Robust Control 1/4/2016

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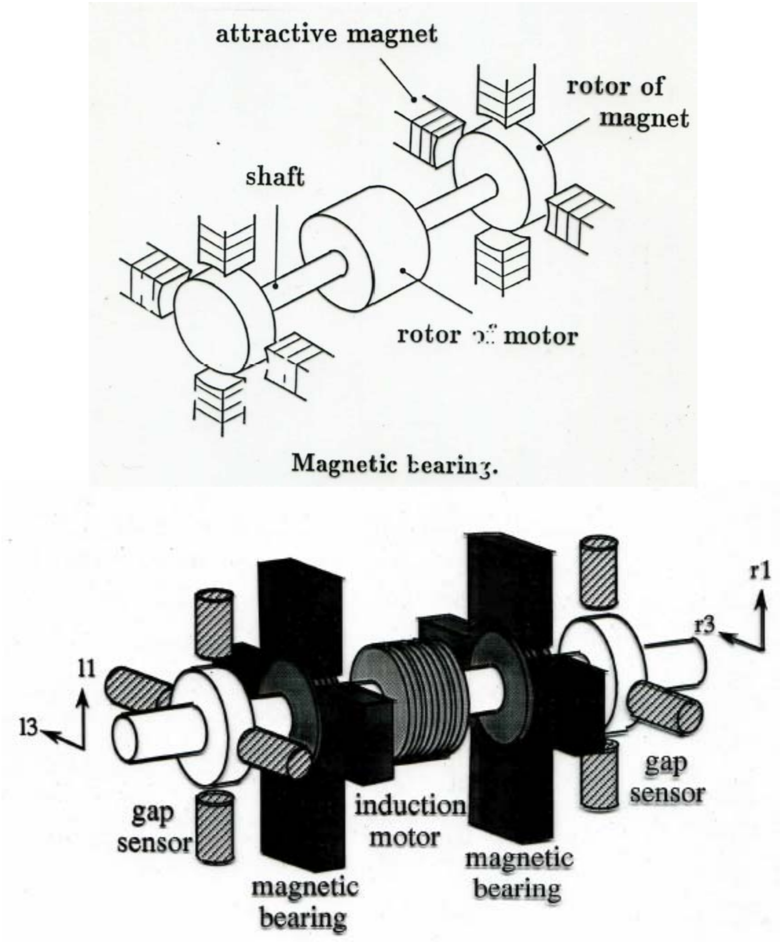
T: Magnetic Bearing: Robust Performance

Reference:

M. Fujita, K. Hatake, F. Matsumura and K. Uchida
An Experimental Evaluation and Comparison of H_∞/μ *Control for a Magnetic Bearing*
12th IFAC World Congress, Sydney, Australia, July 18-23, 1993.

Magnetic Bearing

Real Physical System



Parameter	Value
Mass of the Rotor : m	$1.39e1$ [kg]
Moment of Inertia about X : J_x	$1.348e{-2}$ [kg m^2]
Moment of Inertia about Y : J_y	$2.326e{-1}$ [kg m^2]
Distance between Center of Mass and Left Electromagnet : l_l	$1.30e{-1}$ [m]
Distance between Center of Mass and Right Electromagnet : l_r	$1.30e{-1}$ [m]
Distance between Center of Mass and Motor : l_m	0 [m]
Steady Attractive Force : $F_{l1,r1}$	$9.09e1$ [N]
Steady Attractive Force : $F_{l2\sim l4,r2\sim r4}$	$2.20e1$ [N]
Steady Current : $I_{l1,r1}$	$6.3e{-1}$ [A]
Steady Current : $I_{l2\sim l4,r2\sim r4}$	$3.1e{-1}$ [A]
Steady Gap : W	$5.5e{-4}$ [m]
Resistance : R	$1.07e1$ [Ω]
Inductance : L	$2.85e{-1}$ [H]

Figure Magnetic Bearing

Assumptions

- The rotor is rigid and has no unbalance
- All Electromagnets are identical
- Attractive force of an electromagnet is in proportion to the square of the ratio of the electric current to the gap length
- The resistance and the inductance of the electromagnet coil are constant and independent of the gap length
- Small deviations from the equilibrium point are treated

Nominal Model

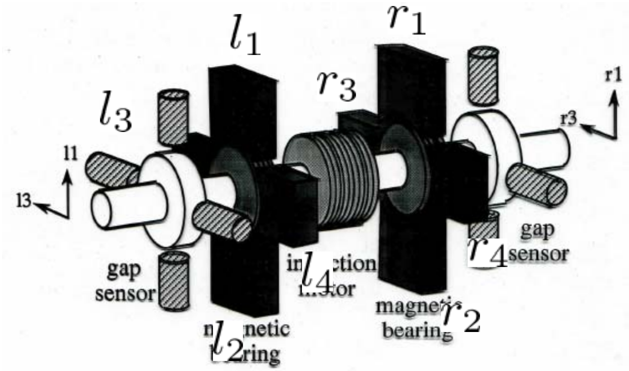
State-Space Representation

$$\begin{bmatrix} \dot{x}_v \\ \dot{x}_h \end{bmatrix} = \begin{bmatrix} A_v & pA_{vh} \\ -pA_{vh} & A_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} B_v & 0 \\ 0 & B_h \end{bmatrix} \begin{bmatrix} u_v \\ u_h \end{bmatrix}$$

$$\begin{bmatrix} y_v \\ y_h \end{bmatrix} = \begin{bmatrix} C_v & 0 \\ 0 & C_h \end{bmatrix} \begin{bmatrix} x_v \\ x_h \end{bmatrix}$$

$$x_v = [g_{l1} \ g_{r1} \ \dot{g}_{l1} \ \dot{g}_{r1} \ i_{l1} \ i_{r1}]^T \qquad u_v = [e_{l1} \ e_{r1}]^T$$

$$x_h = [g_{l3} \ g_{r3} \ \dot{g}_{l3} \ \dot{g}_{r3} \ i_{l3} \ i_{r3}]^T \qquad u_h = [e_{l3} \ e_{r3}]^T$$



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g: deviations from the steady gap lengths between the electromagnets and the rotor

i: deviations from the steady currents of the electromagnets

e: deviations from the steady voltages of the electromagnets

- The subscripts "*v*" and "*h*" stand for the vertical motion and horizontal motion of the magnetic bearing.

Mathematical Model

$$A_v = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 5.9365e+04 & -2.9332e+02 & 0 & 0 & -6.2251e+01 & 3.0759e-01 \\ -2.9332e+02 & 5.9365e+04 & 0 & 0 & 3.0759e-01 & -6.2251e+01 \\ 0 & 0 & 0 & 0 & -3.7544e+01 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.7544e+01 \end{bmatrix}$$

$$A_h = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2.3136e+04 & -1.1432e+02 & 0 & 0 & -4.1048e+01 & 2.0282e-01 \\ -1.1432e+02 & 2.3136e+04 & 0 & 0 & 2.0282e-01 & -4.1048e+01 \\ 0 & 0 & 0 & 0 & -3.7544e+01 & 0 \\ 0 & 0 & 0 & 0 & 0 & -3.7544e+01 \end{bmatrix}$$

$$A_{vh} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3.0344e-03 & 3.0344e-03 & 0 & 0 \\ 0 & 0 & 3.0344e-03 & -3.0344e-03 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B_v = B_h = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3.5088 & 0 \\ 0 & 3.5088 \end{bmatrix}$$

$$C_v = C_h = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Nominal Model

- Gyroscopic effect : $p \neq 0$
- If $p = 0$ (ignore gyroscopic effect)

  - (v) Vertical plant G_v

$$G_v(s) = C_v(sI - A_v)^{-1}B_v$$

  - (h) Horizontal plant G_h

$$G_h(s) = C_h(sI - A_h)^{-1}B_h$$

- Nominal model

$$G = \begin{bmatrix} G_v & 0 \\ 0 & G_h \end{bmatrix}$$

Model Uncertainty

- Perturbation (gyro effect $p \neq 0$) $G_p(s) = (I + \Delta_p)$

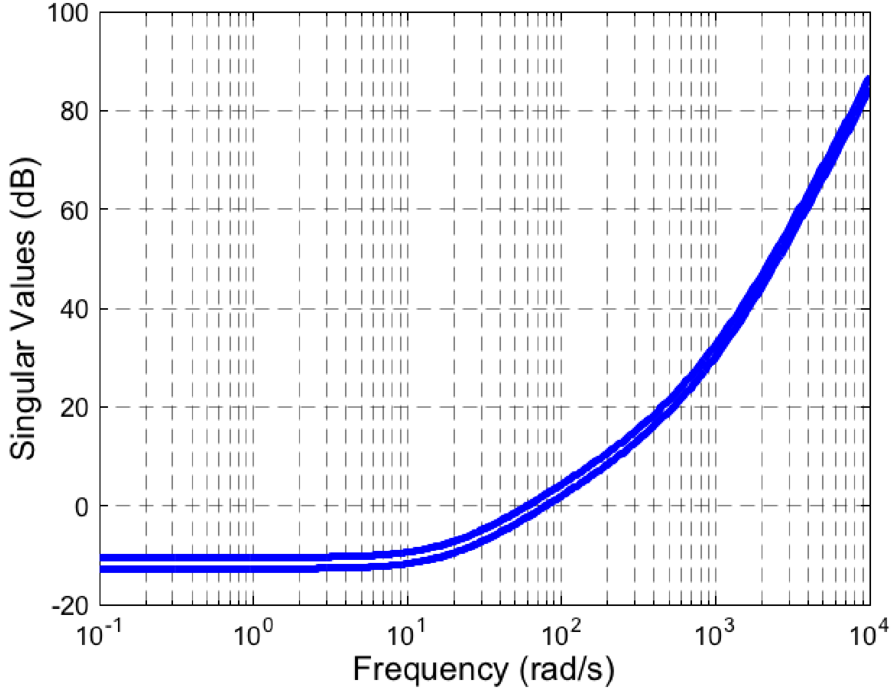
- Uncertainty weight

$$W_T(s) = (1 + \frac{s}{2\pi \cdot 19})(1 + \frac{s}{2\pi \cdot 500})(1 + \frac{s}{2\pi \cdot 1500}) \begin{bmatrix} 0.23 & 0 & 0 & 0 \\ 0 & 0.23 & 0 & 0 \\ 0 & 0 & 0.23 & 0 \\ 0 & 0 & 0 & 0.23 \end{bmatrix}$$

- Robust stability

$$\|W_T GK(I - GK)^{-1}\|_\infty$$

$$= \|W_T T\|_\infty < 1$$



Performance

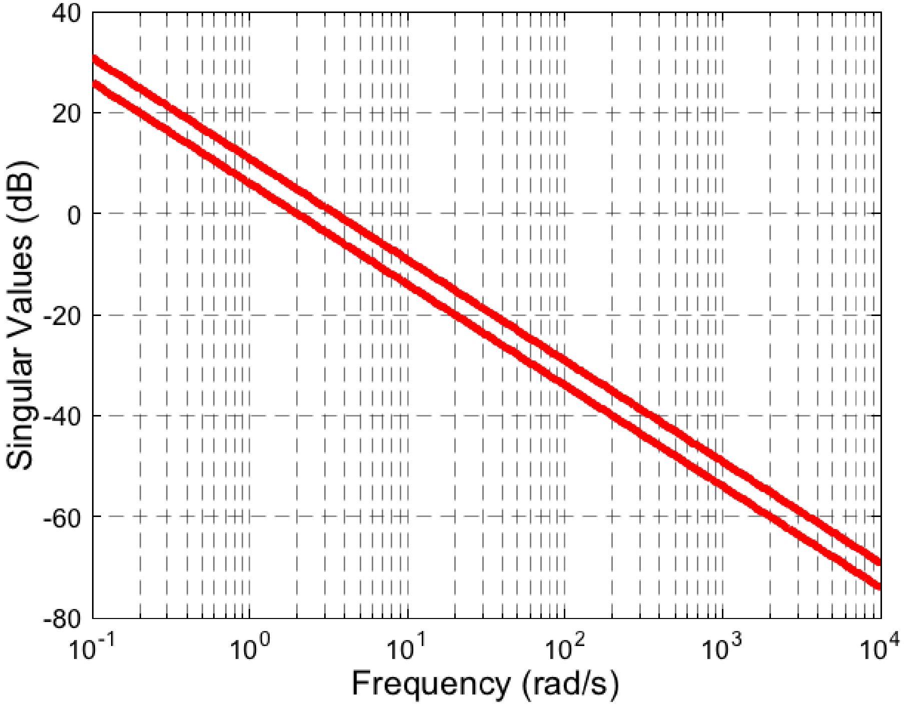
- Performance weight

$$W_S(s) = \frac{1}{1 + s/(2\pi \cdot 0.01)} \begin{bmatrix} 200 & 0 & 0 & 0 \\ 0 & 200 & 0 & 0 \\ 0 & 0 & 350 & 0 \\ 0 & 0 & 0 & 350 \end{bmatrix}$$

- Nominal performance

$$\|W_S(I - GK)^{-1}\|_\infty$$

$$= \|W_S S\|_\infty < 1$$



Loop Shaping

- For frequencies: $\underline{\sigma}(GK) >> 1$

$$\bar{\sigma}((I - GK)^{-1}) = \frac{1}{\underline{\sigma}(I - GK)} \leq \frac{1}{\underline{\sigma}(GK) - 1}$$
$$\approx \frac{1}{\underline{\sigma}(GK)}$$

- For frequencies: $\bar{\sigma}(GK) << 1$

$$\bar{\sigma}(GK(I - GK)^{-1}) = \frac{1}{\underline{\sigma}((GK)^{-1} - I)} \leq \frac{1}{\underline{\sigma}((GK)^{-1}) - 1}$$
$$\approx \bar{\sigma}(GK)$$

- Loop shaping

$$\underline{\sigma}(GK) > \bar{\sigma}(W_s): \qquad \underline{\sigma}(GK) >> 1$$
$$\bar{\sigma}(GK) > \bar{\sigma}(W_T^{-1}): \qquad \bar{\sigma}(GK) << 1$$

Loop Shaping Design Procedure

[Step 1] Loop Shaping

Selecting shaping functions W_1 and W_2, the singular values of the nominal plant G are shaped to have a desired open loop shape. Let G_s represent this shaped plant,

$$G_s = W_2 G W_1$$

W_1 and W_2 should be selected such that G_s has no hidden unstable modes.

Loop Shaping Design Procedure

[Step 2] Robust Stabilization

The maximum stability margin $\varepsilon_{\max}$ is calculated.

If $\varepsilon_{\max} << 1$, return to Step 1 and W_1 and W_2 are reselected.

Otherwise, ε is appropriately selected as $\varepsilon \leq \varepsilon_{\max}$, and an H_∞ controller K_∞ is synthesized for G_s.

[Step 3] Final Controller

The final controller K can be obtained by the combination of W_1, W_2 and K_∞ as

$$K = W_1 K_\infty W_2$$

Loop Shaping Design Procedure

- Normalized left coprime factorization

$$G = M^{-1}N \qquad\qquad MM^* + NN^* = I$$

- Uncertainties

$$G_\Delta = M_\Delta^{-1} N_\Delta = (M + M_{\Delta_M})(N + N_{\Delta_N})$$

$$\Delta = [\Delta_N,\ \Delta_M] \in RH_\infty \qquad \|\Delta\|_\infty < \varepsilon$$

- Robust stabilizing problem

$$\left\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \right\|_\infty \le \varepsilon^{-1} = \gamma$$

Loop Shaping Design Procedure

$$\left\| \begin{array}{cc} (I-GK)^{-1} & (I-GK)^{-1}G \\ K(I-GK)^{-1} & K(I-GK)^{-1}G \end{array} \right\|_\infty \leq \varepsilon^{-1} = \gamma$$

$$\varepsilon_{\max} = \inf_{K stabilizing} \left\| \left[\begin{array}{c} K \\ I \end{array} \right] (I-GK)^{-1} M^{-1} \right\|_\infty$$

$$= (1 - \|N, M\|_H^2)^{-1/2}$$

$$\|N, M\|_H^2 = \lambda_{max}(PQ) = \lambda_{max}(ZX(I+ZX)^{-1})$$

$$K = \left[\begin{array}{c|c} A^c + \gamma^2 W_1^{*-1} Z C^* (C + DF) & \gamma^2 W_1^{*-1} Z C^* \\ \hline B^* X & -D^* \end{array} \right]$$

$$F = -S^{-1}(D^*C + B^*X) \qquad A^c = A + BF$$

$$W_1 = I + (XZ - \gamma^2 I)$$

Design for vertical motion

$$W_{1v}(s) = \frac{1300(1+s/(2\pi\cdot 5))(1+s/(2\pi\cdot 35))(1+s/(2\pi\cdot 50))}{(1+s/(2\pi\cdot 0.01))(1+s/(2\pi\cdot 700))(1+s/(2\pi\cdot 1200))}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$W_{2v}(s) = 10000\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\varepsilon_{\mathrm{max}v} = 0.19944 \qquad \varepsilon_v^{-1} = \gamma_v = 5.25$$

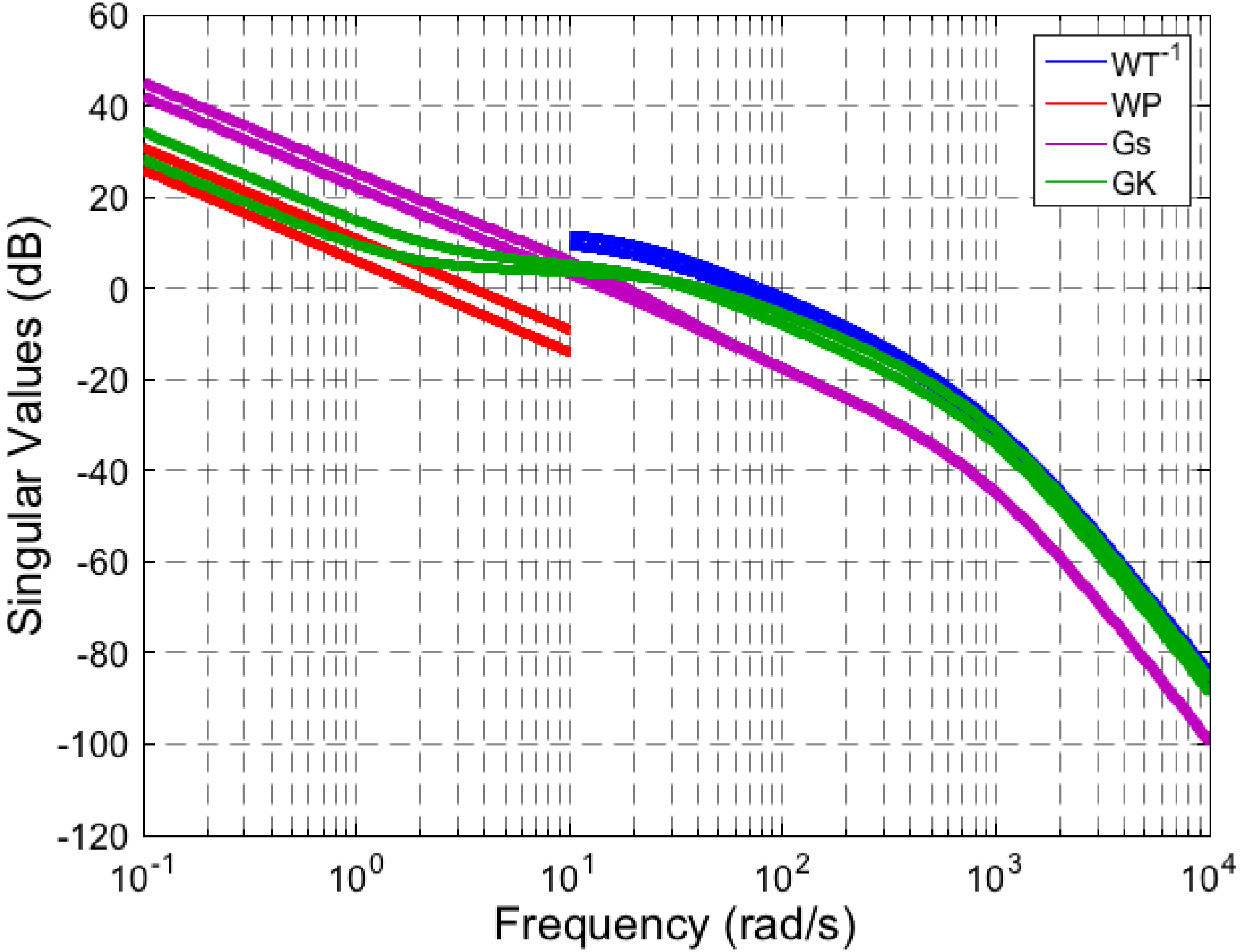
Design for horizontal motion

$$W_{1h}(s) = \frac{1100(1+s/(2\pi\cdot 5))(1+s/(2\pi\cdot 25))(1+s/(2\pi\cdot 40))}{(1+s/(2\pi\cdot 0.01))(1+s/(2\pi\cdot 700))(1+s/(2\pi\cdot 1200))}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$W_{2h}(s) = 10000\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

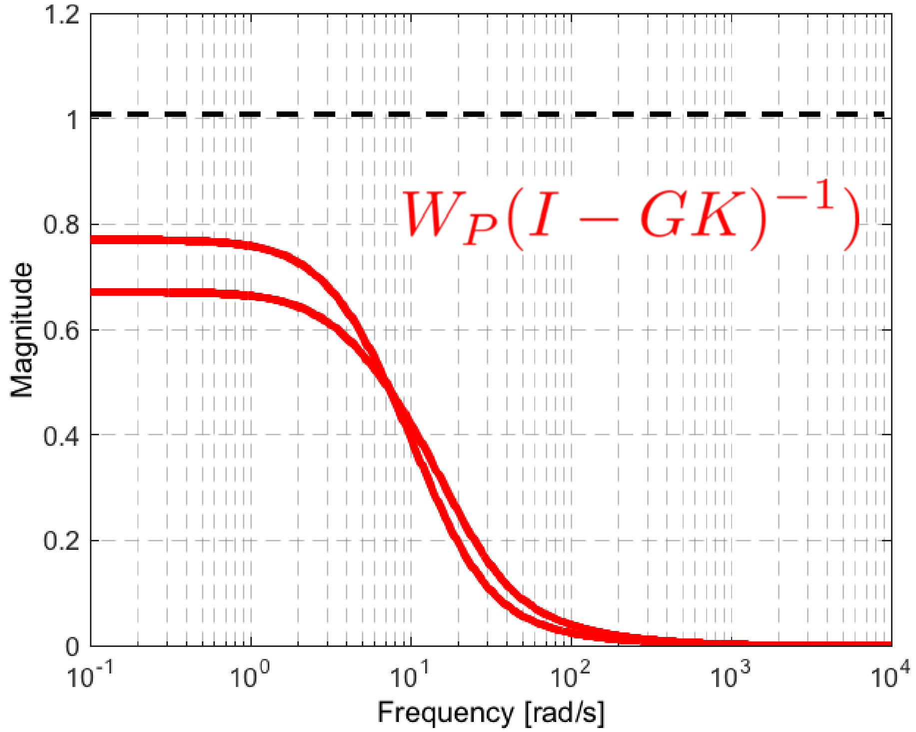
$$\varepsilon_{\mathrm{max}h} = 0.27432 \qquad \varepsilon_h^{-1} = \gamma_h = 3.75$$

Loop Transfer Function



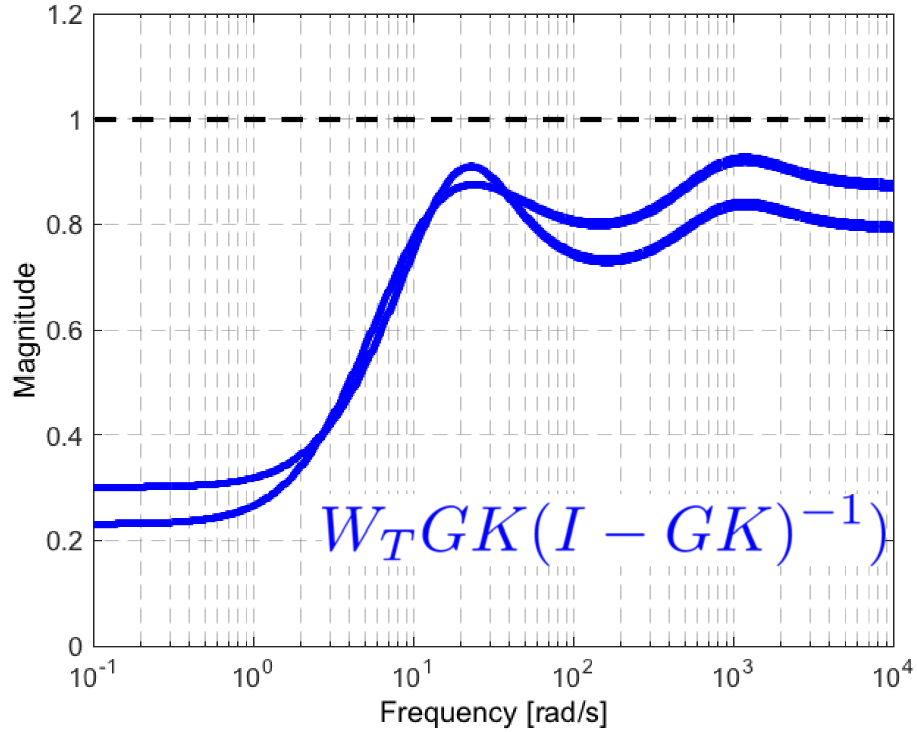
Nominal Performance and Robust Stability

NP Test



$$\bar{\sigma}(W_P(I - GK)^{-1}) < 1$$

RS Test



$$\bar{\sigma}(W_T GK(I - GK)^{-1}) < 1$$

Interconnection Structure

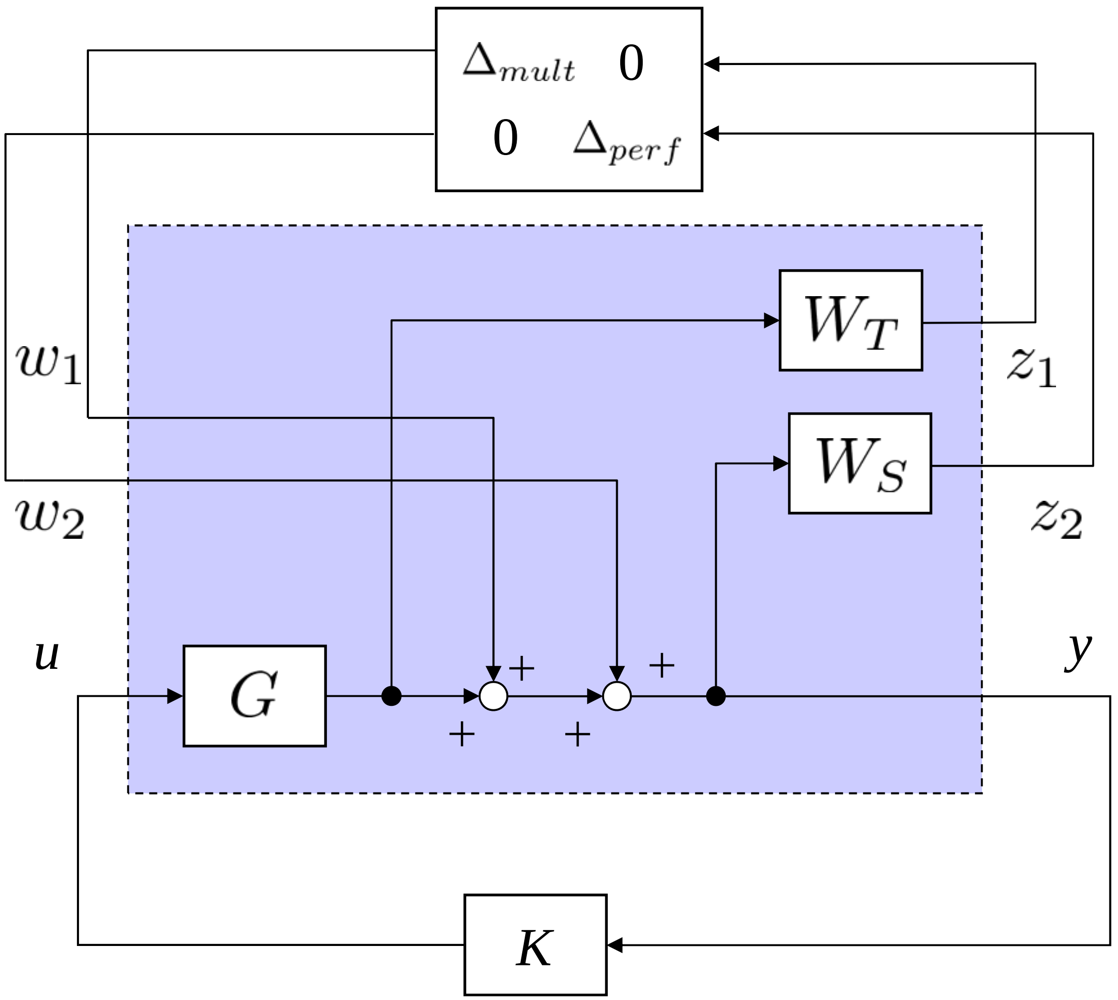
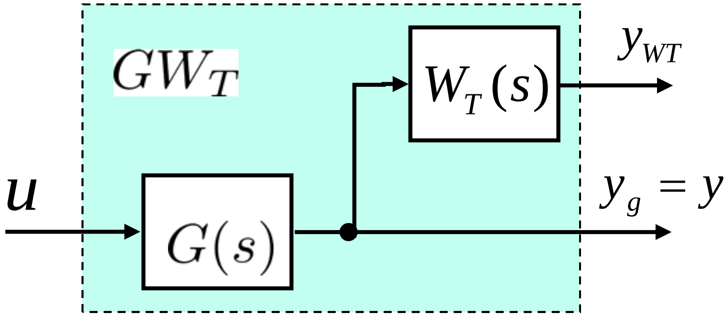


Fig. Feedback Structure

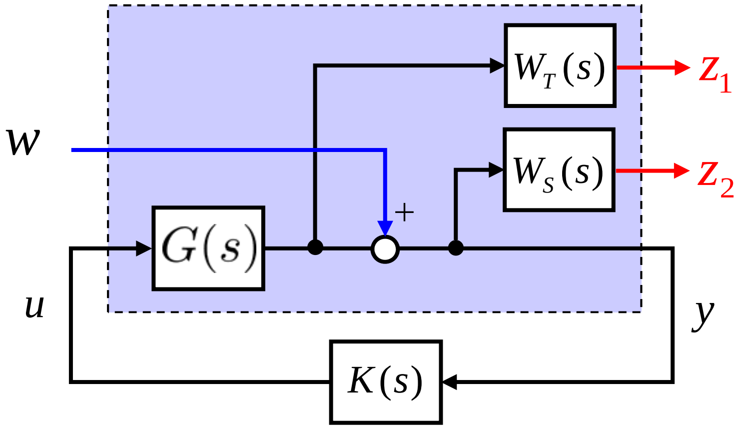
Mixed Sensitivity Design

$$\left\| \begin{array}{c} W_S(I-GK)^{-1} \\ W_T GK(I-GK)^{-1} \end{array} \right\| < \gamma_{min} \qquad \gamma_{min} = 0.87$$



MATLAB Command

```
systemnames = 'G WT';
inputvar = '[ u(4) ]';
outputvar = '[ WT; G ]';
input_to_G = '[ u ]';
input_to_WT = '[ G ]';
GWT = sysic;
```



MATLAB Command

```
% Generalized Plant
systemnames = 'WP GWT';
inputvar = '[ w(4); u(4) ]';
outputvar = '[ WP; GWT(1:4); GWT(5:8)+w ]';
input_to_WP = '[ GWT(5:8)+w ]';
input_to_GWT = '[ u ]';
Gmix = sysic;
nmeas = 4; ncon = 4;
[Kmix, CLmix, gammix, infomix] =
hinfsyn(Gmix, nmeas, ncon, 'tolgam', 0.1);
```

μ -Controller

- Set of Plants

$$\mathcal{G} := \{(I + \Delta_{mult} W_T) G_{nom} : \|\Delta_{mult}\|_\infty \leq 1\}$$

- Robust Stability

$$\|W_T G_{nom} K (I - G_{nom} K)^{-1}\|_\infty < 1$$

- Robust Performance

$$\|W_S (I - (I + \Delta_{mult} W_T) G_{nom} K)^{-1}\|_\infty < 1$$

μ -Controller: Structured Singular Value

- Linear Fractional Transformation

$$F_l(P,K) := P_{11} + P_{12}K(I - P_{22}K)P_{21} \qquad P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$$

- Block Structure

$$\mathbf{\Delta} := \left\{ \begin{bmatrix} \Delta_{mult} & 0 \\ 0 & \Delta_{perf} \end{bmatrix} : \Delta_{mult} \in C, \mathbf{\Delta}_{perf} \in C \right\}$$

- Structured Singular Value

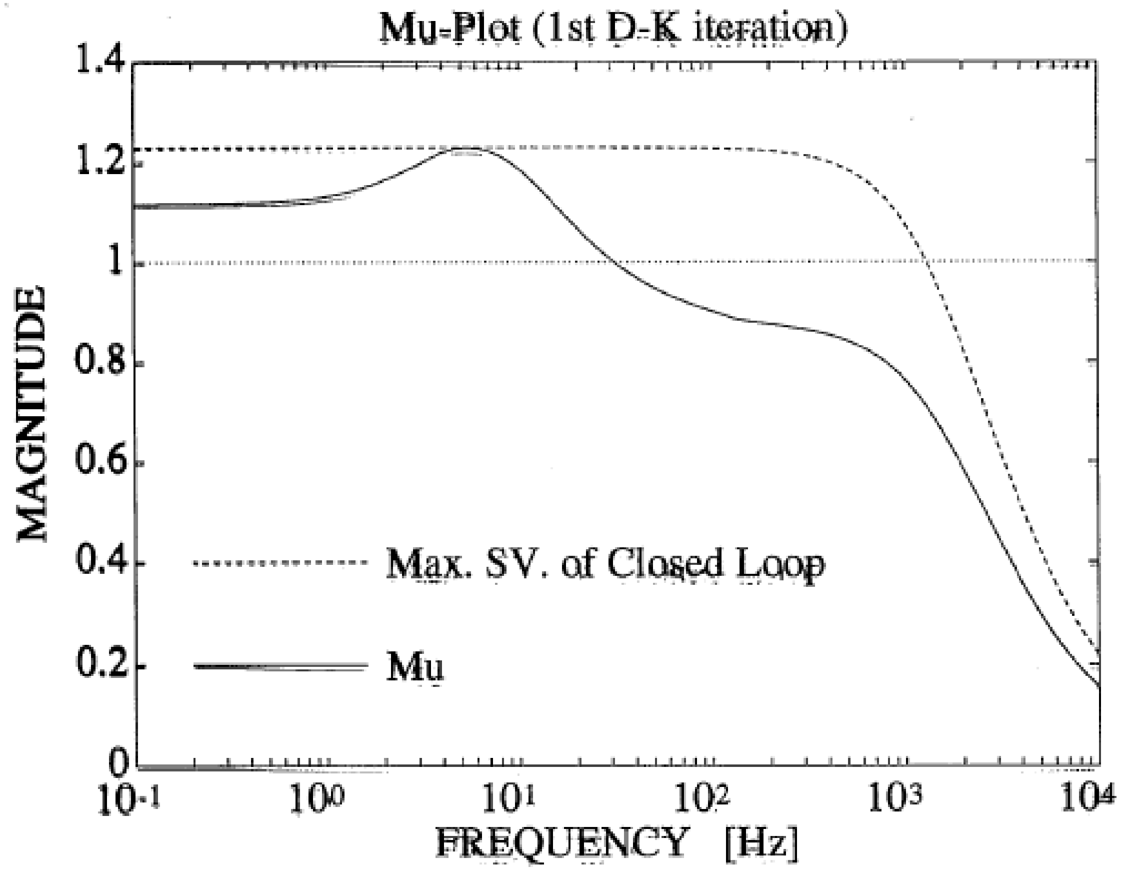
$$\mu_\Delta(M) := \frac{1}{\min\{\bar{\sigma}(\Delta) : \Delta \in \mathbf{\Delta}, \det(I - M\Delta) = 0\}}$$

- Robust Performance Test

$$\sup_{\omega \in R} \mu_\Delta(F_l(P,K)(j\omega)) < 1$$

μ -Controller: D-K Iteration

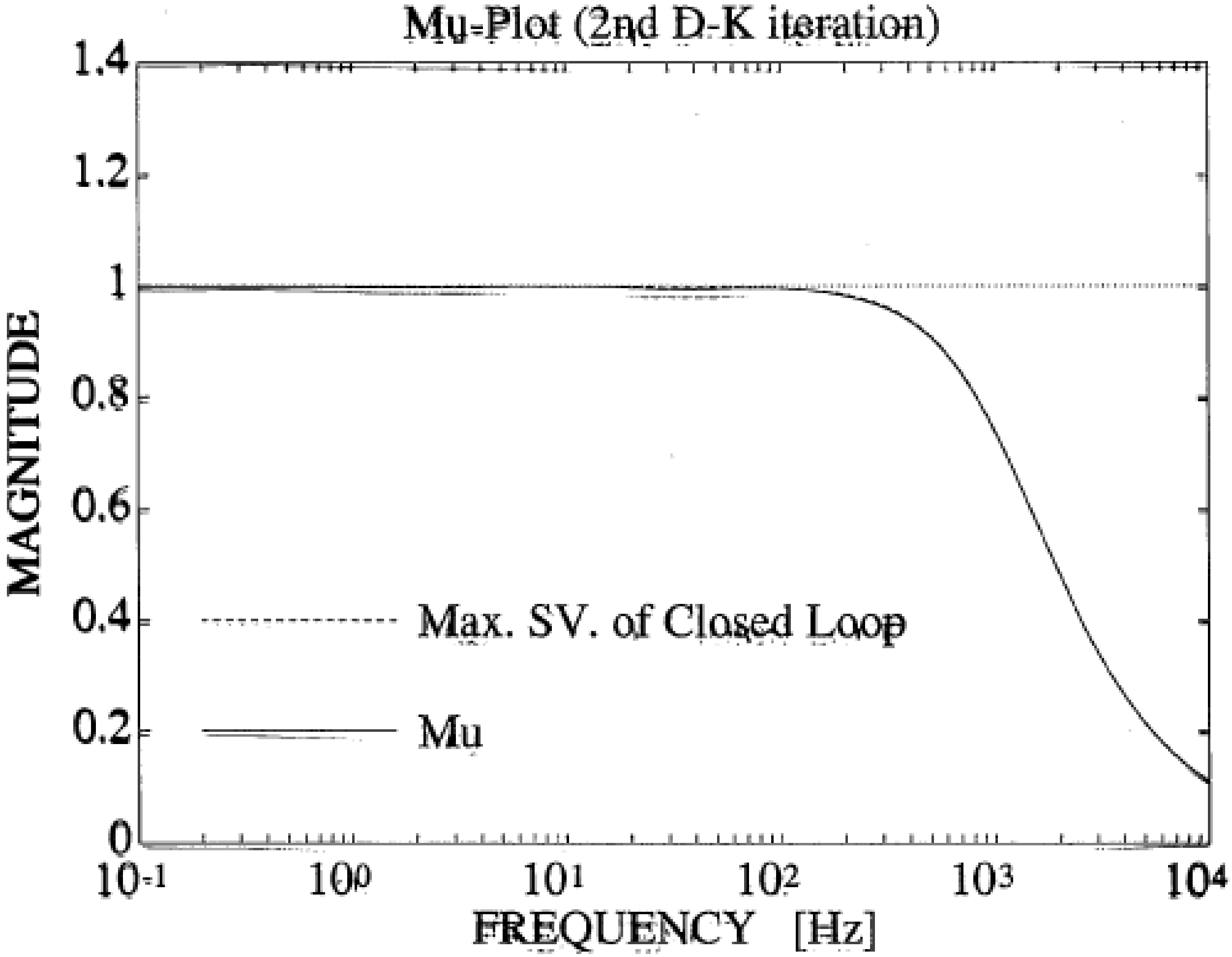
- D-K Iteration : 1st



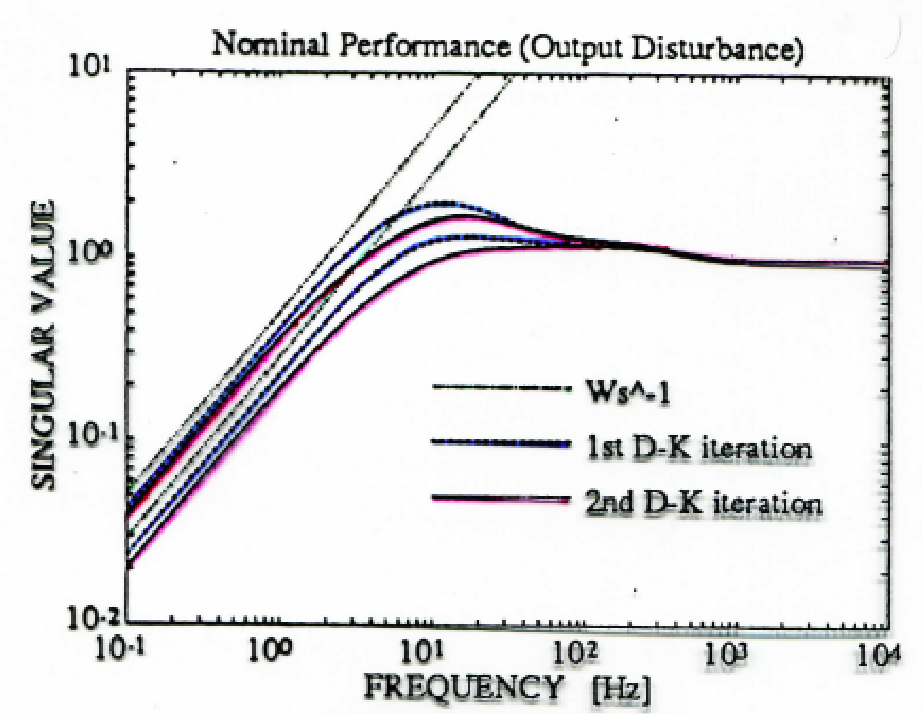
- 2nd order fit for the *D*-scaling

μ -Controller: D-K Iteration

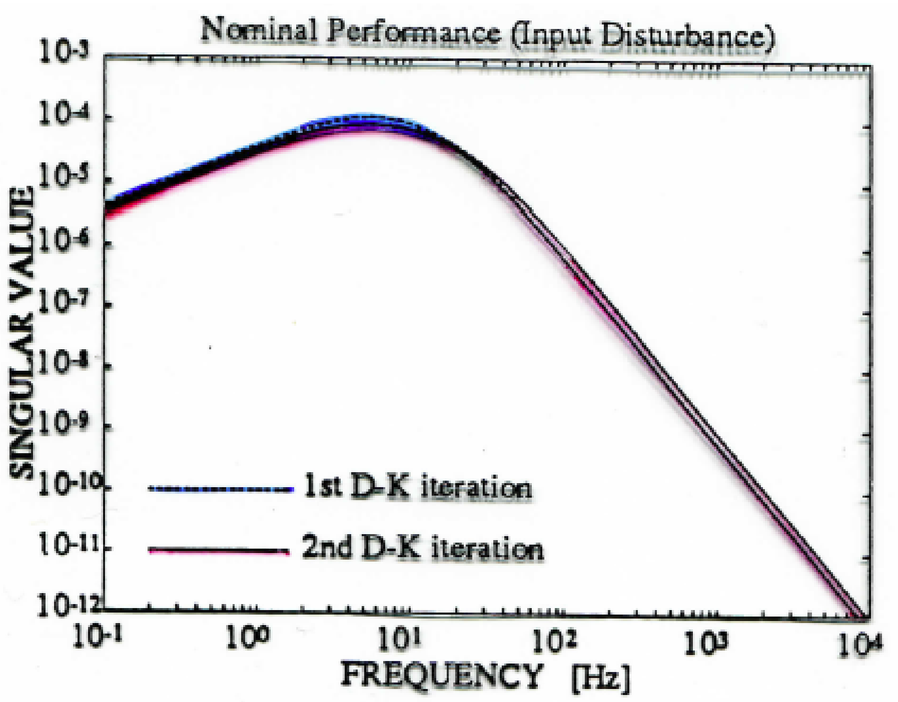
- D-K Iteration : 2nd



μ -Controller: Analysis

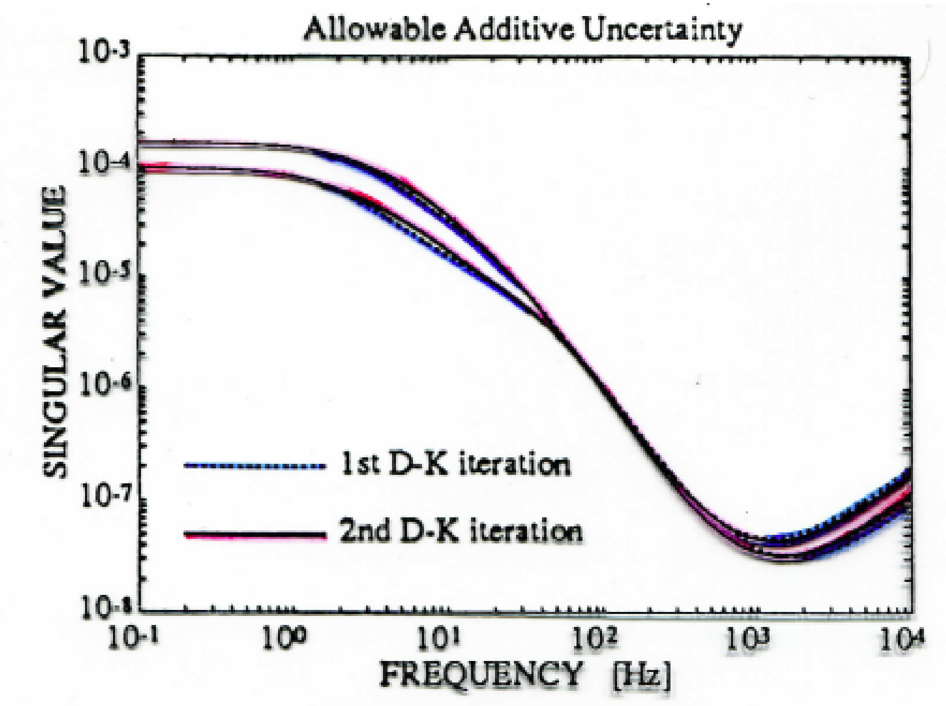


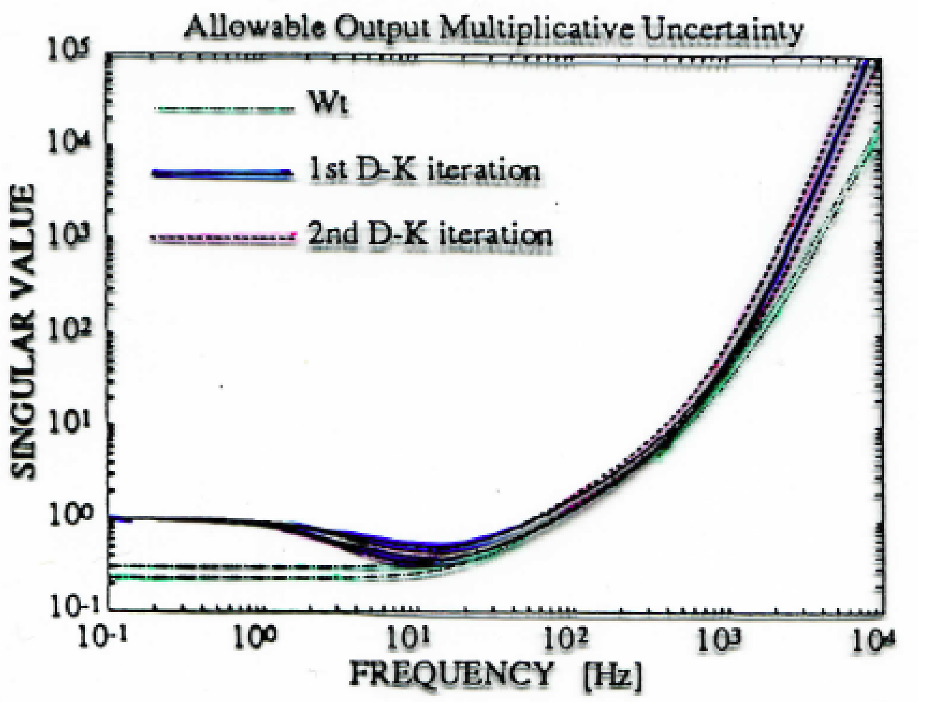
$$\sigma((I - GK)^{-1})$$



$$\sigma((I - GK)^{-1}G)$$

μ -Controller: Analysis





$$\frac{1}{\sigma(K(I-GK)^{-1})}$$

$$\frac{1}{\sigma(GK(I-GK)^{-1})}$$

Digital Implementation and Experiments

- Sampling Time

$$K_{LSDP} : T = 184\ \mu s$$

$$K_{mixed} : T = 148\ \mu s$$

$$K_{\mu} : T = 338\ \mu s$$

- Discretization : Tustin Transform

$$s = \frac{2(z-1)}{T(z+1)}$$

MATLAB Command

```
G = tf(num, den);
Gd = c2d(G, T, 'tustin');
```

Digital Implementation and Experiments

Additional weight is about 3.3 kg

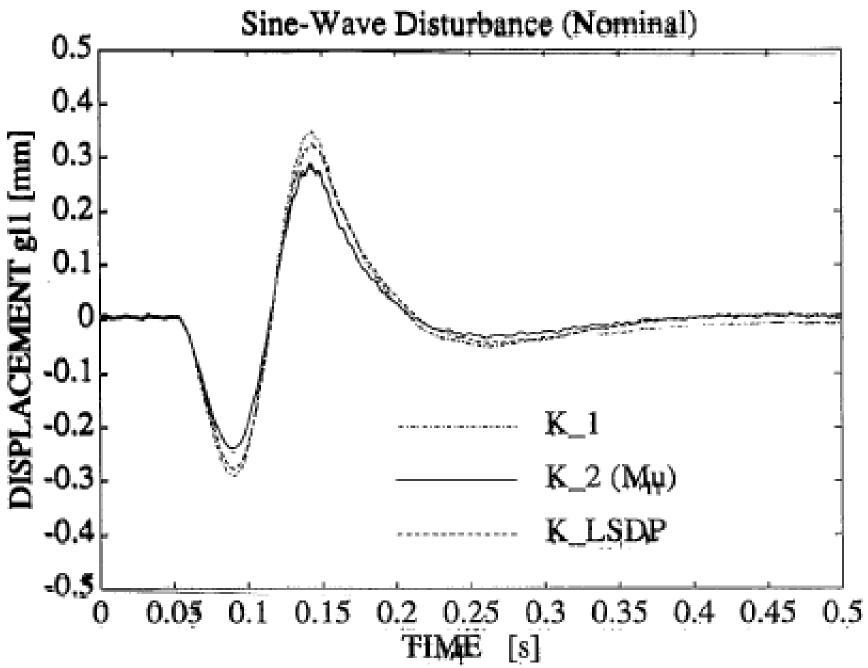
Experiment 1

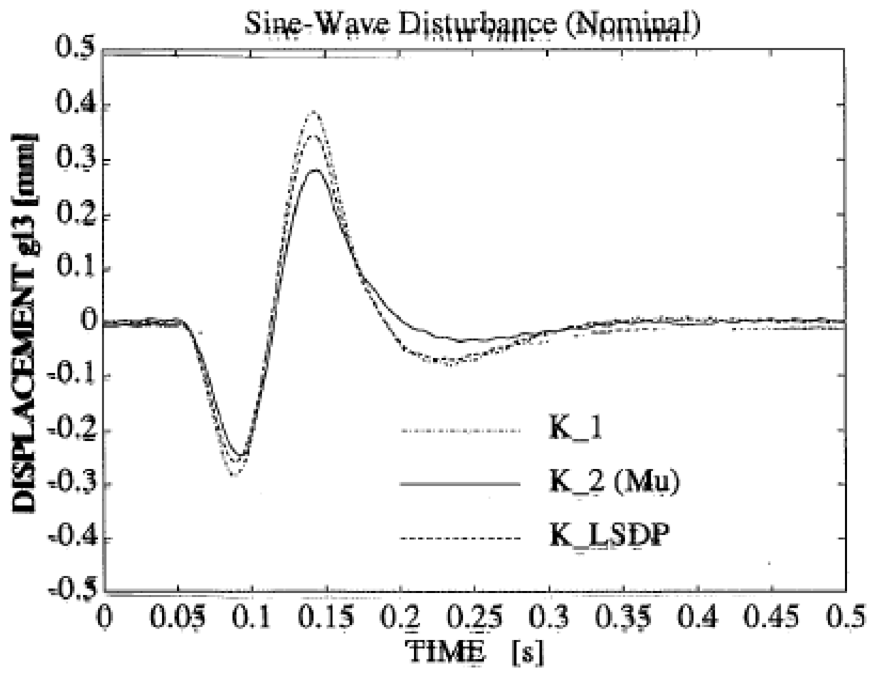
- Sine-wave type signal of only one cycle
- The frequency of the sine-wave is 10 Hz
- The peak value is 6 V for the vertical case and 4.5 V for the horizontal case

Experiment 2

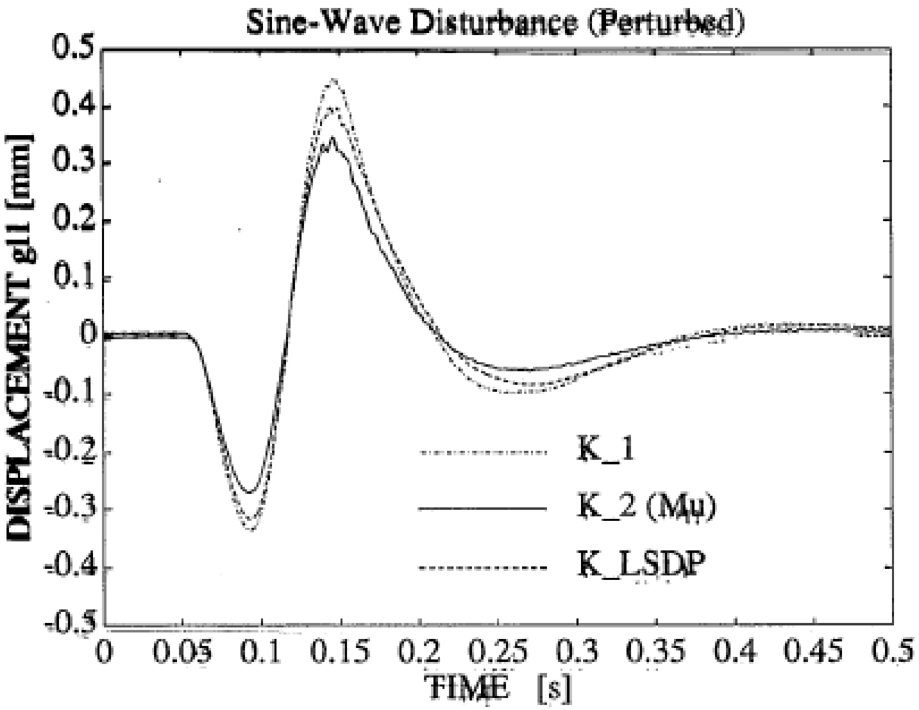
- Step type signal
- Applied voltage is 5 V

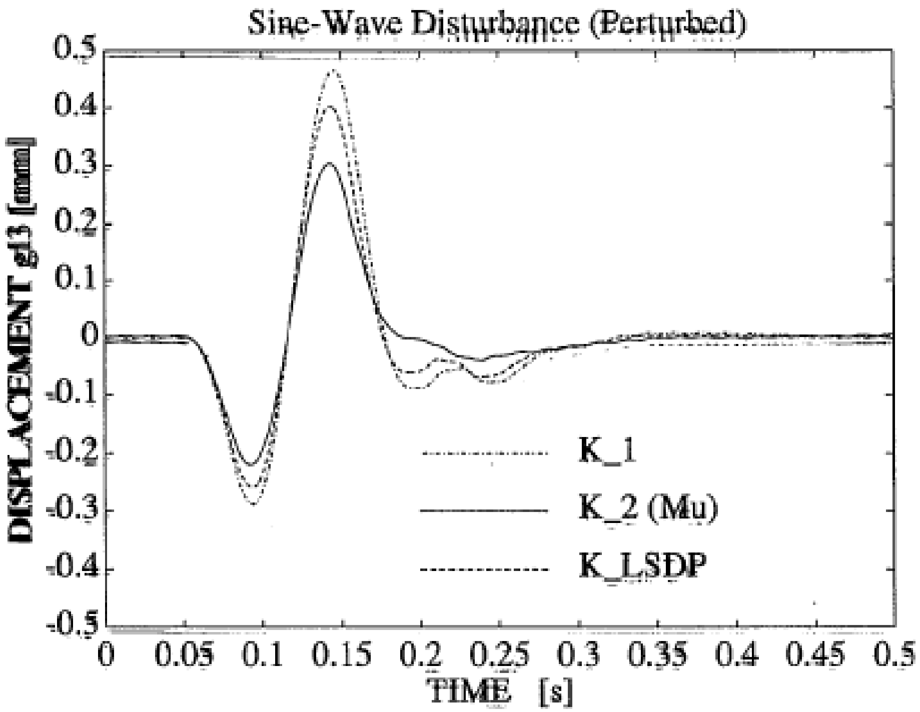
Results of Experiment 1 : Nominal



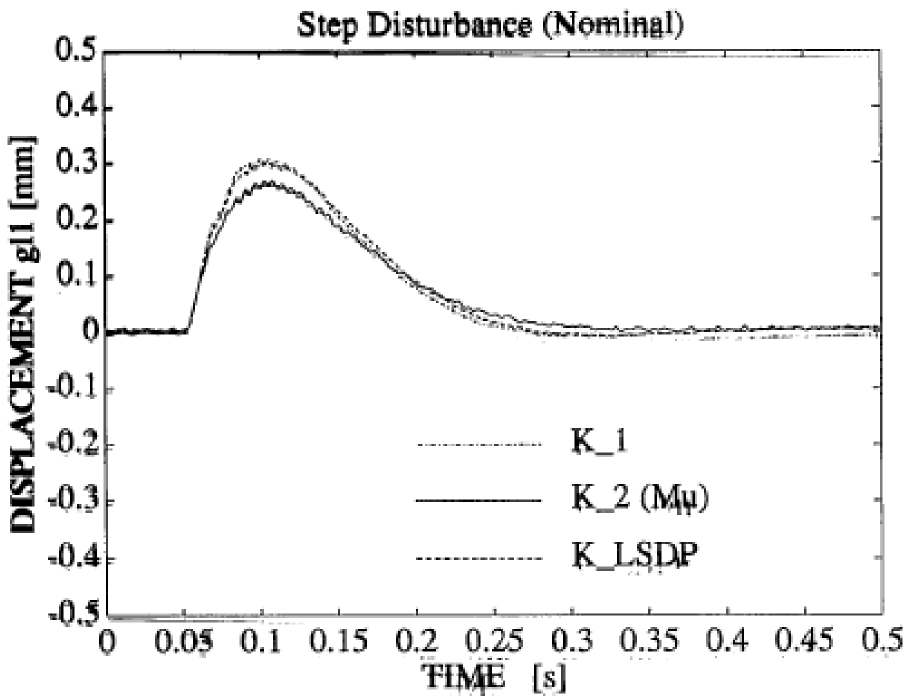


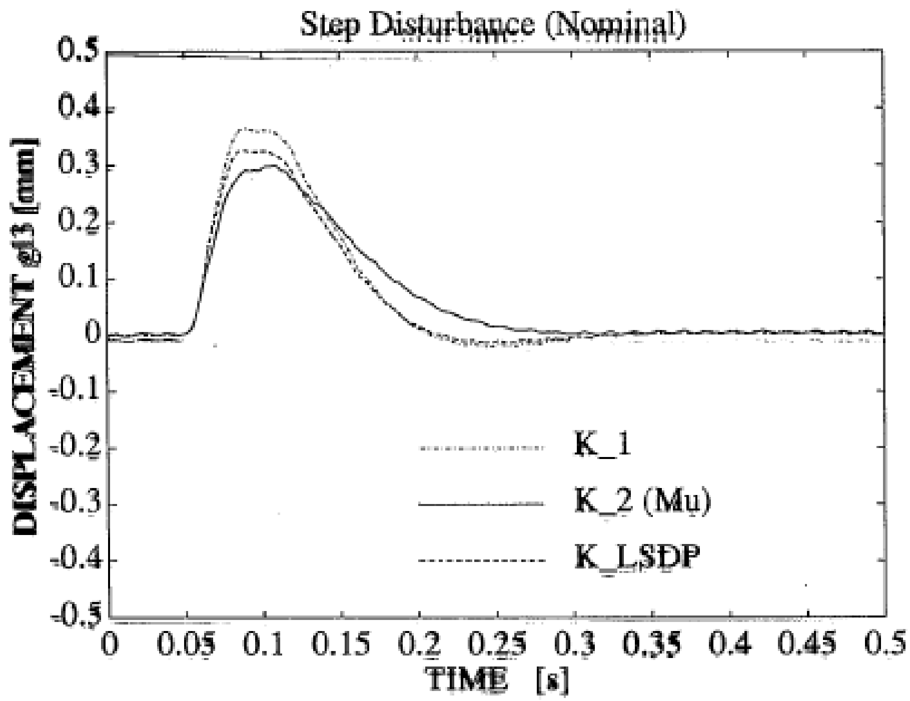
Results of Experiment 1 : Perturbed



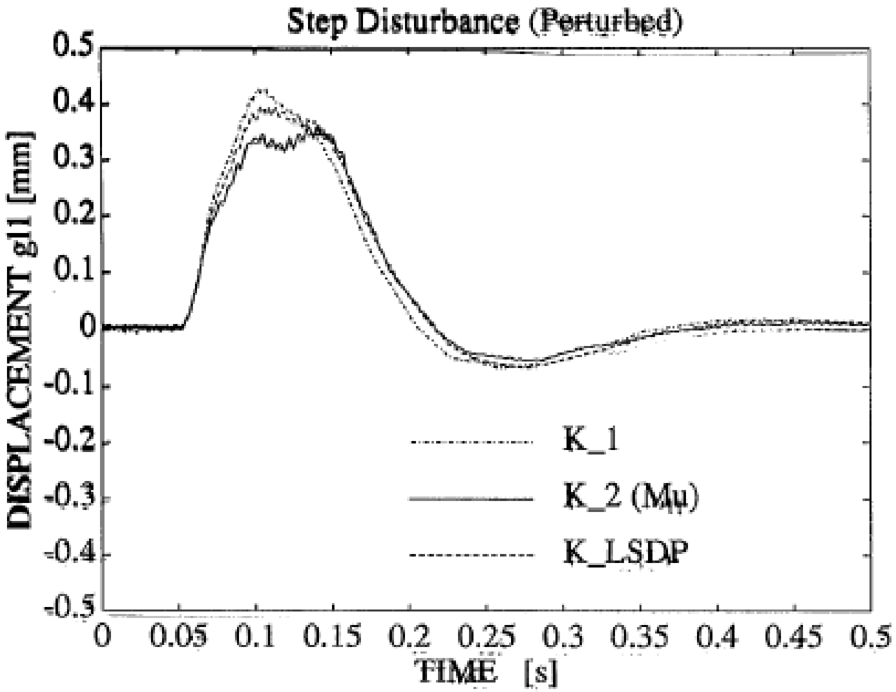


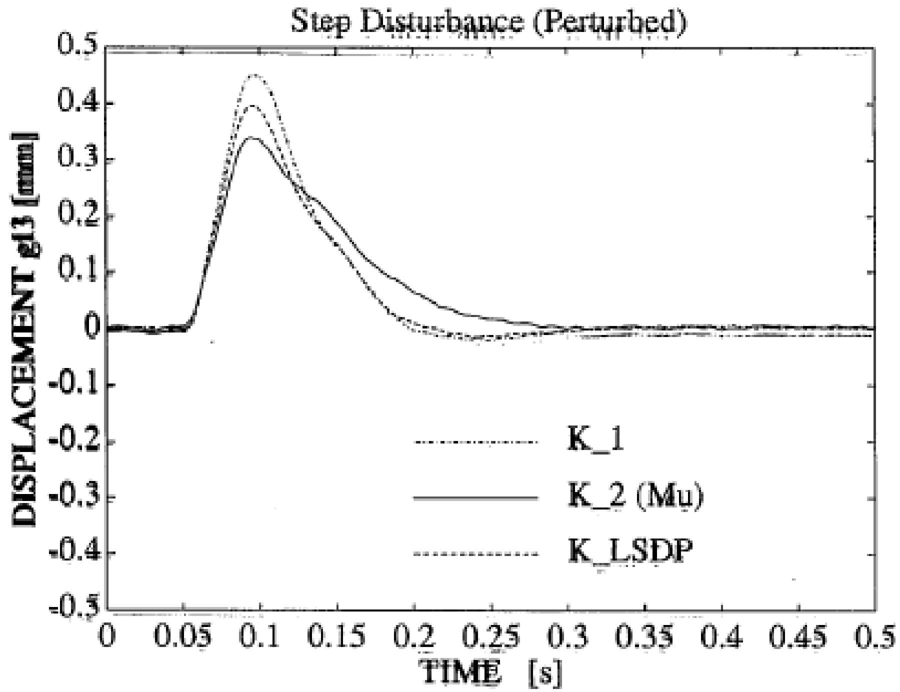
Results of Experiment 2 : Nominal





Results of Experiment 2: Perturbed





Unbalance Control

- Free parameter

$$K = F_L(K_a, \phi) := K_{11} + K_{12}\Phi(I - K_{22}\Phi)^{-1}K_{21}$$

- Condition

$$\Phi(\pm j\omega_0) = K_{22}(\pm j\omega_0)^{-1}$$

Unbalance Control

- Design for vertical motion

$$W_{1v}(s) = \frac{1300(1+s/(2\pi\cdot 5))(1+s/(2\pi\cdot 35))(1+s/(2\pi\cdot 50))}{(1+s/(2\pi\cdot 0.01))(1+s/(2\pi\cdot 700))(1+s/(2\pi\cdot 1200))}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$W_{2v}(s) = 10000\left(1+\frac{10s}{s^2+\omega_0^2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\varepsilon_{\mathrm{max}v} = 0.19926$$

$$\varepsilon_v^{-1} = \gamma_v = 5.25$$

- Design for horizontal motion

$$W_{1h}(s) = \frac{1100(1+s/(2\pi\cdot 5))(1+s/(2\pi\cdot 25))(1+s/(2\pi\cdot 40))}{(1+s/(2\pi\cdot 0.01))(1+s/(2\pi\cdot 700))(1+s/(2\pi\cdot 1200))}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

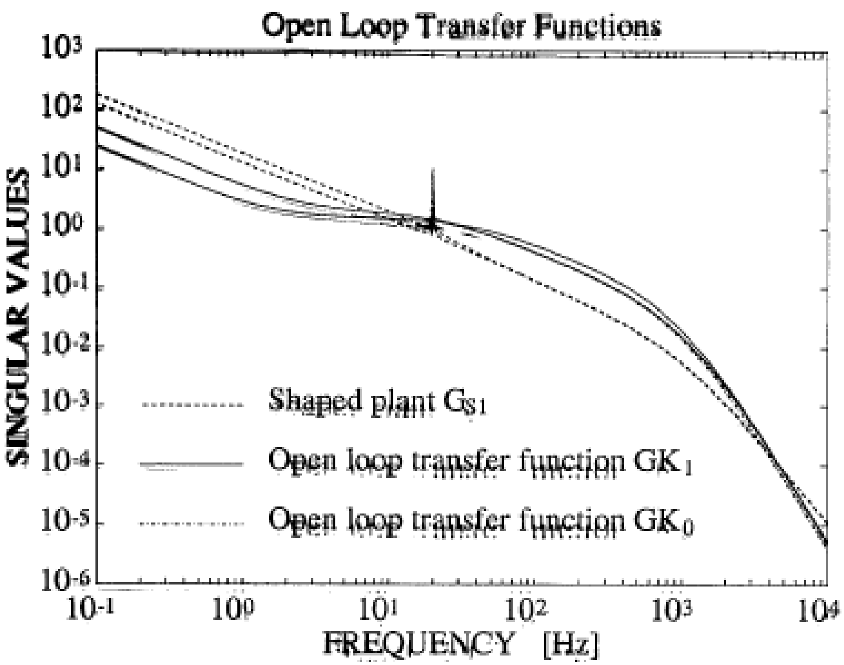
$$W_{2h}(s) = 10000\left(1+\frac{10s}{s^2+\omega_0^2}\right)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

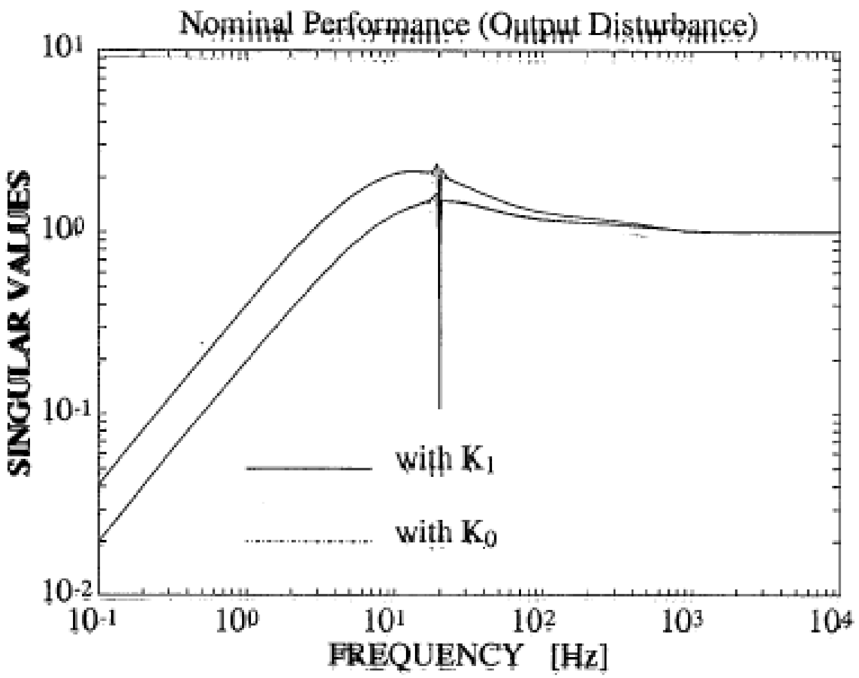
$$\varepsilon_{\mathrm{max}h} = 0.27276$$

$$\varepsilon_h^{-1} = \gamma_h = 3.75$$

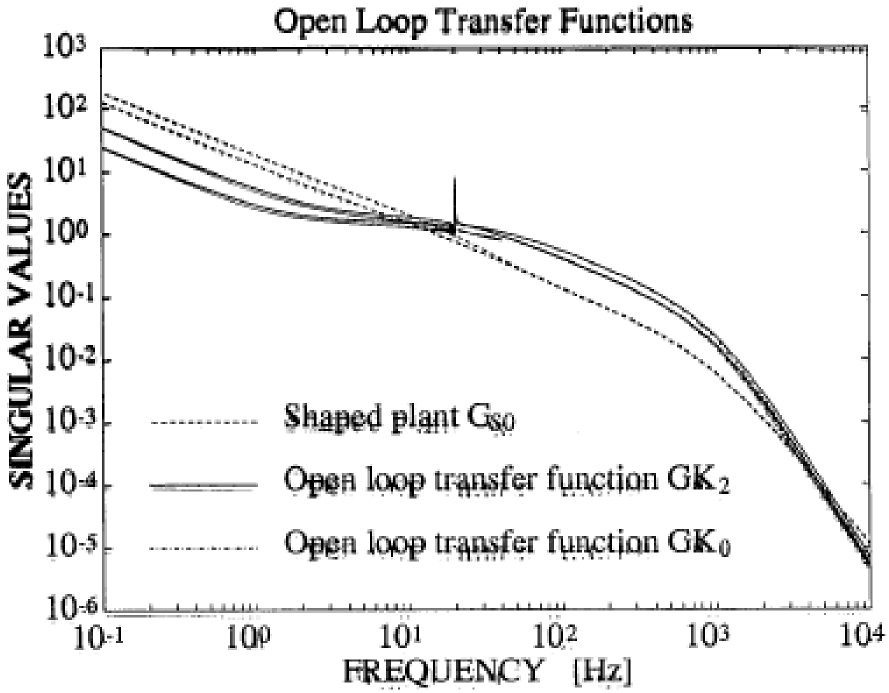
- Rotational speed $\omega_0 = 40\pi$ (1200 rpm)

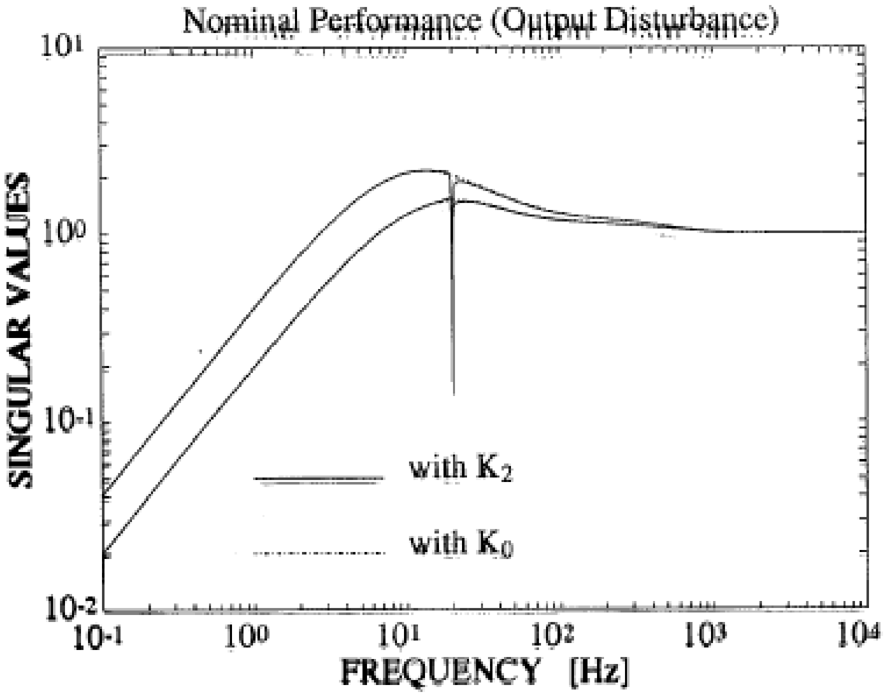
Unbalance Control: Experimental Results



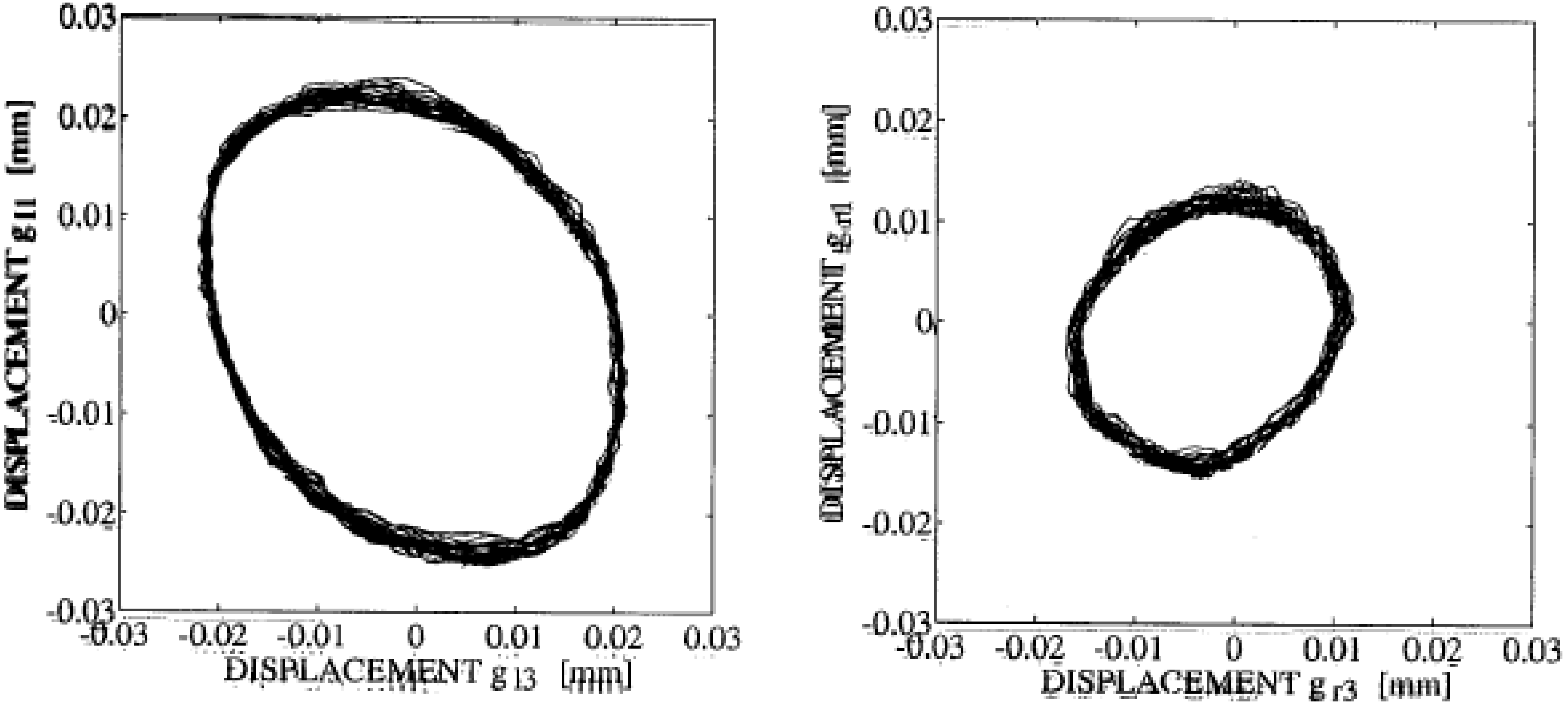


Unbalance Control: Experimental Results

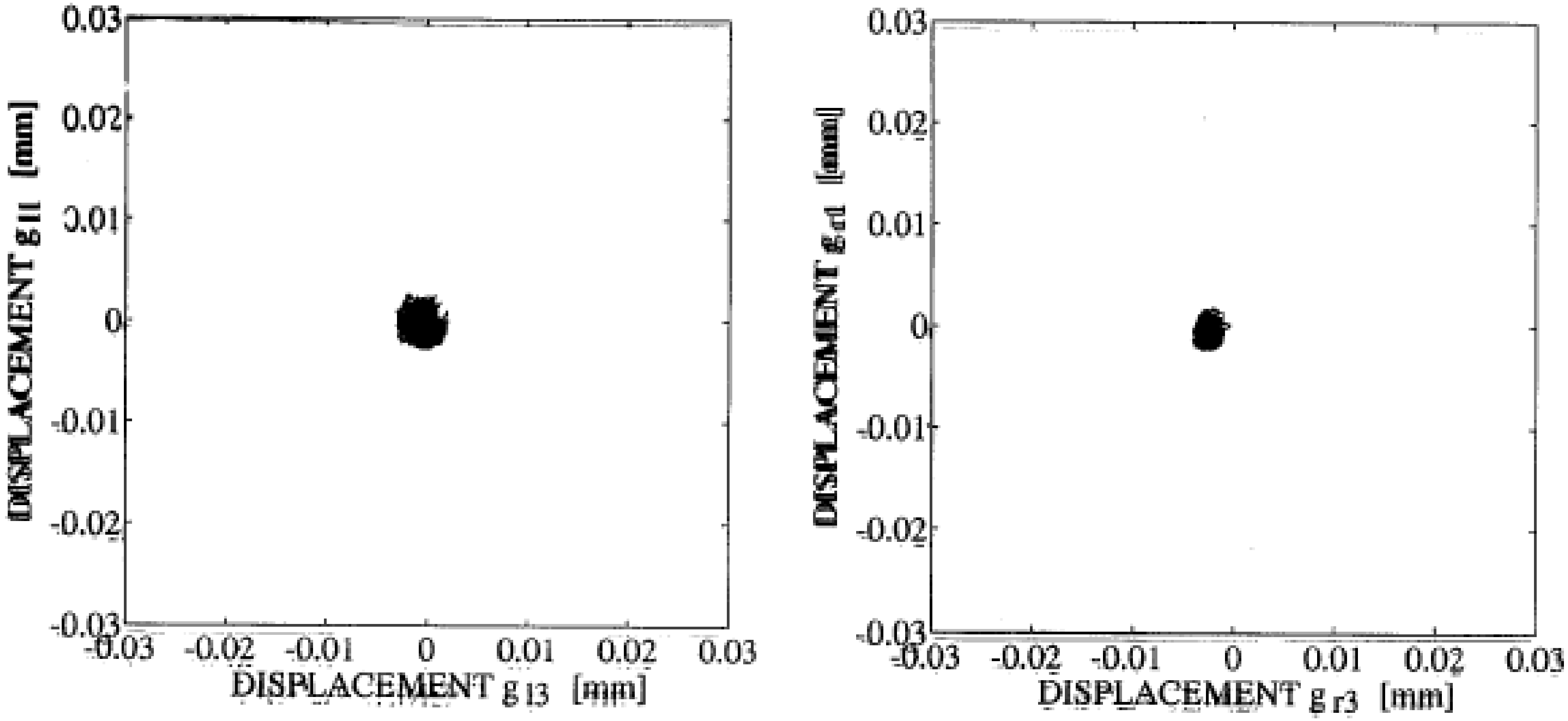




Unbalance Control: Experimental Results



Unbalance Control: Experimental Results



Unbalance Control: Experimental Results