_2$$

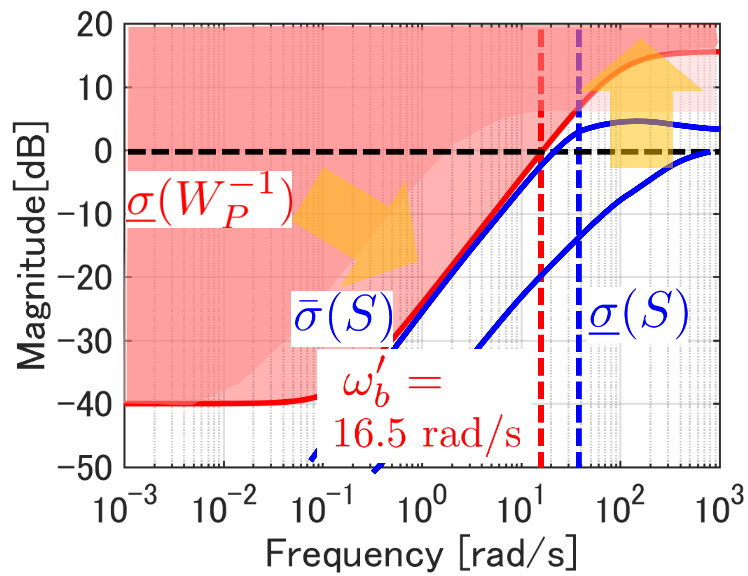
$$w_p(s) = \frac{\frac{1}{M_S}s + \omega_b}{s + \omega_b A} = \frac{0.5s + 1.5}{s + 0.015}$$

$$\left[\begin{array}{l} \omega_b = 1.5 (\approx 0.67(x + \sqrt{4x^2 + 3y^2})) \\ M_s = 2, A = 0.01 \end{array} \right]$$

MATLAB Command

```
Ms = 2; A = 0.01; wb = 1.5;
wP = tf([1/Ms wb], [1 wb*A]);
WP = eye(2)*wP;
```





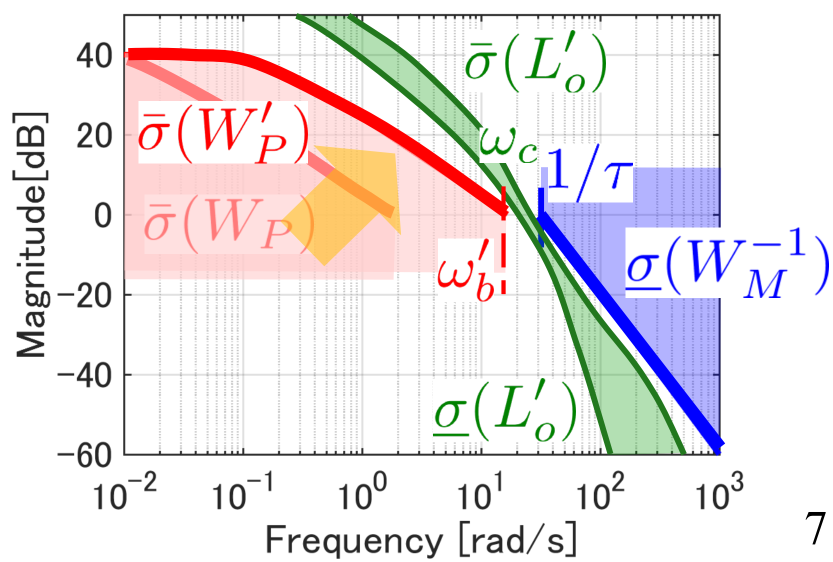
Update Larger ω_b

$\omega_b = 1.5 \to 16.5$

$M_s = 2 \to 6$ (Trade-off)

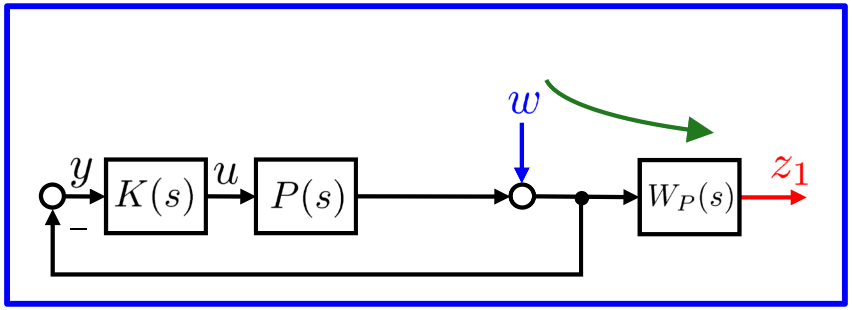
$A = 0.01 > 0$

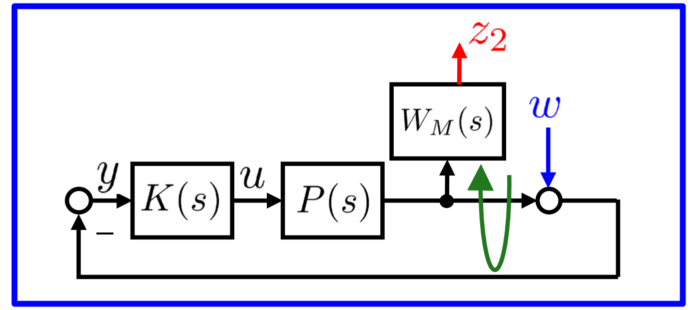
$$w_p'(s) = \frac{0.165s + 16.5}{s + 0.01}$$

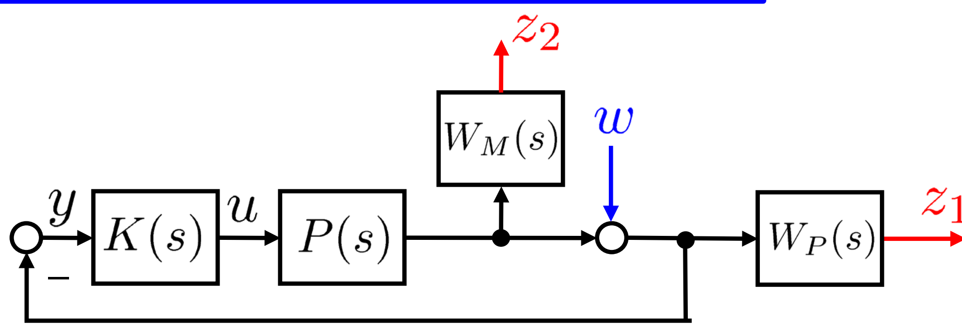


HiMAT: Control Problem Formulation

Nominal Performance $\|W_P(s)S_o(s)\|_\infty$ Robust Stability $\|W_M(s)T_o(s)\|_\infty$

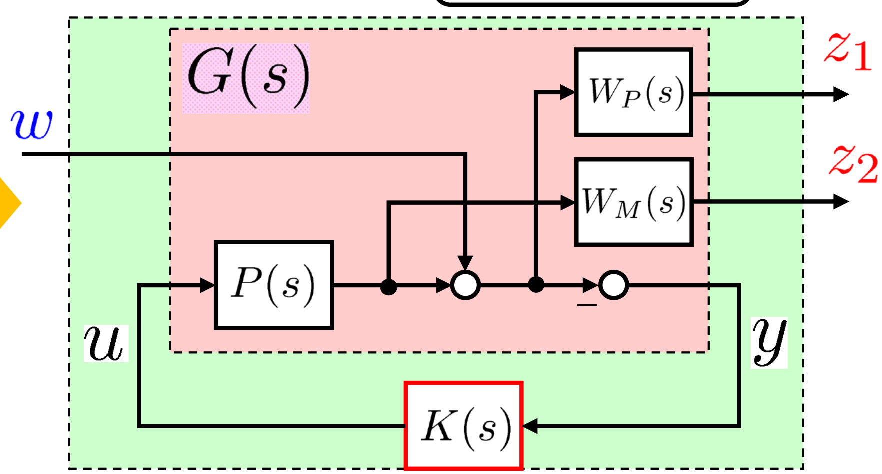






Generalized Plant

sysic



MATLAB Command

```
%Generalized Plant%
systemnames = 'Pnom WP WM';
inputvar = '[w(2);u(2)]';
outputvar = '[WP;WM;-w-Pnom]';
input_to_Pnom= '[u]';
input_to_WP = '[w+Pnom]';
input_to_WM = ' [Pnom]';
G = sysic;
```

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} W_P(s)S_o(s) \\ -W_M(s)T_o(s) \end{bmatrix} w$$

$$= F_l(G, K)$$

Mixed Sensitivity

HiMAT: γ-iteration to obtain H_∞ Controller

[SP05, p. 358]

Find K such that $\|F_l(G,K)\|_\infty < \gamma$

Check 1)

Appropriately sub-optimal

(Default settings)

$\gamma = 0.9538 < 1 \quad (\gamma_{opt} = 0.9510)$

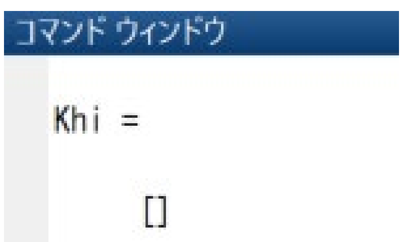
Check 2) $\gamma = 0.9$ No Solution

Resetting value of Gamma min based on D_11, D_12, D_21 terms

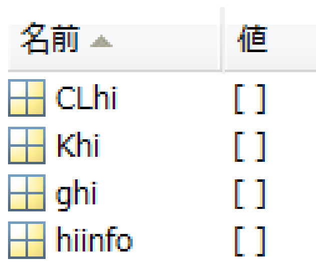
Test bounds: 0.1667 < gamma <= 0.9538

gamma	hamx_eig	xinf_eig	hamy_eig	yinf_eig	nrho_xy	p/f
0.954	2.1e-02	1.8e-05	1.6e-01	1.0e-22	0.9915	p
0.560	2.1e-02	-1.1e+06#	1.6e-01	-6.4e-20	0.2088	f
0.757	2.1e-02	-8.2e+06#	1.6e-01	-1.4e-20	0.6112	f
0.855	2.1e-02	-5.0e+07#	1.6e-01	-4.1e-20	2.9142#	f
0.905	2.1e-02	1.9e-05	1.6e-01	-4.4e-23	3.1083#	f
0.929	2.1e-02	1.8e-05	1.6e-01	-4.7e-20	1.5096#	f
0.941	2.1e-02	1.8e-05	1.6e-01	-2.5e-20	1.1979#	f
0.948	2.1e-02	1.8e-05	1.6e-01	-1.1e-19	1.0852#	f

Gamma value achieved: 0.9538



Data Structure



There is No Controller

MATLAB Command

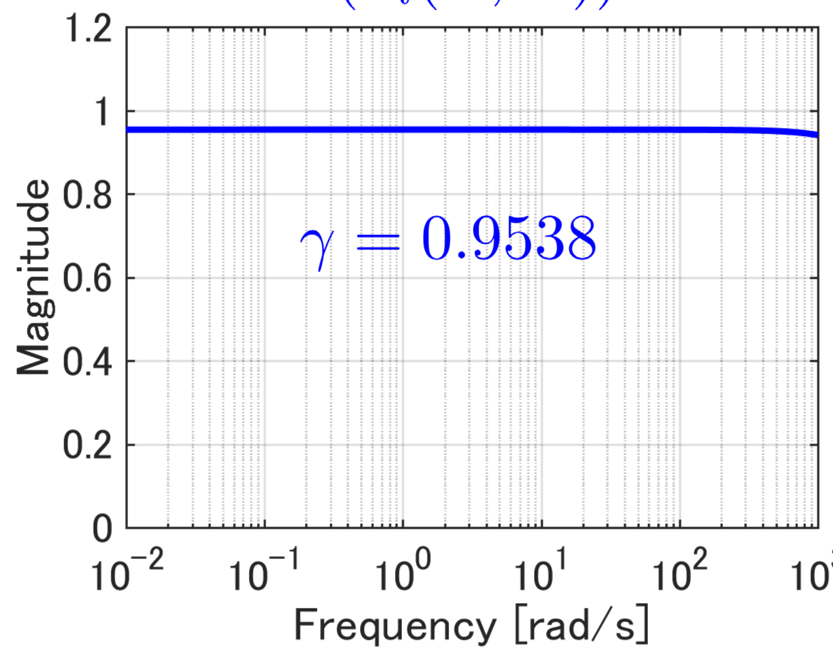
```
[Khi,CLhi,ghi,hiinfo] = …
hinfsyn(G,nmeas,ncon,'Display', 'on');
```

MATLAB Command

```
[SV,w]=sigma(CLhi);
figure; semilogx(w,SV)
```

Closed-loop Transfer Function

$\bar{\sigma}(F_l(G,K))$



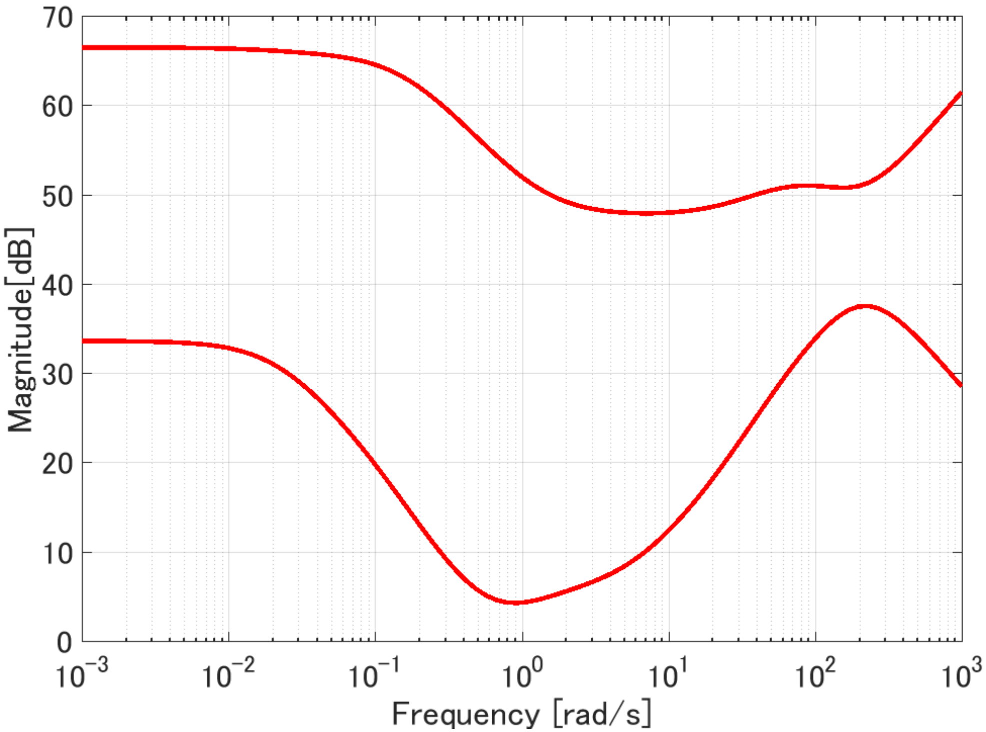
HiMAT: Controller

MATLAB Command
figure sigma(Khi)

$$K(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$

[Ex.]

$$K_{11}(s) = \frac{-2.101 \times 10^6 (s+46.24)(s+30)(s+1.506)}{(s+1.857\times 10^4)(s+5600)(s+120.4)} \times \frac{(s+0.165)(s+0.02513)(s^2-1453s+1.174\times 10^6)}{(s+60.39)(s+47.57)(s+0.165)^2(s+0.02098)}$$



Poles

$-1.8549 \cdot 10^4, -5.6005 \cdot 10^3$
$-1.2045 \cdot 10^2$
$-60.3871, -47.5747$
$-0.165, -0.165, -0.0210$

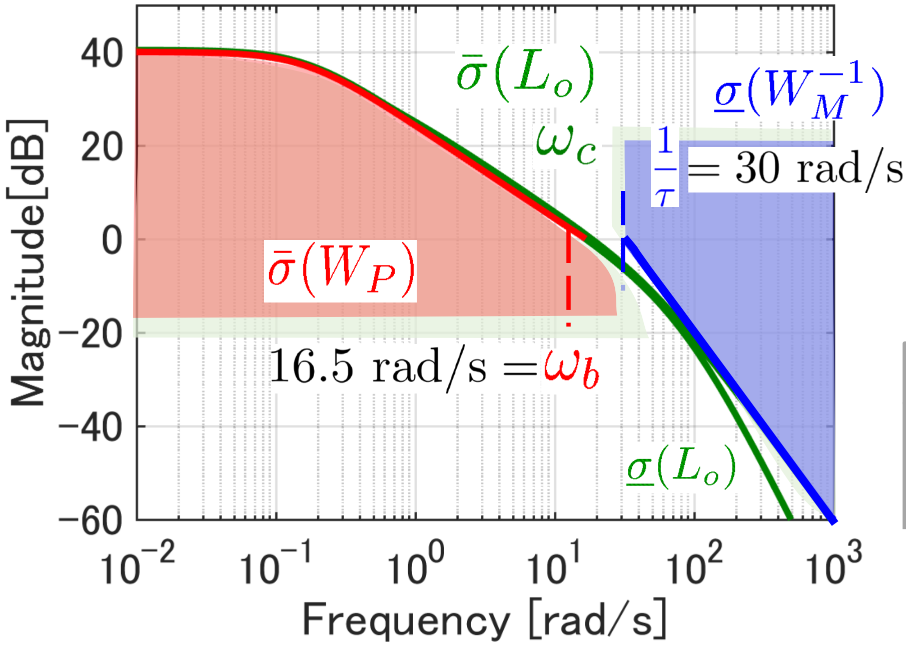
Zeros

$-0.6127 \pm 0.3018j$
$-30, -30$
$-5.6757, -0.2578$

Order 8

➡ Model Reduction

HiMAT: Open-loop Frequency Response



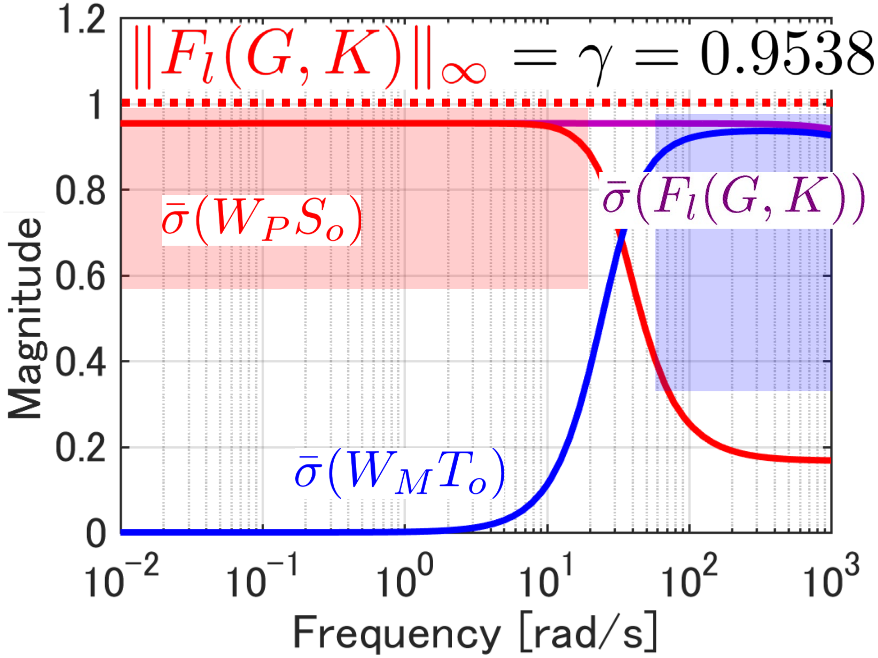
$\gamma = 0.9538 < 1$

(corresponding maximum stability margin)

Loop Transfer Function

MATLAB Command

```
figure
sigma(Fhi.Lo,WP, inv(WM),WP/ghi,ghi*inv(WM))
```



NP/RS Test

Nominal Performance (NP)

$\|W_P S_o\|_\infty = 0.9538 < 1$

Robust Stability (RS)

$\|W_M T_o\|_\infty = 0.9358 < 1$

MATLAB Command

```
[SV,w]=sigma(WP*Fhi.So);
figure; semilogx(w,SV)
[SV,w]=sigma(WM*Fhi.To);
figure; semilogx(w,SV)
```

HiMAT: Closed-loop Performance

MATLAB Command
pole(CLhi) zero(CLhi) figure; pzmap(CLhi)

Nominal Stability (NS)

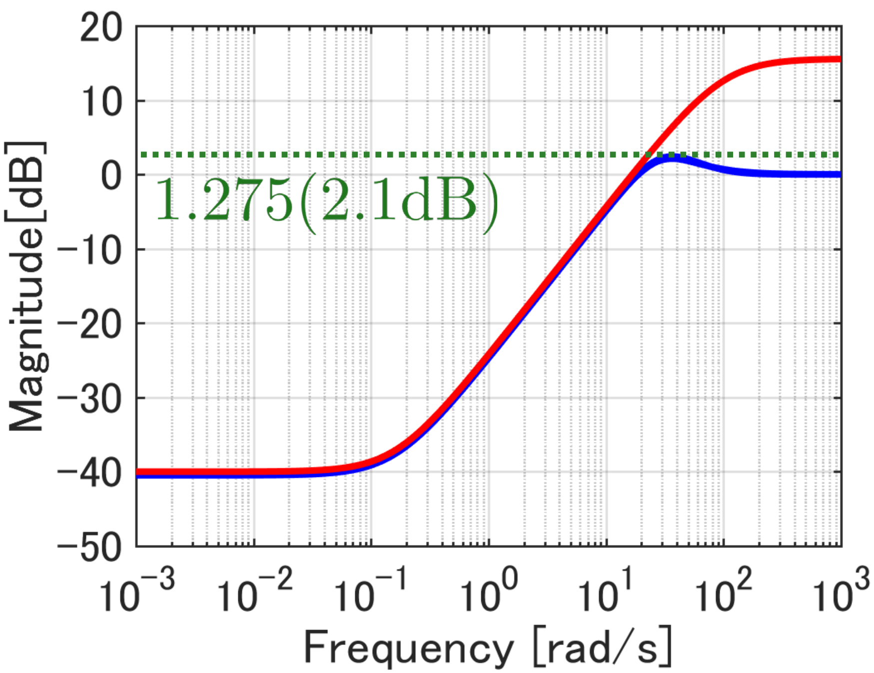
Poles of $F_l(G, K)$

$$p = -0.165, -0.165, -0.0210, -0.2578, -5.6757, -30, -30$$
$$-1.3346 \cdot 10^2, -5.6007 \cdot 10^3, -1.8549 \cdot 10^4$$
$$-0.6898 \pm 0.2488j, -22.441 \pm 17.871j, -23.720 \pm 19.701j$$

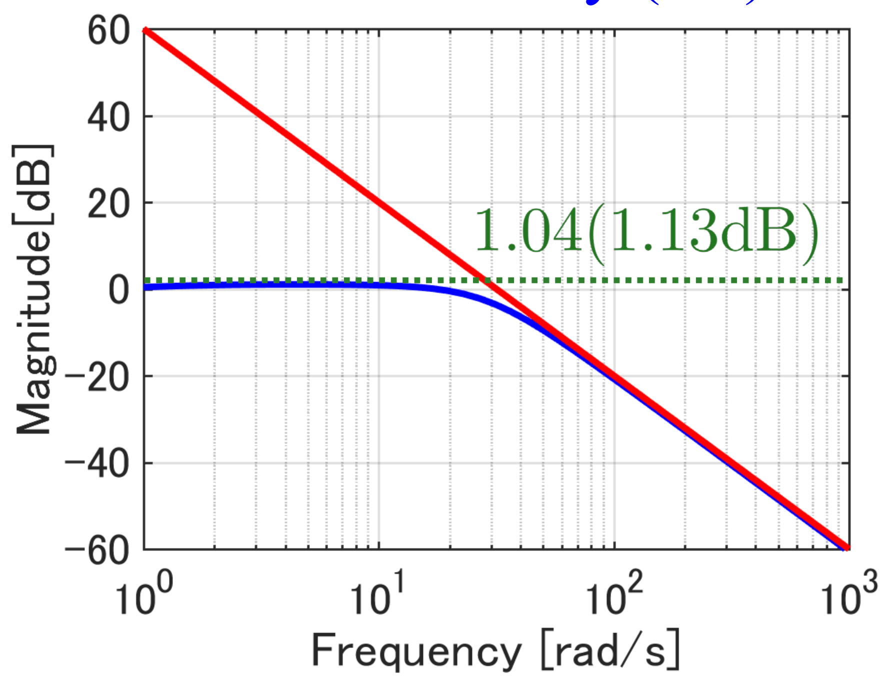
Zeros of $F_l(G, K)$

$$-0.0210$$

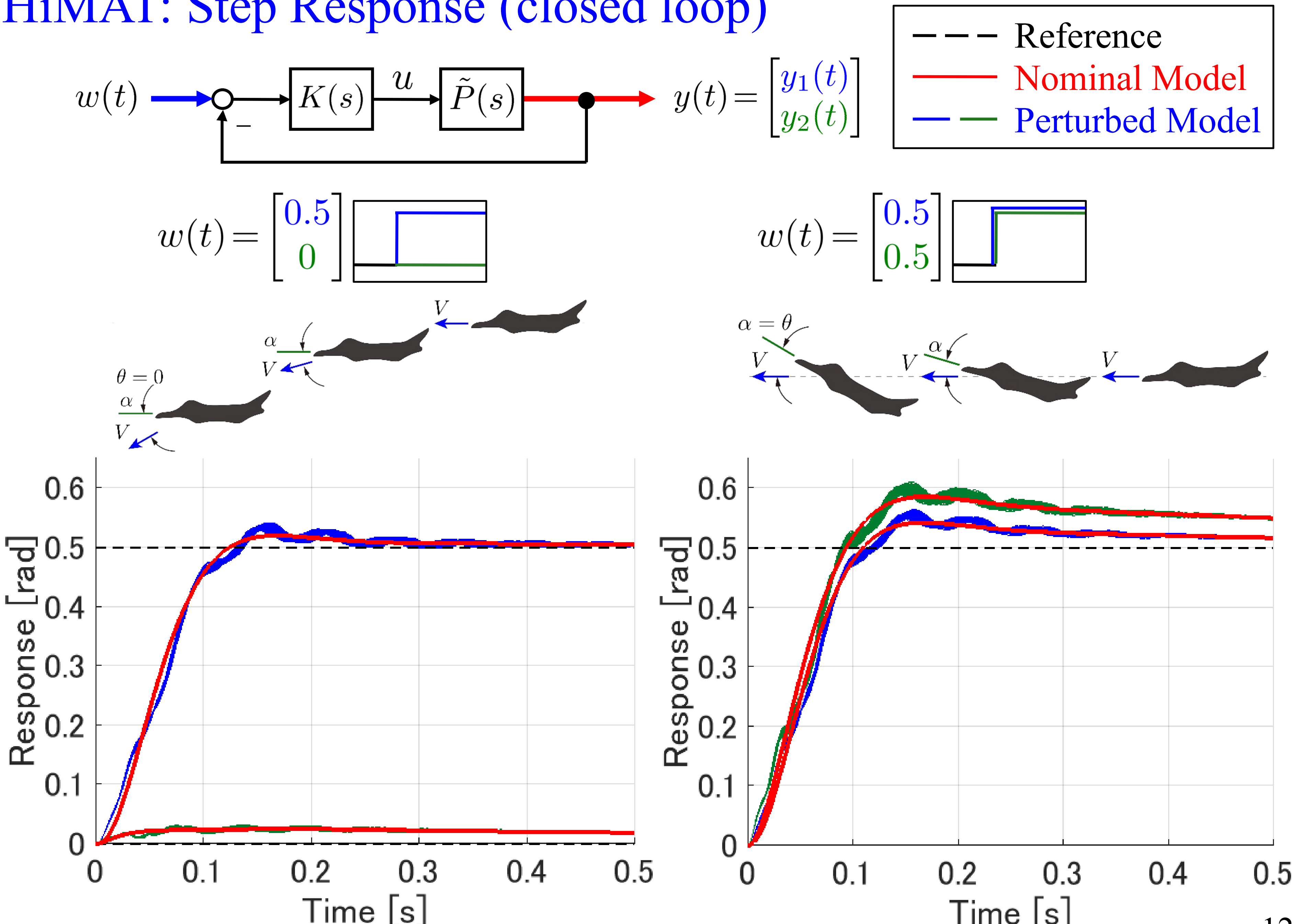
Nominal Performance (NP)



Robust Stability (RS)



HiMAT: Step Response (closed loop)



Today's MATLAB Commands

MATLAB Command

```
wM = tf([1 0 0], [1000]);
WM = eye(2)*wM;
```

MATLAB Command

```
Ms = 2; A = 0.01; wb = 1.5;
wP = tf([1/Ms wb], [1 wb*A]);
WP = eye(2)*wP;
```

MATLAB Command

```
%Generalized Plant%
systemnames = 'Pnom WP WM';
inputvar = '[w(2);u(2)]';
outputvar = '[WP;WM;-w-Pnom]';
input_to_Pnom= '[u]';
input_to_WP = '[w+Pnom]';
input_to_WM = ' [Pnom]';
G = sysic;
```

MATLAB Command

```
[Khi,CLhi,ghi,hiinfo] = …
hinfsyn(G,nmeas,ncon,'Display', 'on');
```

MATLAB Command

```
[SV,w]=sigma(CLhi);
figure; semilogx(w,SV)
```

MATLAB Command

```
figure
sigma(Fhi.Lo,WP, inv(WM),WP/ghi,ghi*inv(WM))
```

MATLAB Command

```
[SV,w]=sigma(WP*Fhi.So);
figure; semilogx(w,SV)
[SV,w]=sigma(WM*Fhi.To);
figure; semilogx(w,SV)
```

MATLAB Command

```
pole(CLhi)
zero(CLhi)
figure; pzmap(CLhi)
```

NASA HiMAT

Real Product

General characteristics

- Crew: None
- Length: 6.86m
- Wingspan: 4.75m
- Height: 1.31m
- Max takeoff weight: 1,542kg

Performance

- Maximum speed: Mach 1.6
- Maneuvering: 8G performance

NASA HiMAT

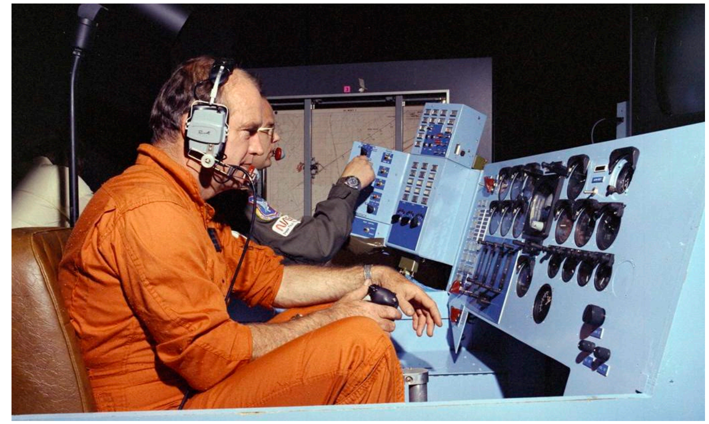
Real Product

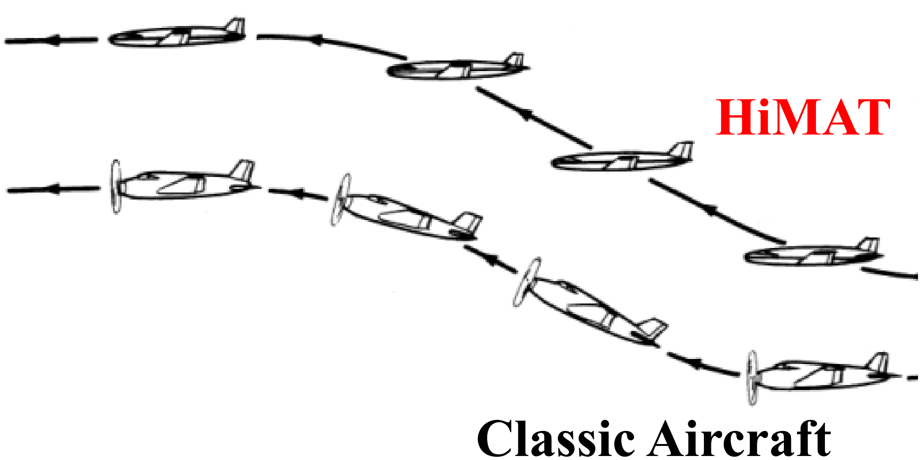
Testing Product

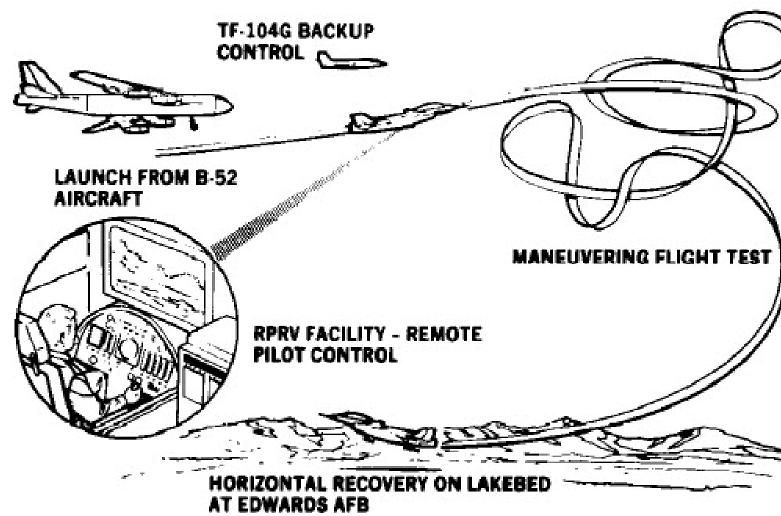
Testing high technology

UAV(Unmanned Aerial Vehicle)

Teleoperation

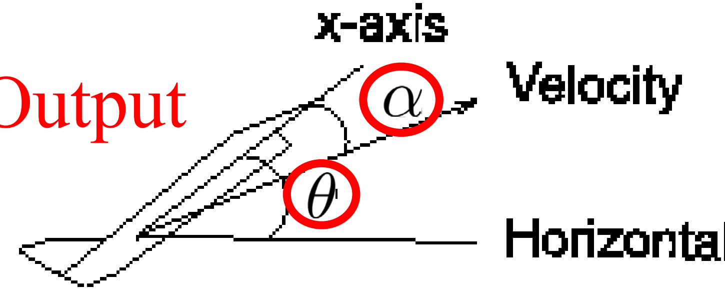




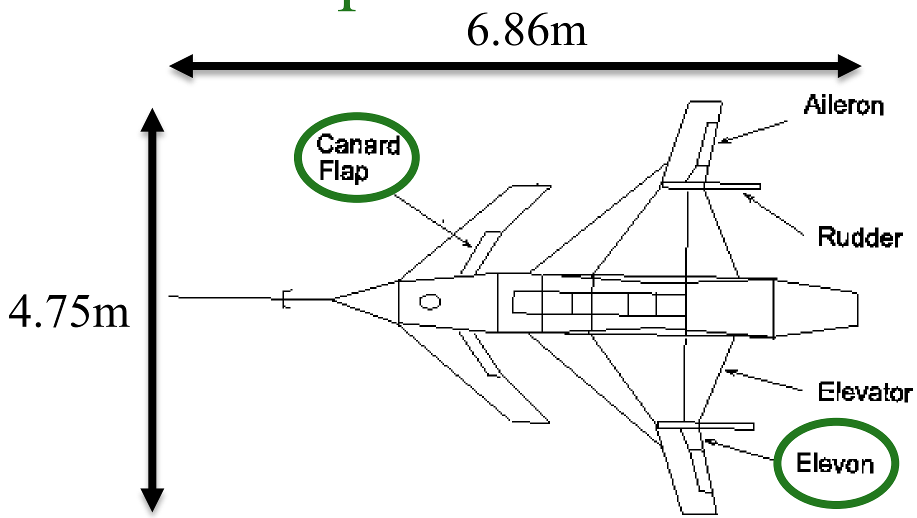


Modeling HiMAT

Output



Control Input



α : angle of attack

θ : attitude angle

δ_e : elevon actuator

δ_c : canard actuator

x_e, x_c } states to represent actuator dynamics

$$x^T = \begin{pmatrix} \dot{\alpha} & \alpha & \dot{\theta} & \theta & x_e & x_c \end{pmatrix}$$

$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \qquad u^T = \begin{pmatrix} \delta_e & \delta_c \end{pmatrix} \qquad y^T = \begin{pmatrix} \alpha & \theta \end{pmatrix}$$

$$= \begin{bmatrix} \frac{-5.124s^3-945.7s^2-18.98s+0.8025}{s^5+34.55s^4+130.4s^3-184.4s^2+35.95s+23.6} & \frac{-948.1s^2-1881s-40.51}{s^5+34.55s^4+130.4s^3-184.4s^2+35.95s+23.6} \\ \frac{-0.149s^3+660.1s^2+12.92s-2.066}{s^5+34.55s^4+130.4s^3-184.4s^2+35.95s+23.6} & \frac{671.9s^2+1290s+30.55}{s^5+34.55s^4+130.4s^3-184.4s^2+35.95s+23.6} \end{bmatrix}$$

HIMAT: Nominal Plant Model

State Space Form (Matrix Representation)

$$G = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$$

$$x^T = \begin{pmatrix} \dot{\alpha} & \alpha & \dot{\theta} & \theta & x_e & x_c \end{pmatrix}$$

$$u^T = \begin{pmatrix} \delta_e & \delta_c \end{pmatrix} \quad y^T = \begin{pmatrix} \alpha & \theta \end{pmatrix}$$

$$A = \begin{pmatrix} -0.023 & -36.62 & -18.90 & -32.09 & 3.251 & -0.763 \\ 0.000 & -1.900 & -0.983 & -0.000 & -0.171 & -0.005 \\ 0.012 & 11.72 & -2.632 & 0.000 & -31.60 & 22.40 \\ 0 & 0 & 1.000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -30.00 & 0 \\ 0 & 0 & 0 & 0 & 0 & -30.00 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 30 & 0 \\ 0 & 30 \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

MATLAB Command

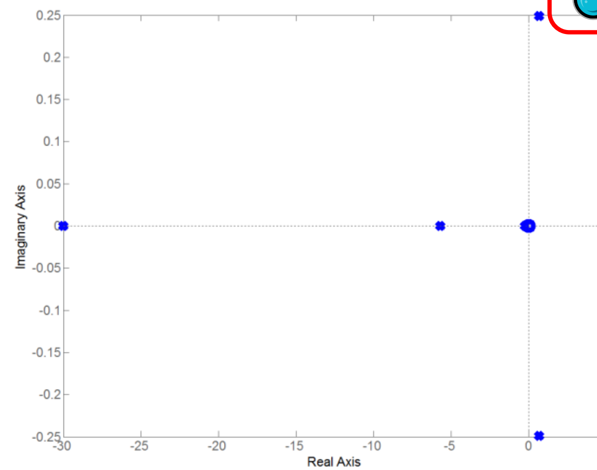
```
% NASA HiMAT model G(s)
ag =[ -2.2567e-02 -3.6617e+01 -1.8897e+01 -3.2090e+01 3.2509e+00 -7.6257e-01;
       9.2572e-05 -1.8997e+00  9.8312e-01  -7.2562e-04 -1.7080e-01 -4.9652e-03;
       1.2338e-02  1.1720e+01 -2.6316e+00  8.7582e-04 -3.1604e+01 2.2396e+01;
                0           0  1.0000e+00            0           0          0;
                0           0           0            0 -3.0000e+01          0;
                0           0           0            0           0 -3.0000e+01];
bg = [ 0 0; 0 0; 0 0; 0 0; 30 0; 0 30];
cg = [ 0 1 0 0 0 0; 0 0 0 1 0 0];
dg = [ 0 0; 0 0];
G=ss(ag,bg,cg,dg);
```

HiMAT: Nominal Plant Model

Controllability ✓

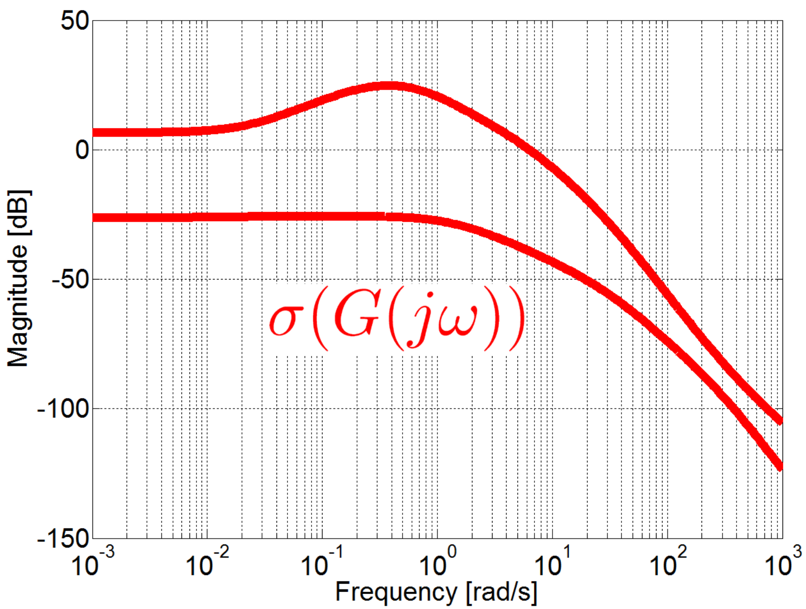
Observability ✓

Poles (Stability) -5.6757, -0.2578, -30, -30,
0.6898 ± 0.2488i
Unstable



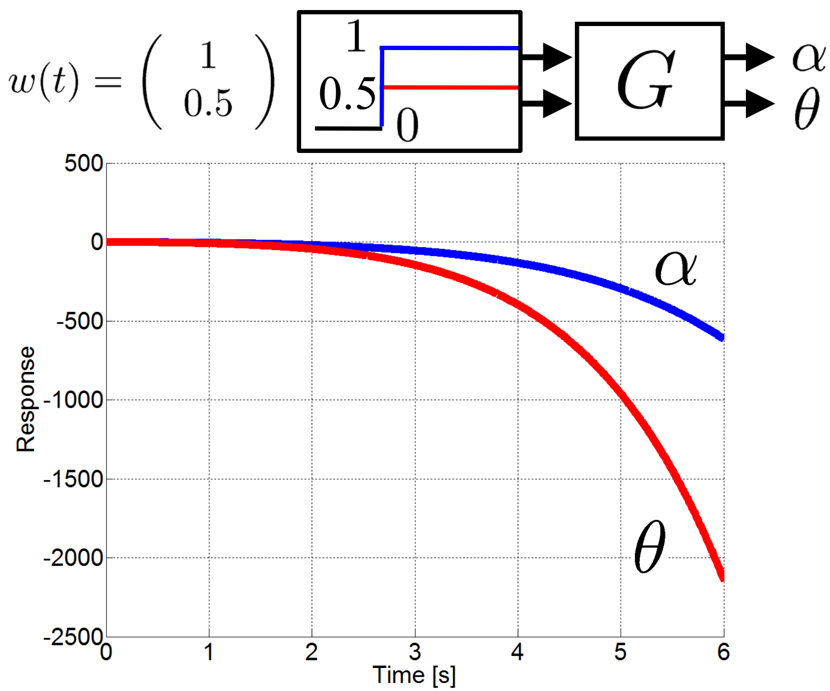
Zeros -0.0210

Frequency Response

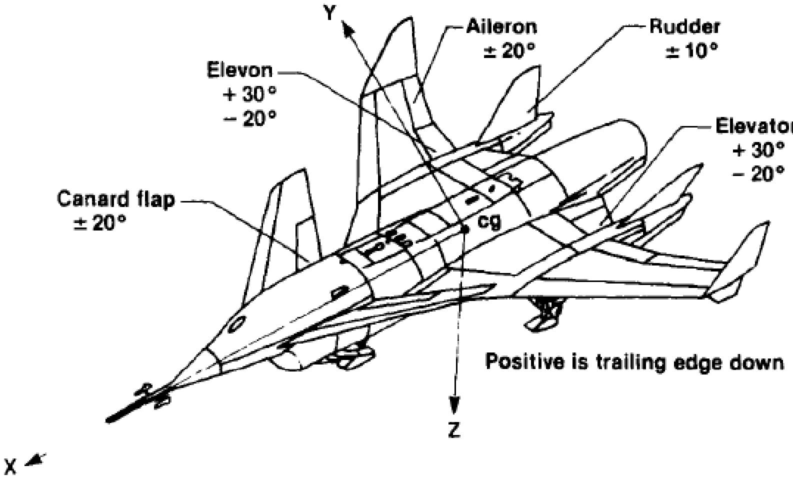


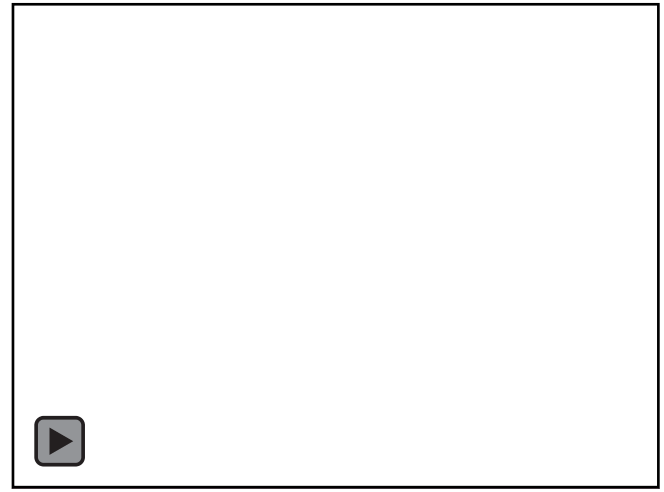
Step Response for Nominal Plant Model

$$w(t) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$

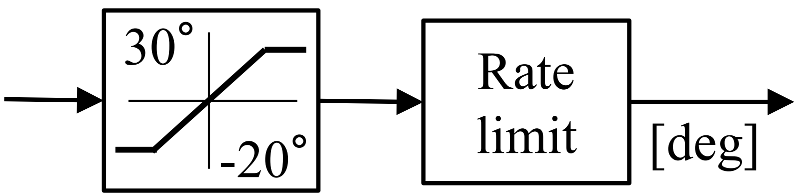


Actuators: Hydraulic serve system



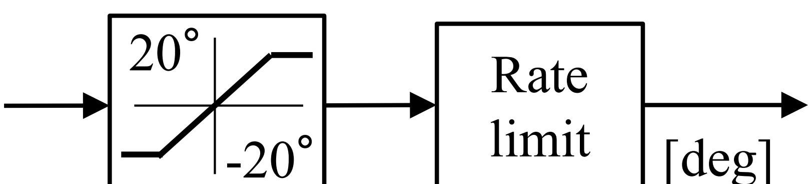


Block diagrams of elevon



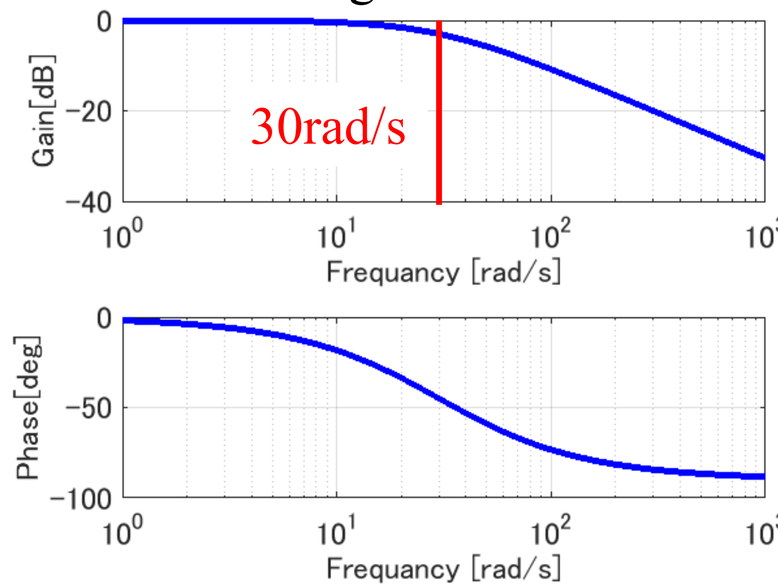
Saturation

Block diagrams of canard



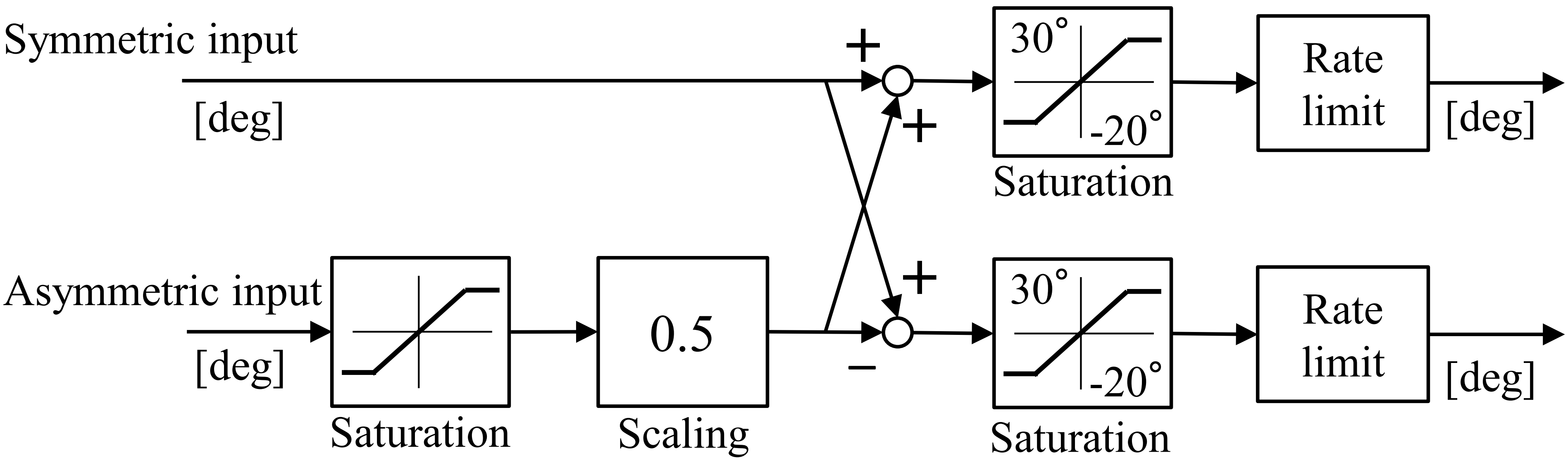
Saturation

Bode diagram of actuators

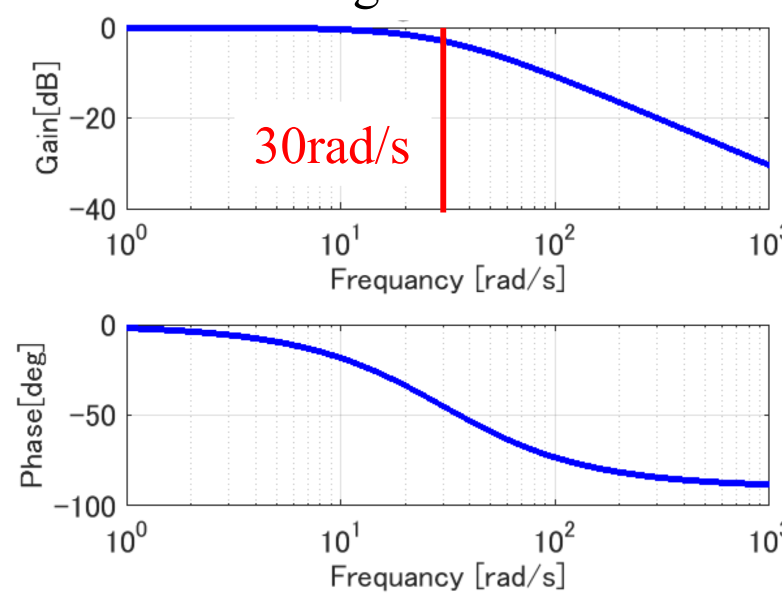


Actuators: Hydraulic serve system

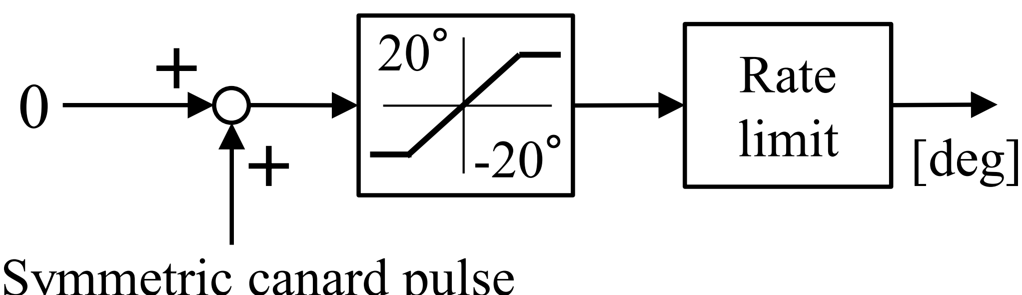
Block diagrams of elevon



Bode diagram of actuators



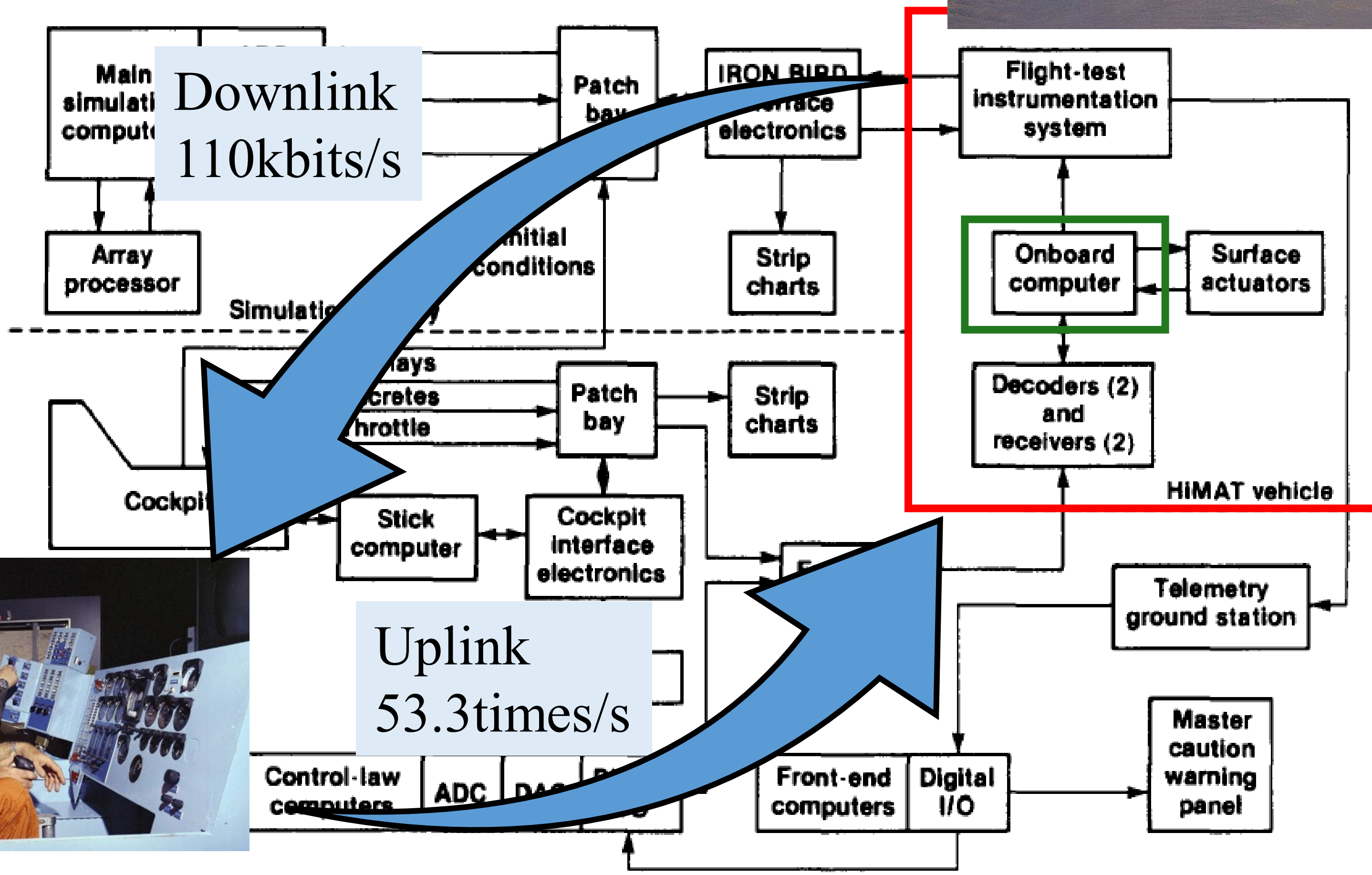
Block diagrams of canard



Loop Time: Control Input time

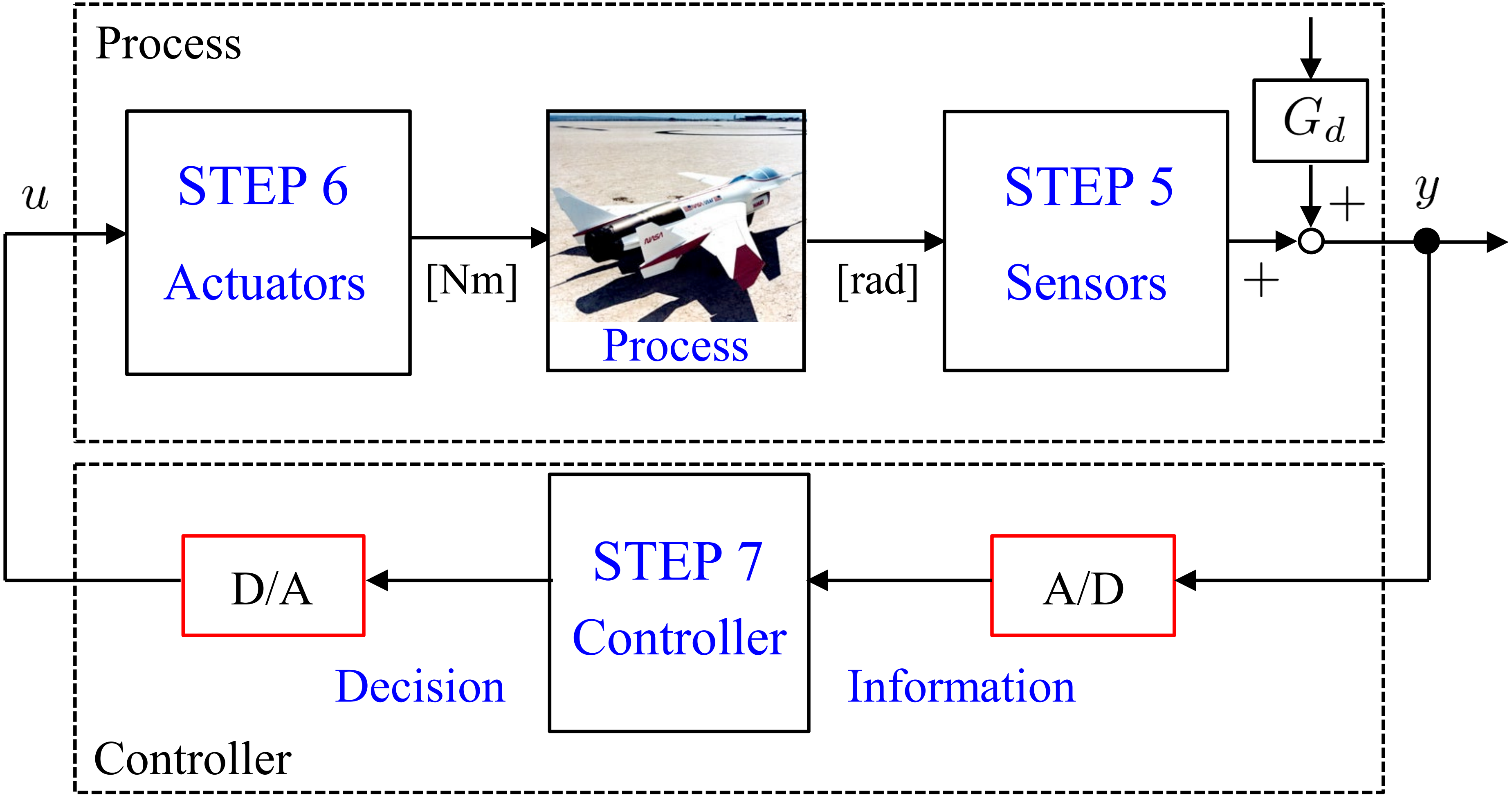
Onboard computer

Processing Time	25 ms (40Hz)



Models: Understand Performance Specifications

G_d : Process Noise



HiMAT: Closed-loop Performance (without update W_P)

Nominal Stability (NS)

Poles of $F_l(G, K)$ ○

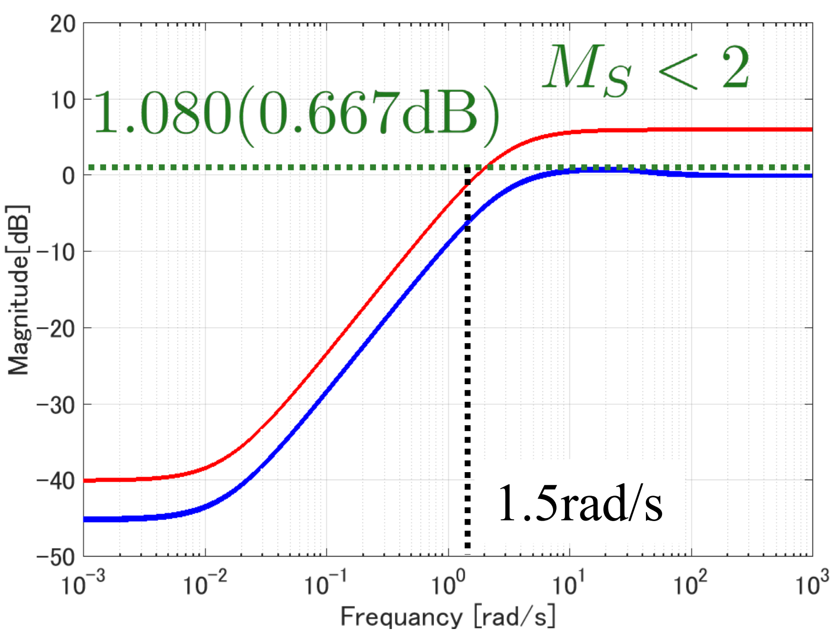
$$p = -0.0150, -0.0150, -0.0210, -0.2578, -2.9920, -3.0011$$
$$-5.6757, -30, -30, -45.8984, -5.7180 \cdot 10^2, -7.5812 \cdot 10^2$$
$$-0.6898 \pm 0.2488j, -34.1085 \pm 28.5745j$$

Zeros of $F_l(G, K)$ ○

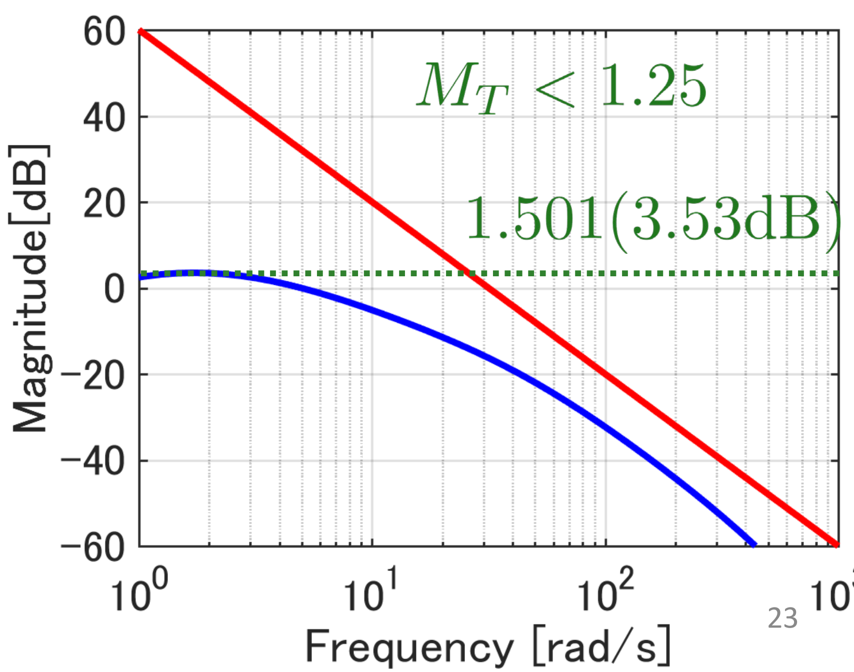
Nothing

$\Rightarrow$ Pole/Zero Cancellations

Nominal Performance (NP)



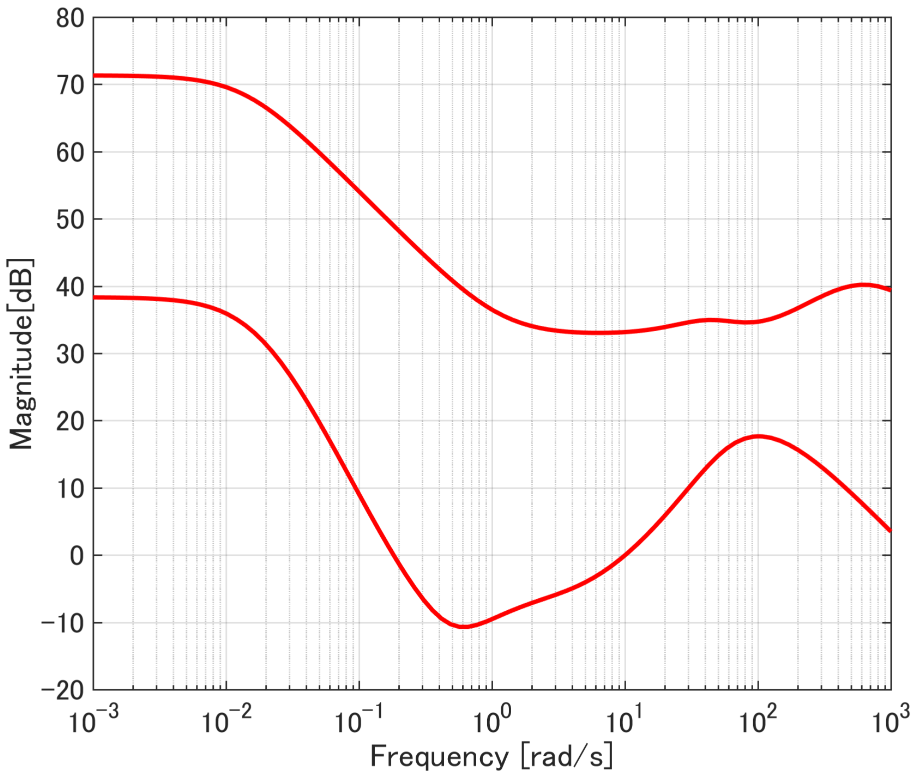
Robust Stability (RS)



HiMAT: Controller (without update W_P)

MATLAB Command
figure sigma(Khi)

$$K(s) = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix}$$



Poles

$$-7.5904 \cdot 10^2, -5.7002 \cdot 10^2$$
$$-35.6460 \pm 30.3447j$$
$$-52.3996, -0.0210$$
$$-0.0148, -0.0148$$

Zeros

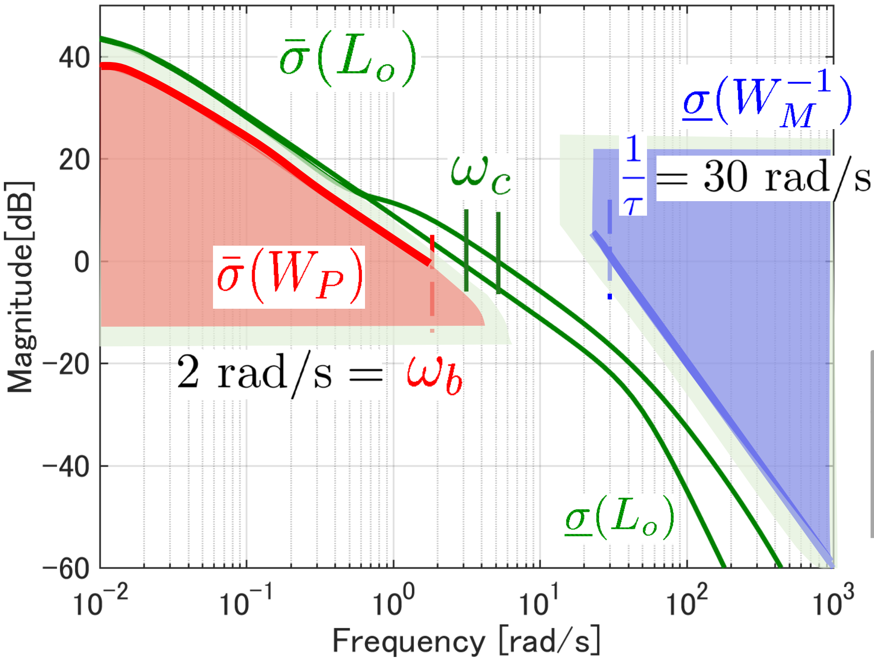
$$-0.3584 \pm 0.3654j$$
$$-30, -30$$
$$-5.6757, -0.2578$$

Order 8

Numerical problems or inaccuracies may be caused too high order

➡ Difficult to implement

HiMAT: Open-loop Frequency Response (without update W_P)



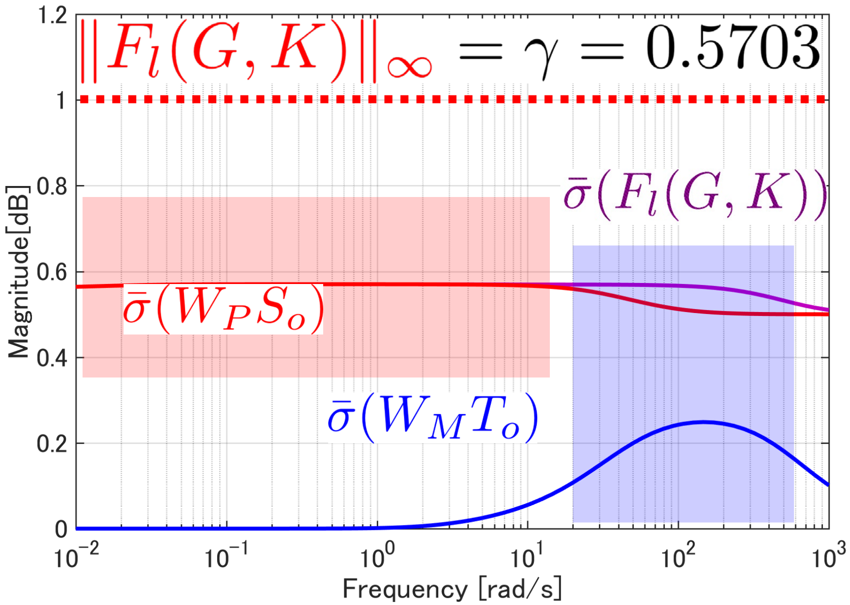
$\gamma = < 1$

(corresponding maximum stability margin)

Loop Transfer Function

MATLAB Command

```
figure
sigma(Fhi.Lo,WP, inv(WM),WP/ghi,ghi*inv(WM))
```



NP/RS Test

Nominal Performance (NP)

$\|W_P S_o\|_\infty = 0.570 < 1$

Robust Stability (RS)

$\|W_M T_o\|_\infty = 0.249 < 1$

MATLAB Command

```
[SV,w]=sigma(WP*Fhi.So);
figure; semilogx(w,SV)
[SV,w]=sigma(WM*Fhi.To);
figure; semilogx(w,SV)
```

HiMAT: γ-iteration to obtain H_∞ Controller (without update W_P)

Find K such that $\|F_l(G,K)\|_\infty < \gamma$

[SP05, p. 358]

Check 1)

Appropriately sub-optimal

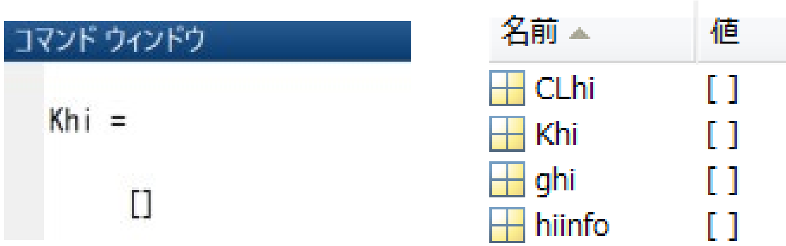
(Default settings)

$\gamma = 0.5703 < 1 \quad (\gamma_{opt} = 0.5438)$

Check 2) $\gamma = 0.5$ No Solution

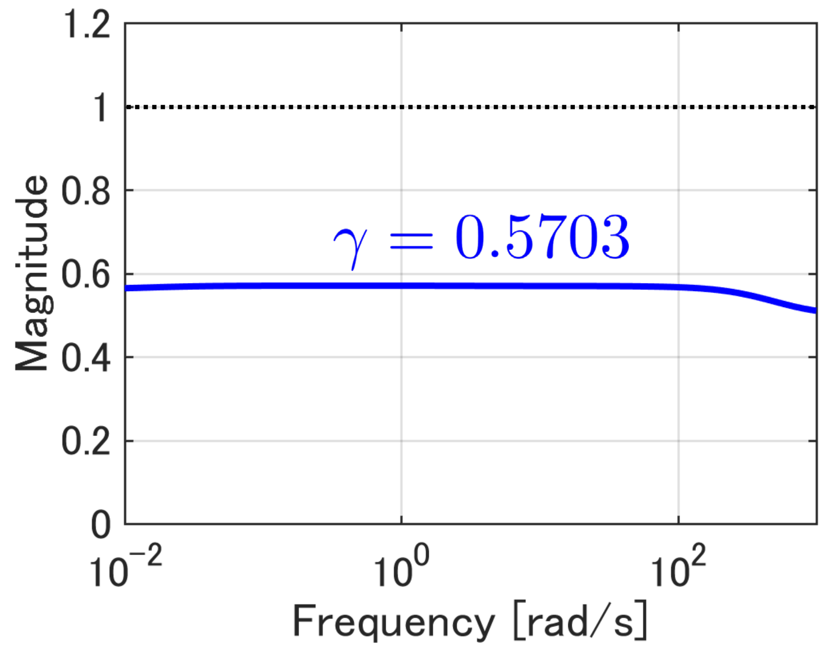
Test) Tune W_M and sub-optimality

Data Structure



There is No Controller

Closed-loop Transfer Function $\bar{\sigma}(F_l(G,K))$



"interested" frequency range

MATLAB Command

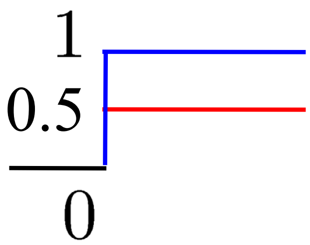
```
[Khi,CLhi,ghi,hiinfo] = …
hinfsyn(G,nmeas,ncon,'Gmax',100,'Gmin',100);
```

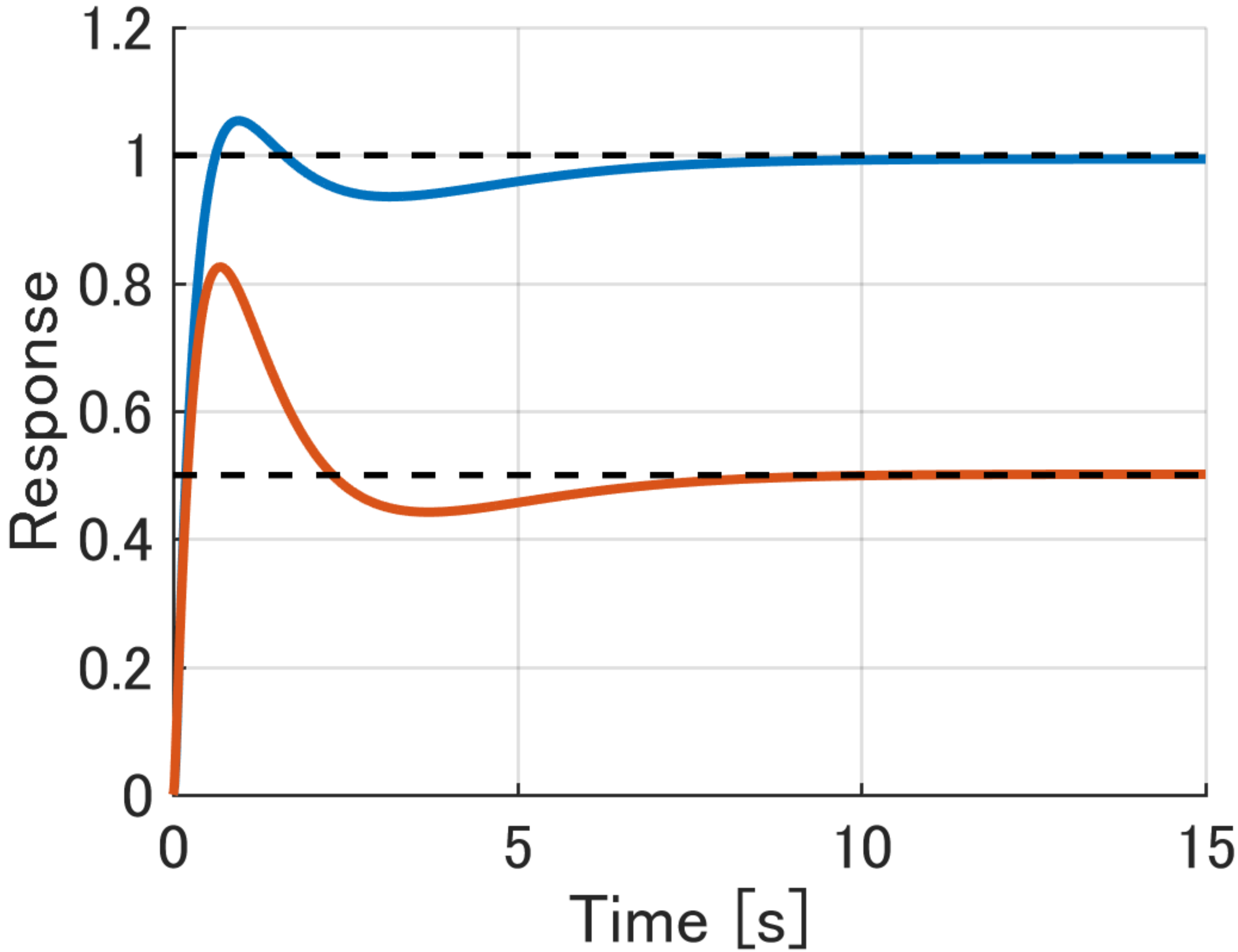
MATLAB Command

```
[SV,w]=sigma(CLhi);
figure; semilogx(w,SV)
```

HiMAT: Step Response of closed loop (without update W_P)

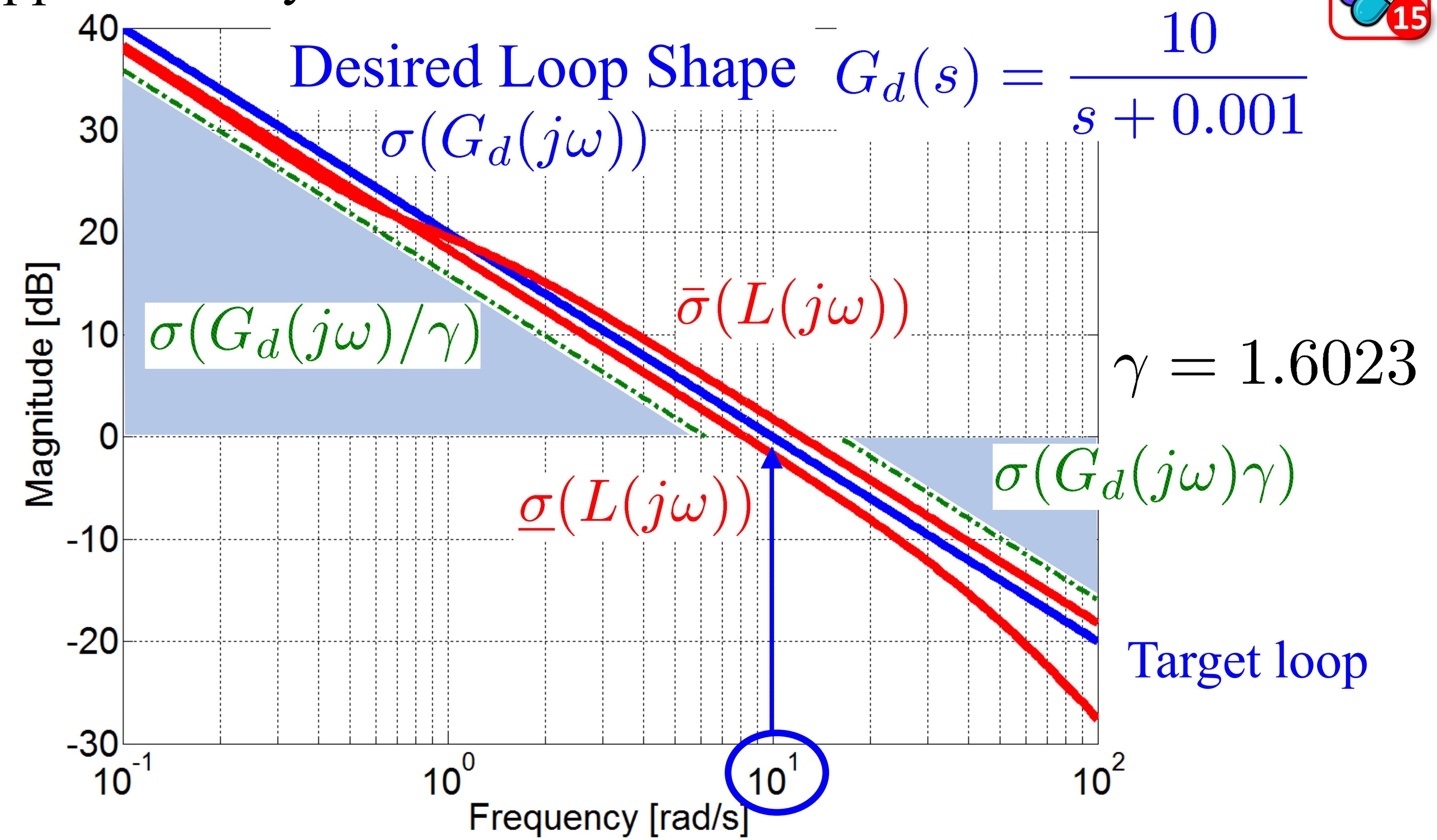
$$w(t) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$





HIMAT: Specifications and Open-loop (Loop Shaping Synthesis)

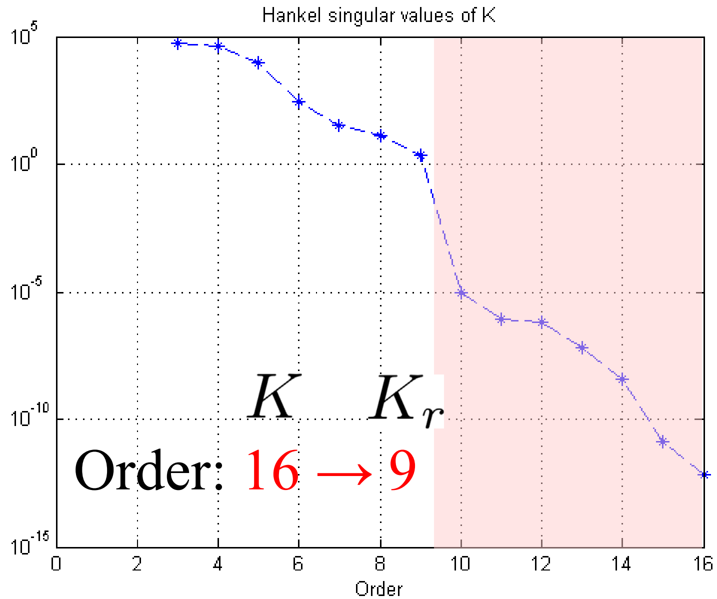
Approximately a bandwidth of 10 rad/s

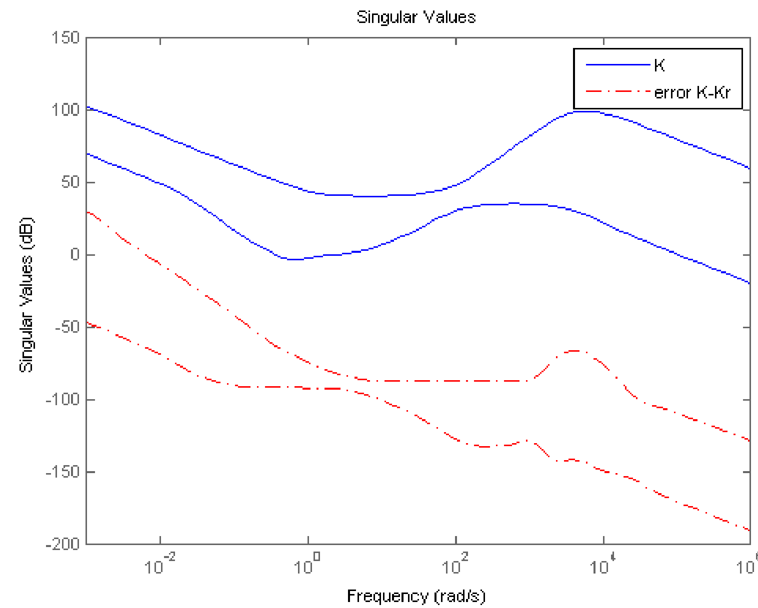


MATLAB Command

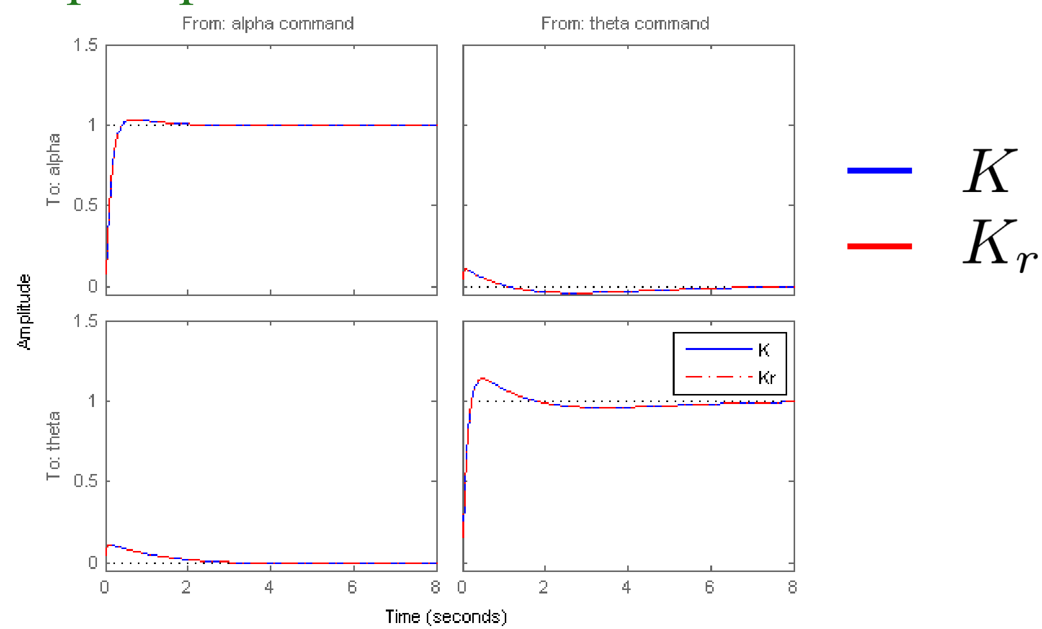
```
Gd= tf( 10, [1 .001] ) ;
[K,CL,GAM,INFO]=loopsyn(G,Gd);
sigma(Gd,‘b',G*K,'r',Gd/GAM,‘g:',Gd*GAM,‘g:',{.1,100})
```

HIMAT: Controller Model Reduction (Loop Shaping Synthesis)





Step responses



MATLAB Command

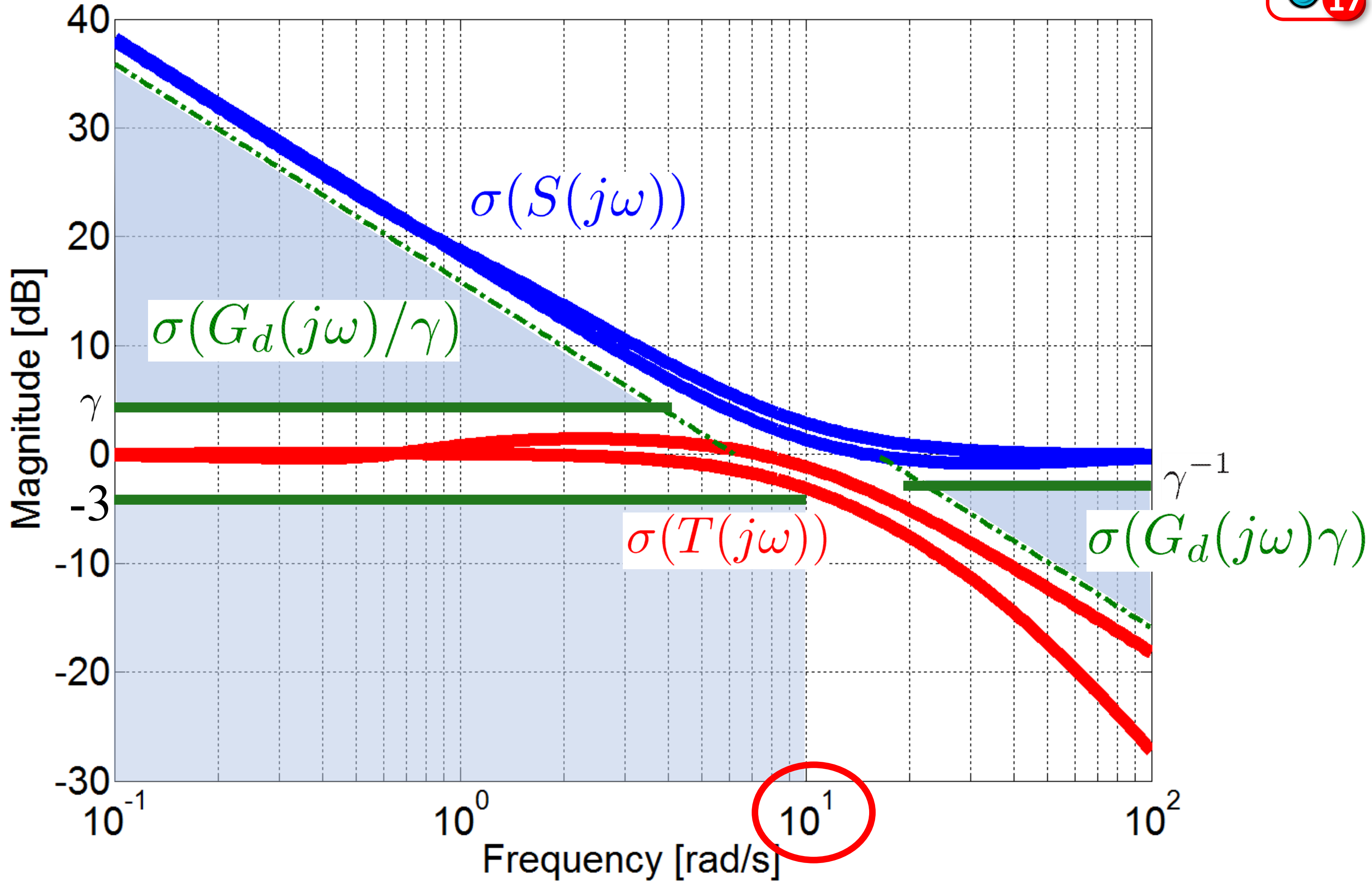
```
size(K)

hsv = hankelsv(K);
semilogy(hsv,'*--'), grid
title('Hankel singular values of K'),
xlabel('Order')

Kr = reduce(K,9); order(Kr)

sigma(K,'b',K-Kr,'r-.')
legend('K','error K-Kr')
```

HIMAT: Sensitivity (Loop Shaping Synthesis)

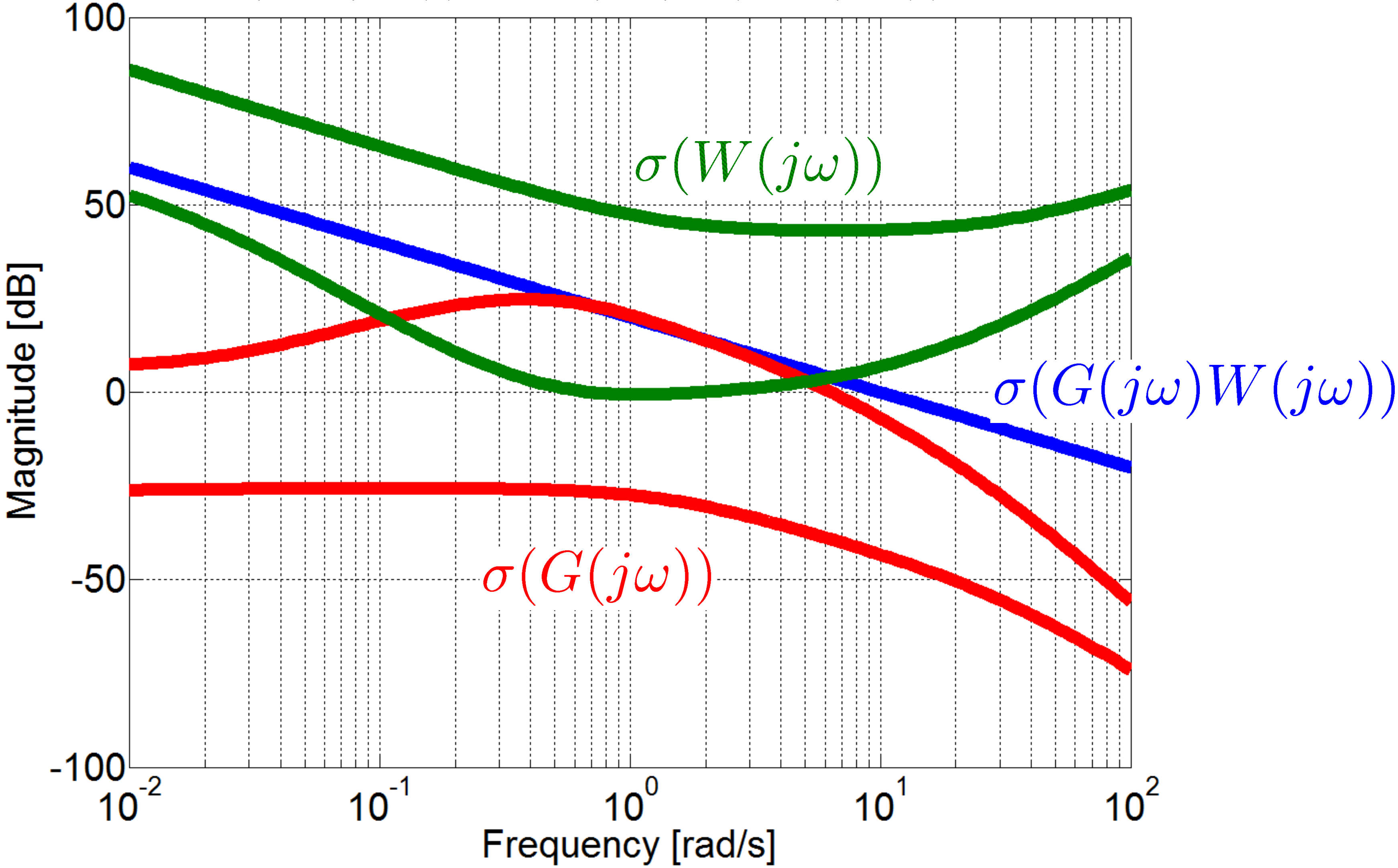


MATLAB Command

```
T = feedback(G*K,eye(2));
S = eye(2)-T;
sigma(inv(S),'m',T,'g',L,'r--',Gd,'b',Gd/GAM,'b:',Gd*GAM,'b:',{.1,100})
```

HIMAT: Designed Weight (Loop Shaping Synthesis)

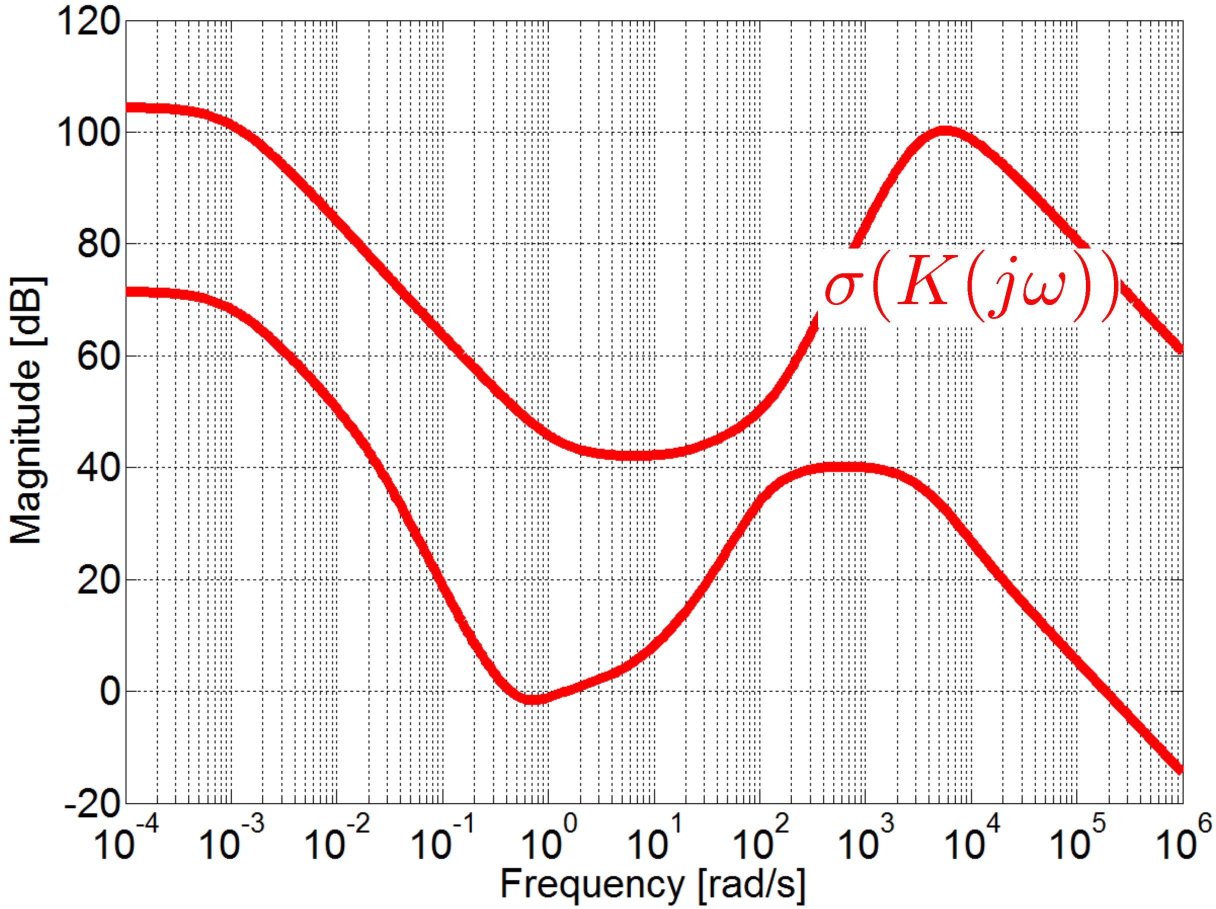
$$\sigma(G_d(j\omega)) \approx \sigma(G(j\omega)W(j\omega)),\ \forall\omega$$



MATLAB Command

```
sigma(INFO.Gs,‘b',G,'r',INFO.W,‘g',{.01,100});
```

HIMAT: Controller (Loop Shaping Synthesis)



Order: 16

MATLAB Command
sigma(K);

Numerical problems or inaccuracies may be caused too high order

➡ Difficult to implement

HIMAT: Step Response of closed loop (Loop Shaping Synthesis)

$$w(t) = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$$