Analysis and Design of Linear Control System –Part2-

Instructor: Prof. Masayuki Fujita

5th Lecture

12 Robust Performance

12.4 Robust Pole Placement: Examples

Example 12.8 (Slow Stable Process Zeros)

(pp. 362--364)

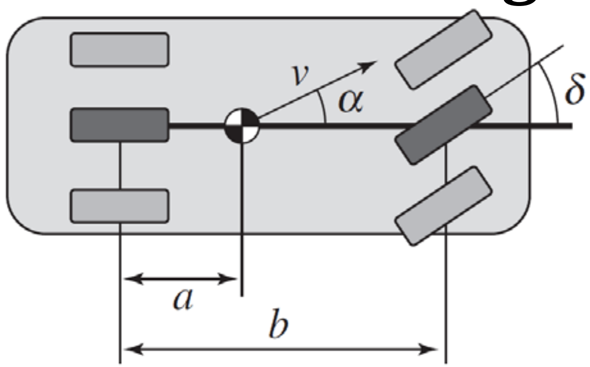
Example 12.9 (Fast Stable Process Poles)

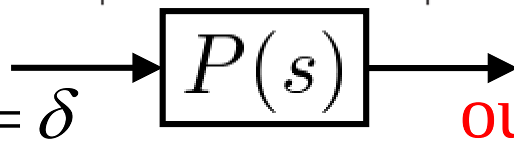
(pp. 364--365)

Keyword : Robust Pole Placement,
Slow Stable Process Zeros,
Fast Stable Process Poles

[Ex. 12.8] Vehicle steering

(Ex. 2.8, 5.12, 7.3, 7.4, 8.6)





input: $u = \delta$ output: y

state: $x = (y, \theta)$

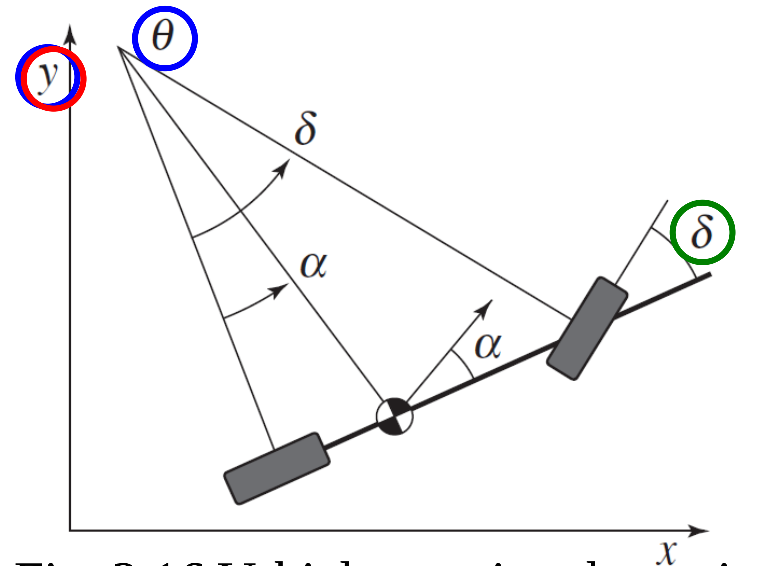


Fig. 2.16 Vehicle steering dynamics

State equation : Plant model (§ 8.3)

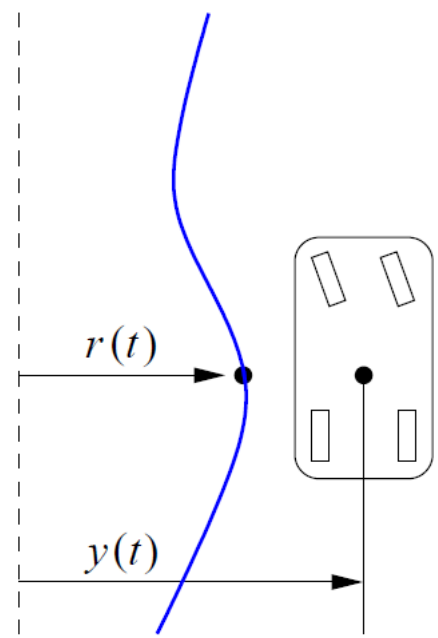
$$\frac{dx}{dt} = Ax + Bu \ , \quad y = Cx + Du$$

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} \gamma \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad D = 0$$

$$\gamma = 0.5$$

Transfer function

$$P(s) = C(sI - A)^{-1}B + D = \frac{\gamma s + 1}{s^2}$$



[Ex. 12.8] Vehicle steering

Controller

Observer

$$\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$$

State feedback

$$u = -K\hat{x} + k_r r$$

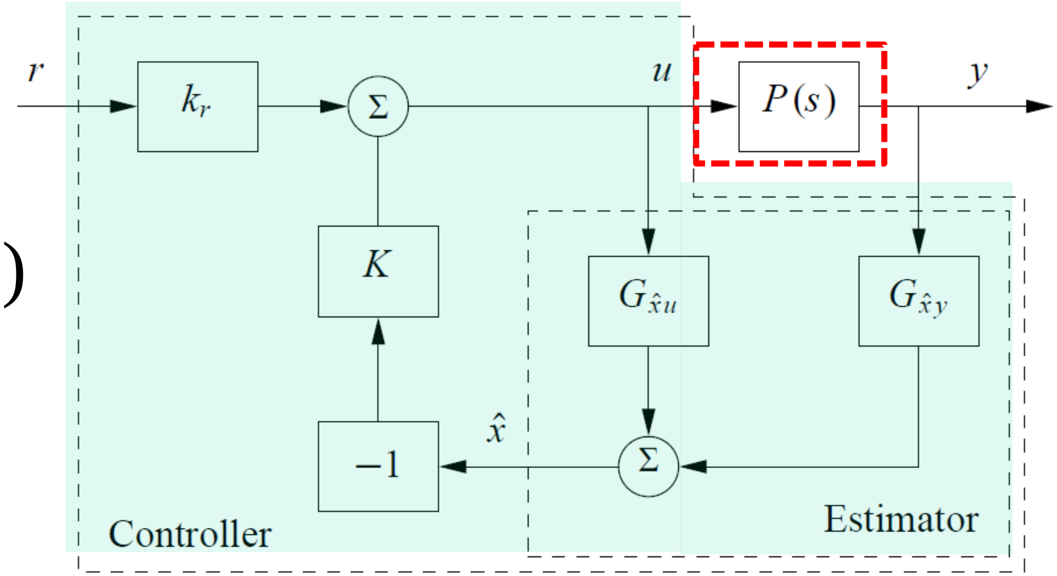


Fig. 8.9 Block diagram for a steering control system

Closed loop system $\tilde{x} = x - \hat{x}$

$$\frac{d}{dt}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - LC \end{bmatrix}\begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} Bk_r \\ 0 \end{bmatrix} r$$

$$y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix}$$

$$\lambda(s) = \det(sI - A + BK)\det(sI - A + LC)$$

Separation Principle

[Ex. 12.8] Vehicle steering　Pole Placement

State feedback gain

$$\det(sI - A + BK) = s^2 + (\gamma k_1 + k_2)s + k_1$$

$$= s^2 + 2\zeta_c\omega_c + \omega_c^2$$

↓

$$k_1 = \omega_c^{\,2} \qquad k_2 = 2\zeta_c\omega_c - \gamma\omega_c^{\,2}$$

Observer gain

$$\det(sI - A + LC) = s^2 + l_1 s + l_2$$

$$= s^2 + 2\zeta_o\omega_o s + \omega_o^2$$

↓

$$l_1 = 2\zeta_o\omega_o \qquad l_2 = \omega_o^{\,2}$$

Faster close loop system

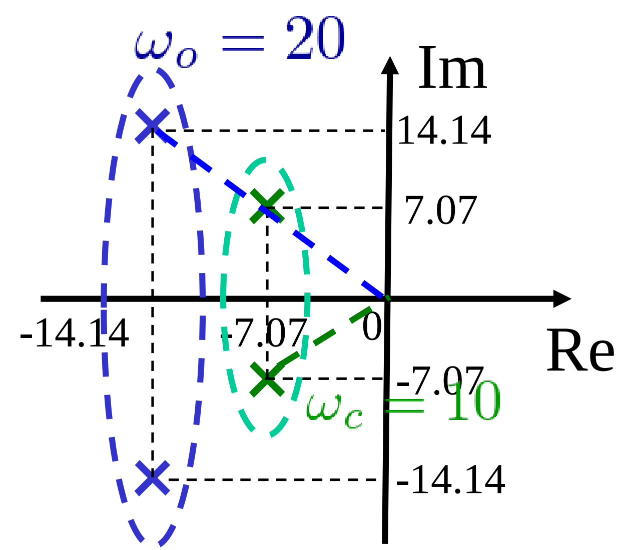
$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 0.707 & \zeta_o = 0.707 \end{cases}$$

$-7.07 \pm 7.07i$　　$-14.14 \pm 14.14i$

↓

$$\begin{cases} k_1 = 100 & l_1 = 28.28 \\ k_2 = -35.86 & l_2 = 400 \end{cases}$$

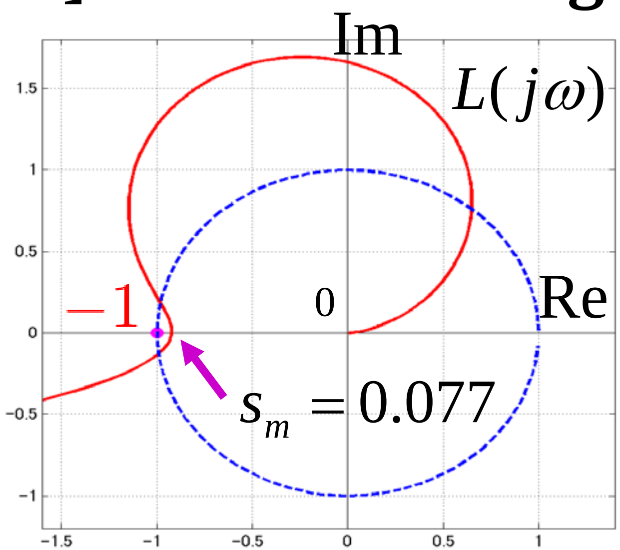
Should be GOOD?



× eig. val. of A - BK

× eig. val. of A - LC

[Ex. 12.8] Vehicle steering



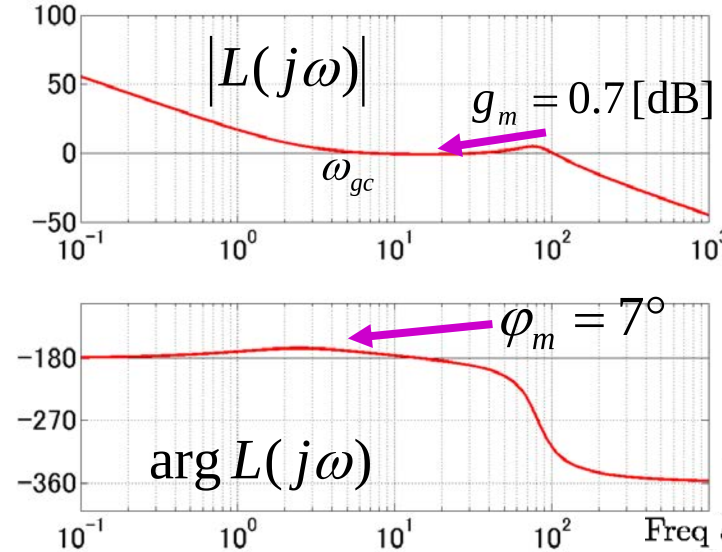
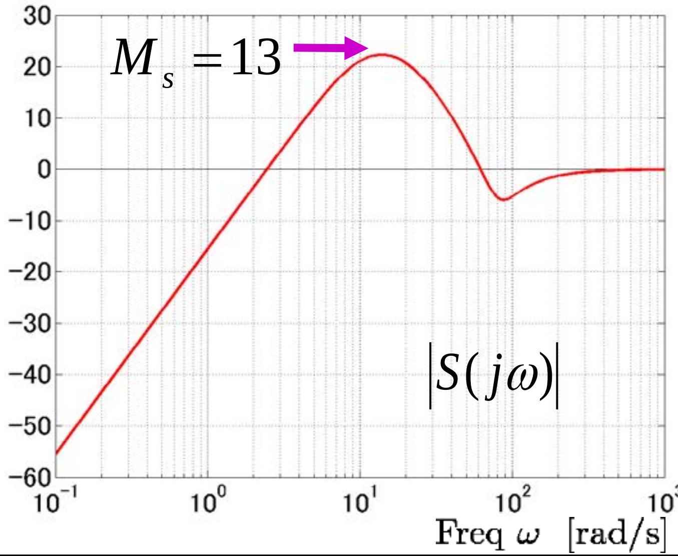


Fig. 12.11



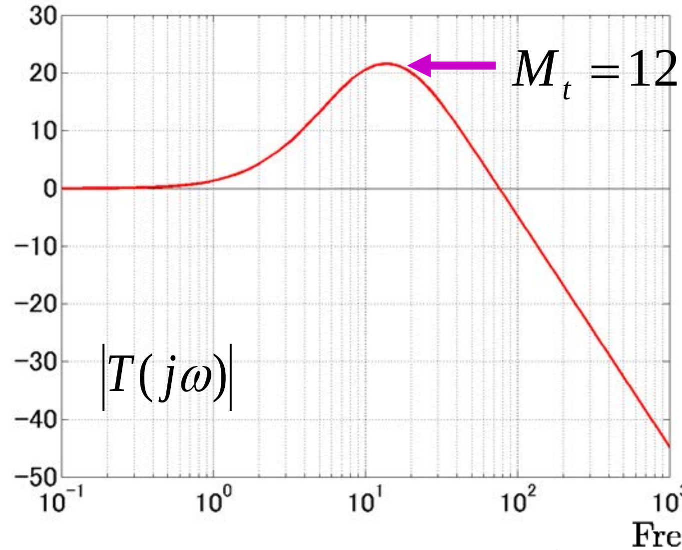
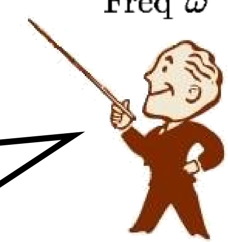


Fig. 12.12

The controller achieves a good control performance, doesn't it?



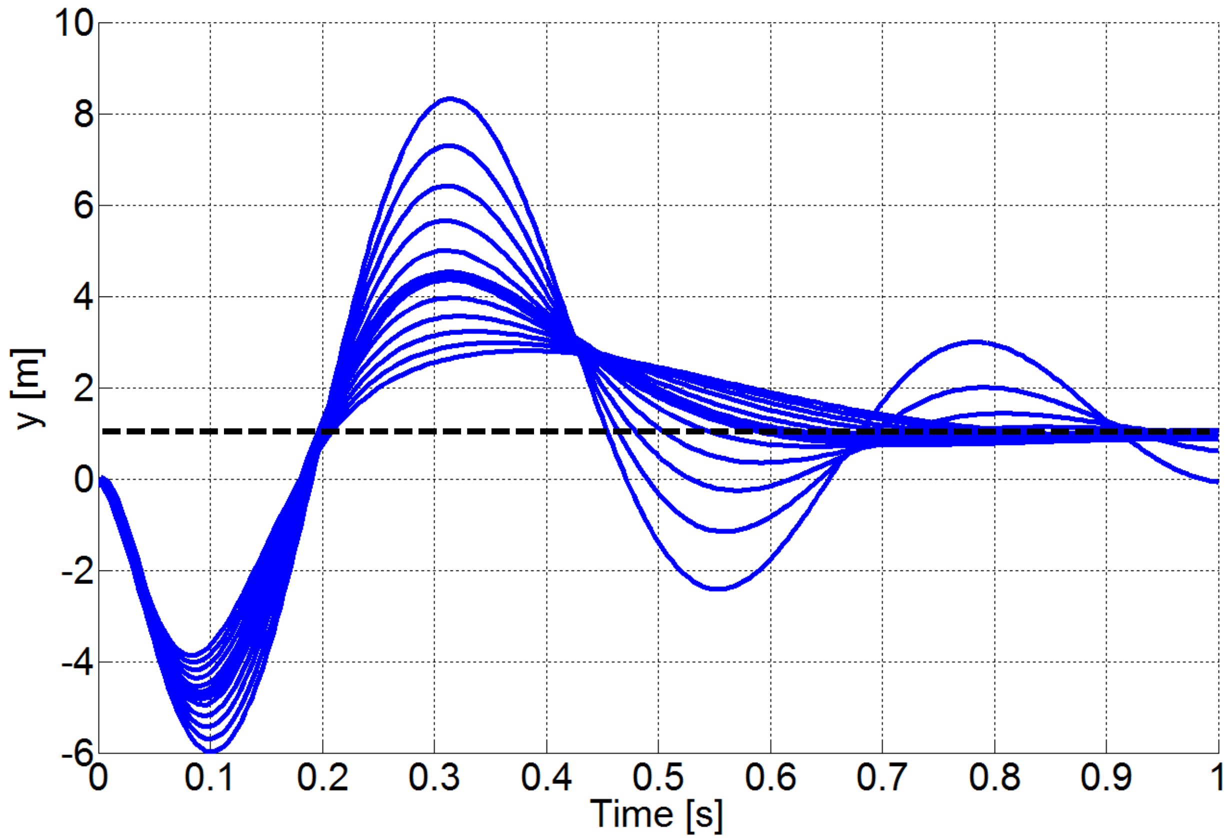
[Ex. 12.8] Vehicle steering

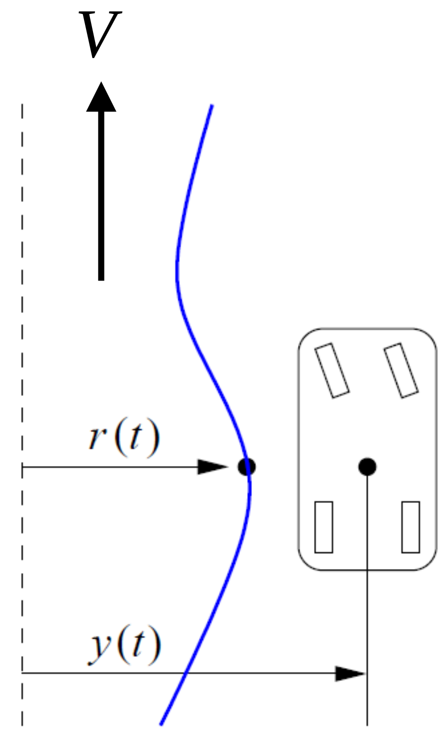
$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 0.707 & \zeta_o = 0.707 \end{cases}$$

$$-7.07 \pm 7.07i \qquad -14.14 \pm 14.14i$$

Nominal $V = 1$

Variation $0.95 \le V \le 1.05$

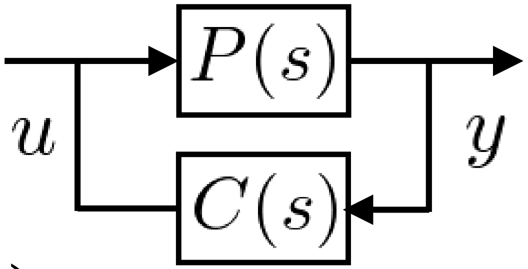




Oops・・・ both performance and robust stability are poor !

[Ex. 12.8] Slow Stable Process Zeros

What happens ?



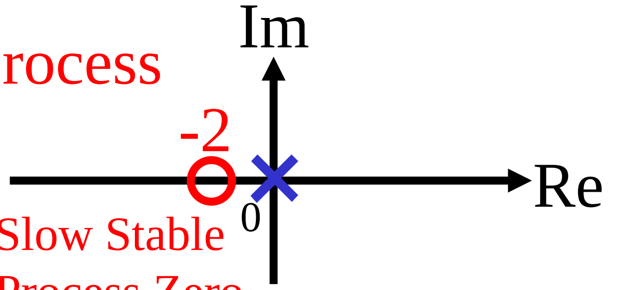
Process (from u to y)

$$P(s) = C(sI - A)^{-1}B + D = \frac{\gamma s + 1}{s^2}$$

$$= \frac{0.5s + 1}{s^2}$$

Pole : $p = 0, 0$

Zero : $z = -2$



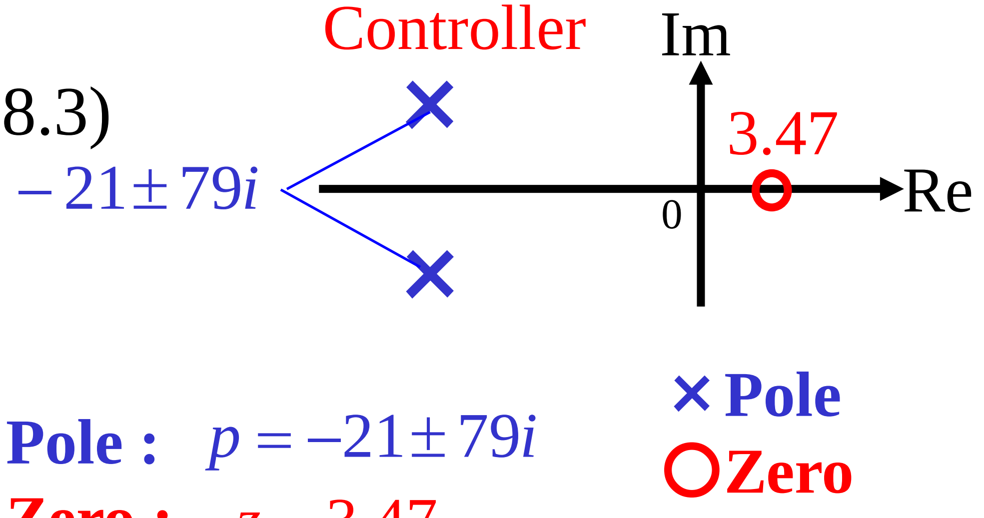
Controller (from y to u) (§ 8.3)

$$C(s) = \frac{KG_{\hat{x}y}(s)}{1 + KG_{\hat{x}u}(s)}$$

$$= \frac{-11516s + 40000}{s^2 + 42.4s + 6657.9}$$

Pole : $p = -21 \pm 79i$

Zero : $z = 3.47$



[Ex. 12.8] Slow Stable Process Zeros

$$T = \frac{PC}{1+PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}$$

$$P = \frac{n_p}{d_p}, \ C = \frac{n_c}{d_c}$$

T has the poles of closed-loop system and its zeros are given by zeros of the process and controller

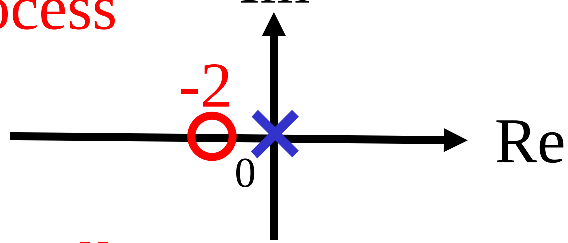
Process (from u to y)

$$P(s) = \frac{0.5s + 1}{s^2} \leftarrow n_P$$

Pole : $p = 0, 0$

Zero : $z = -2$

Process



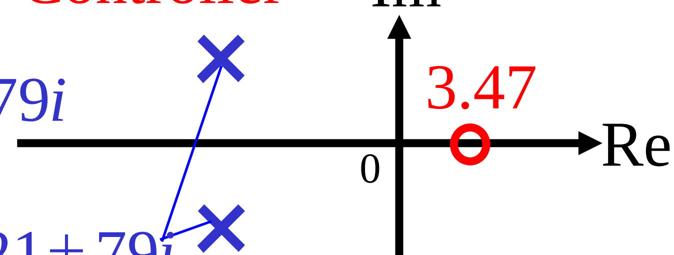
Controller (from y to u)

$$C(s) = \frac{-11516s + 40000}{s^2 + 42.4s + 6657.9} \leftarrow n_c$$

Pole : $p = -21 \pm 79i$

Zero : $z = 3.47$

Controller



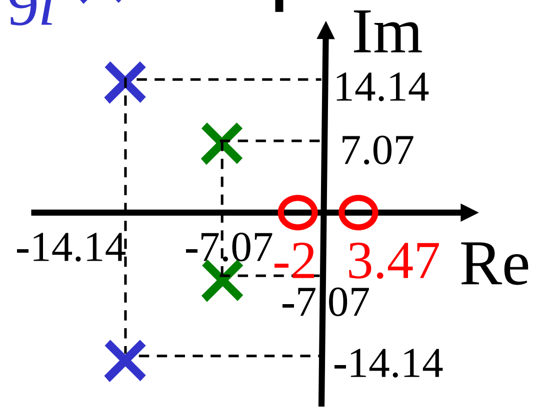
Closed –loop (from r to y)

$$T(s) = \frac{-5758(s+2)(s-3.47)}{(s^2 + 14.14s + 100)(s^2 + 28s + 400)}$$

Pole : $d_p d_c + n_p n_c = 0$

$$p = -7.07 \pm 7.07i$$
$$-14.14 \pm 14.14i$$

Zero : $z = -2, 3.47$



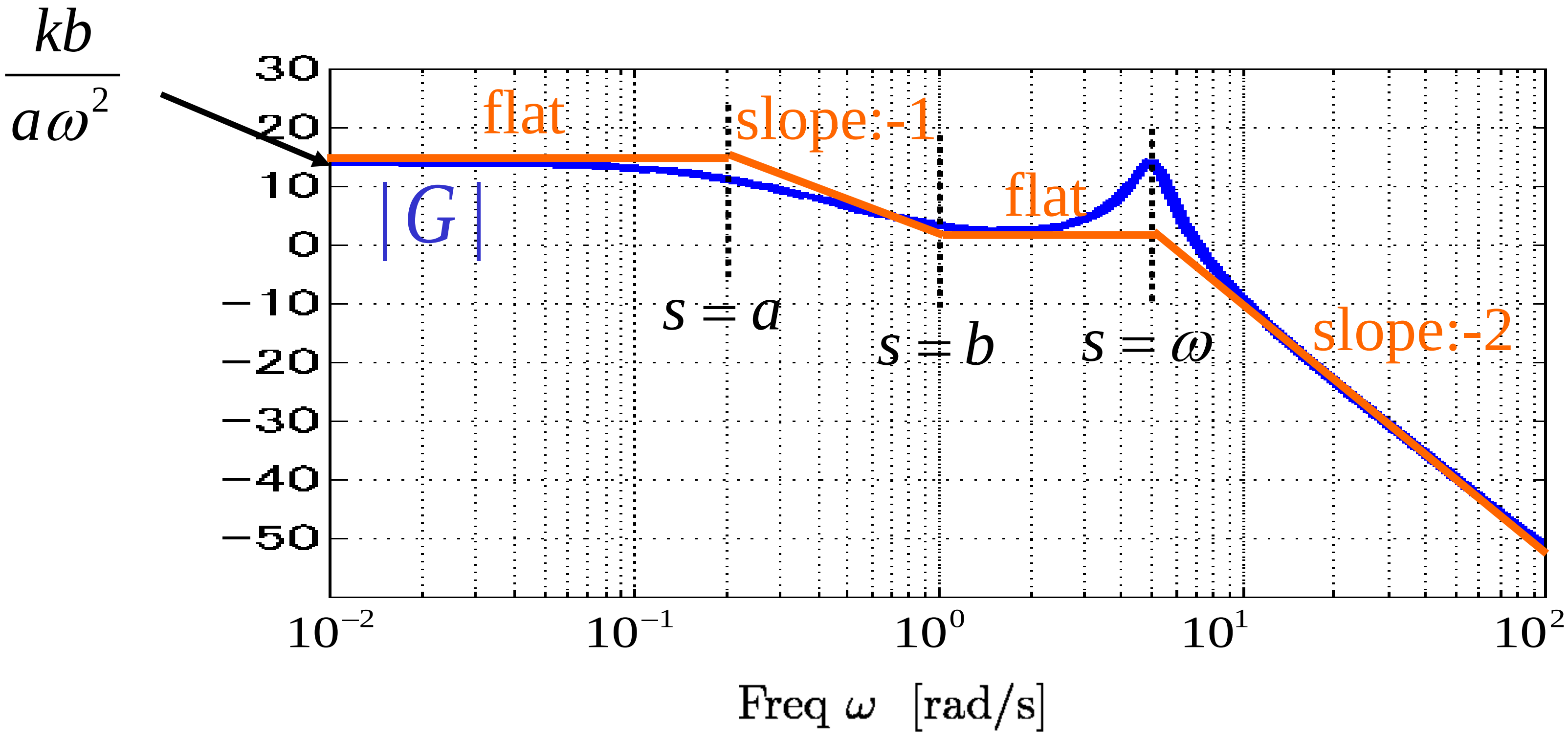
[Ex. 8.8]

Process: $G(s) = \dfrac{k(s+b)}{(s+a)(s^2+2\zeta\omega s+\omega^2)}$

$a = 0.2, b = 1$

$\omega = 5, \zeta = 0.1$

$k = 25$



Pole/Zero $\Leftrightarrow$ Frequency Shape

[Ex. 12.8] Slow Stable Process Zeros

$$T = \frac{PC}{1+PC} = \frac{n_p n_c}{d_p d_c + n_p n_c}$$

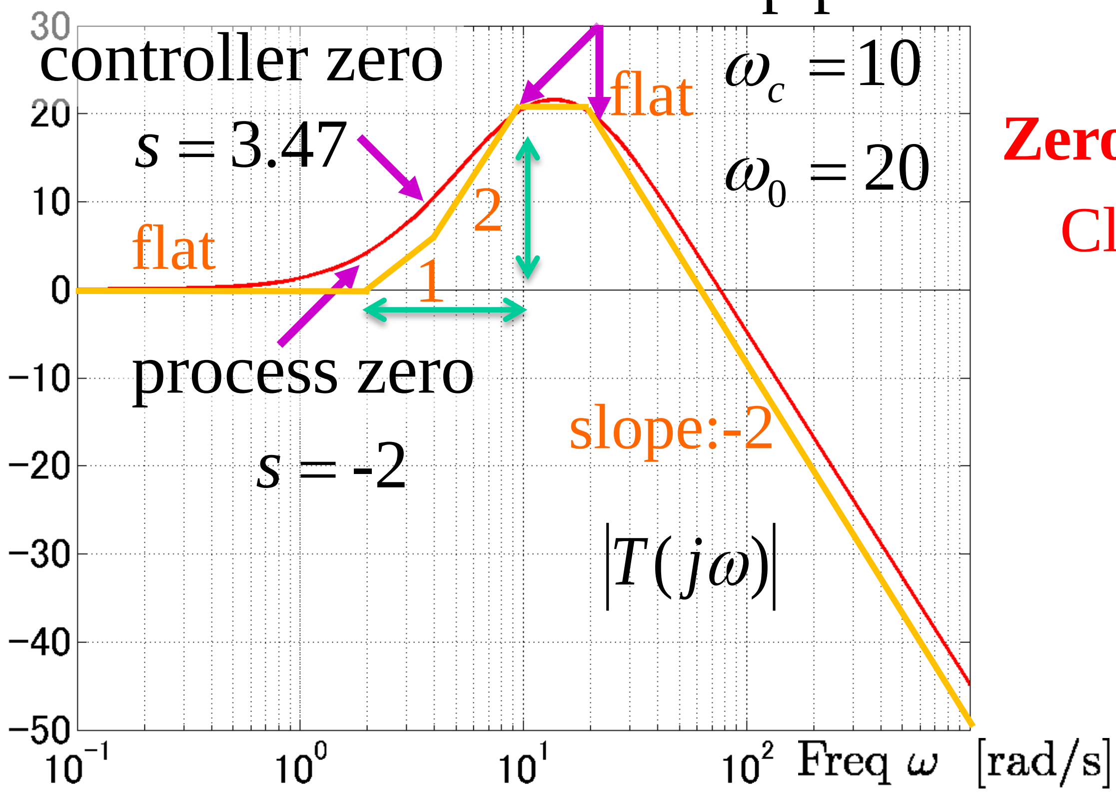
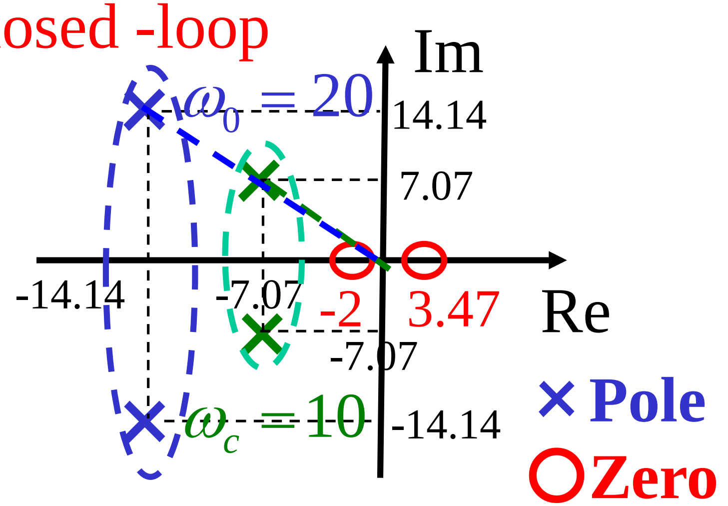


Fig. 12.12

Pole : $-7.07 \pm 7.07i \quad (\omega_c = 10)$

$-14.14 \pm 14.14i \quad (\omega_o = 20)$

Zero : $-2, 3.47$



The magnitude of the peak depends on the ratio of the zeros and the poles of the transfer function.

➡ **assign a closed loop pole close to the slow process zero**

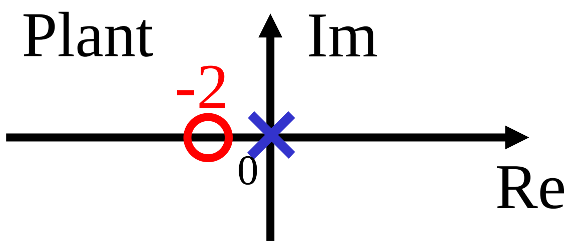
[Ex. 12.8] Slow Stable Process Zeros

Process

$$P(s) = \frac{0.5s+1}{s^2}$$

Pole : $p = 0, 0$

Zero : $z = -2$



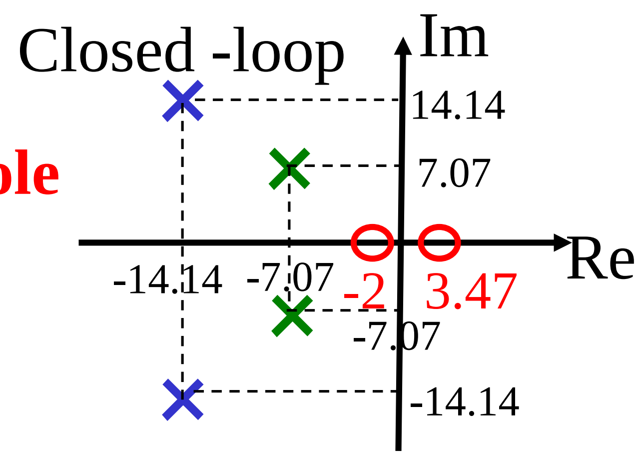
*process stable zero $z = \underline{-2}$

Faster close loop system

$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 0.707 & \zeta_o = 0.707 \end{cases} \rightarrow$$

closed loop pole

$$-7.07 \pm 7.07i$$
$$-14.14 \pm 14.14i$$



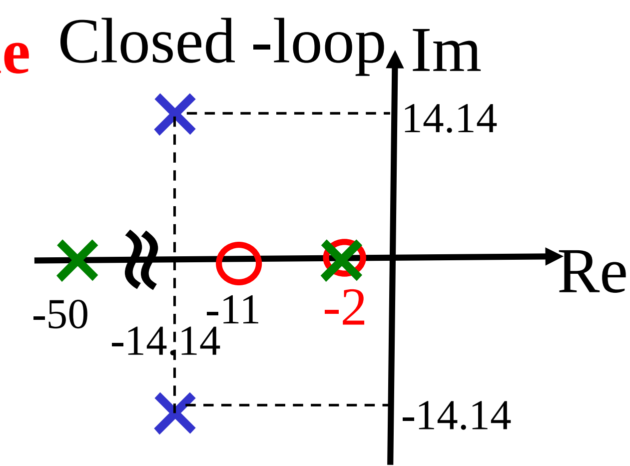
Assign a closed loop pole close to the slow process zero.

$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 2.6 & \zeta_o = 0.707 \end{cases} \rightarrow$$

closed loop pole

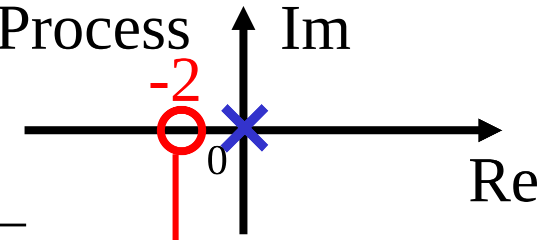
$$-2, -50$$
$$-14.14 \pm 14.14i$$

$$\left(\begin{aligned} \det(sI - A + BK) &= s^2 + 2 \cdot 2.6 \cdot 10s + 10^2 \\ &= (s+2)(s+50) \end{aligned} \right)$$



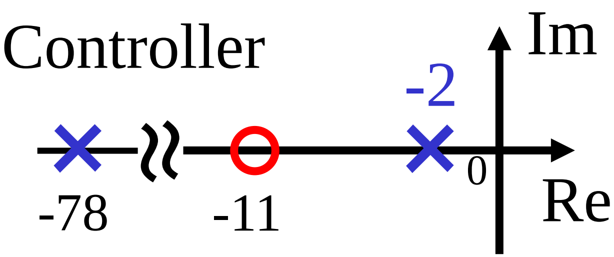
[Ex. 12.8] Slow Stable Process Zeros

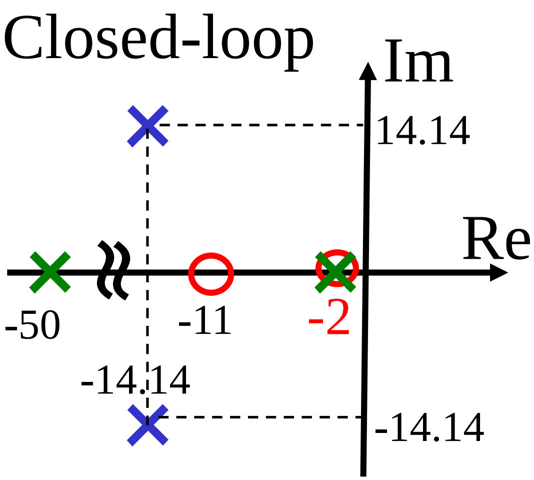
Process: $P(s) = \dfrac{0.5s+1}{s^2}$



$\begin{cases} \omega_c = 10 \\ \zeta_c = 2.6 \end{cases}$ $\quad \omega_0 = 20$, $\zeta_o = 0.707$ $\rightarrow C(s) = \dfrac{3628(s+11.02)}{(s+2)(s+78.28)}$ *process stable zero

$$T(s) \approx \frac{1814(s+11)(s+2)}{(s+2)(s+50)(s^2+28s+400)}$$





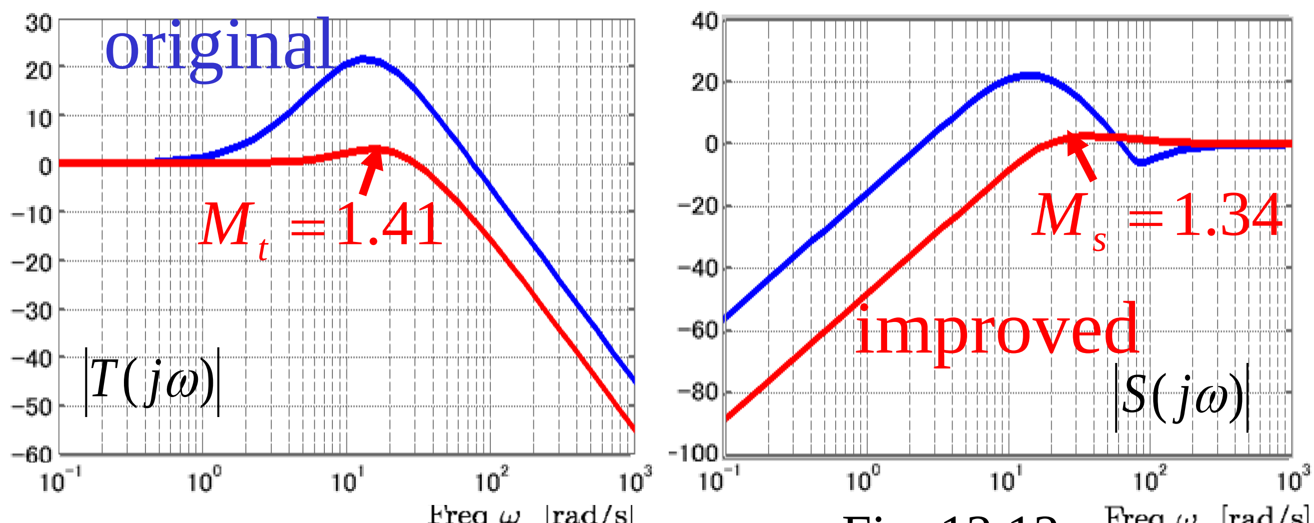


Fig. 12.12

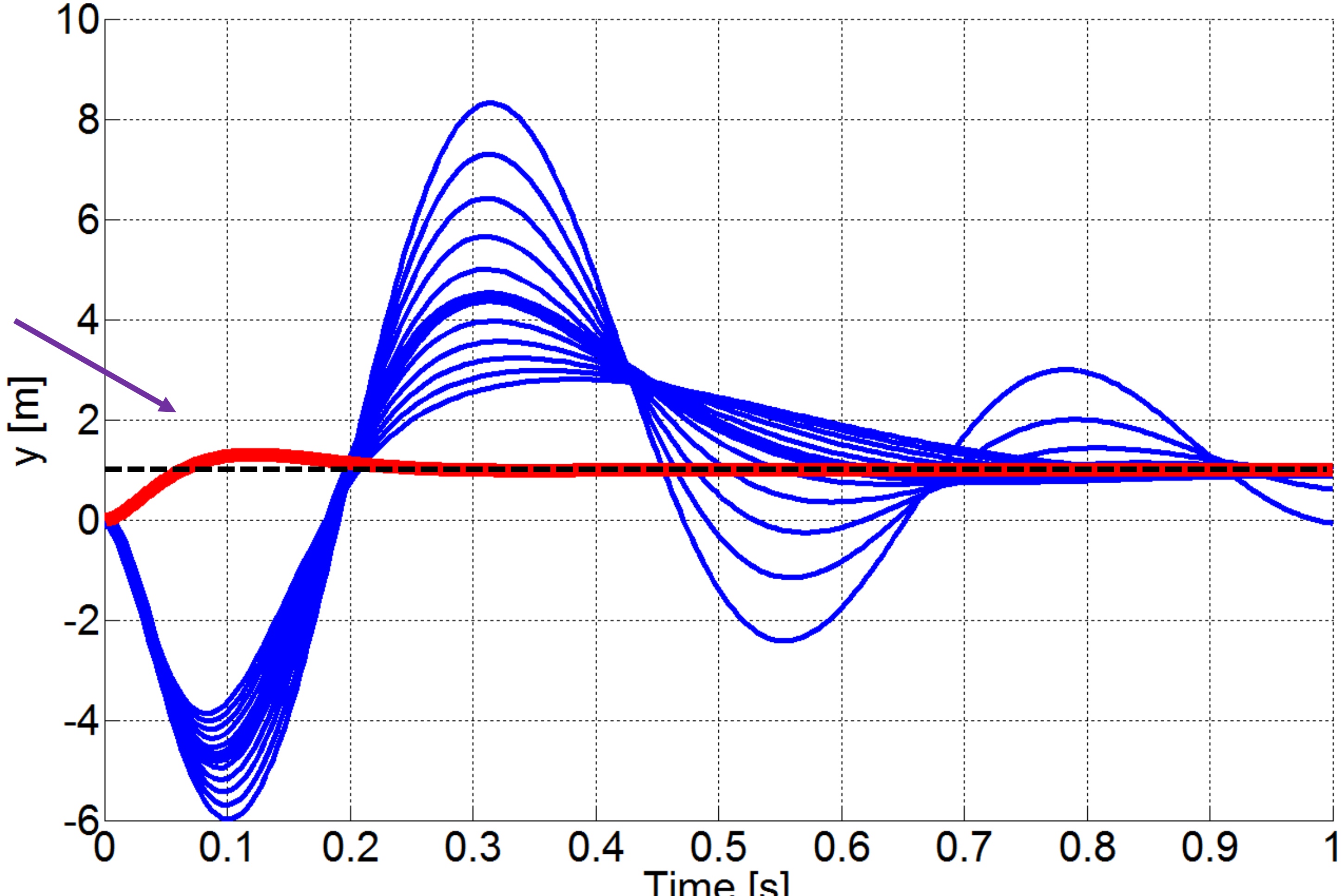
Assign a closed loop pole close to the slow process zero

[Ex. 12.8] Slow Stable Process Zeros

$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 0.707 & \zeta_o = 0.707 \end{cases}$$

$$\begin{cases} \omega_c = 10 & \omega_0 = 20 \\ \zeta_c = 2.6 & \zeta_o = 0.707 \end{cases}$$

2DOF controller



[Ex. 12.9] Fast Stable Process Poles

Process

$$P(s) = \frac{b}{s+a}$$

Pole : $p = -a$

PI controller

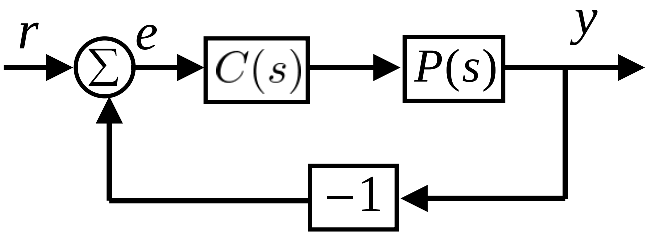
$$C(s) = \frac{k_p s + k_i}{s}$$

Pole : $p = 0$

Zero : $z = -k_i / k_p$

Loop transfer function

$$L(s) = \frac{b(k_p s + k_i)}{s(s+a)}$$



Closed loop characteristic polynomial

$$s(s+a) + b(k_p s + k_i) = s^2 + (a + bk_p)s + k_i b$$

desired closed loop poles : $-p_1 \quad -p_2 \quad s^2 + (p_1 + p_2)s + p_1 p_2$

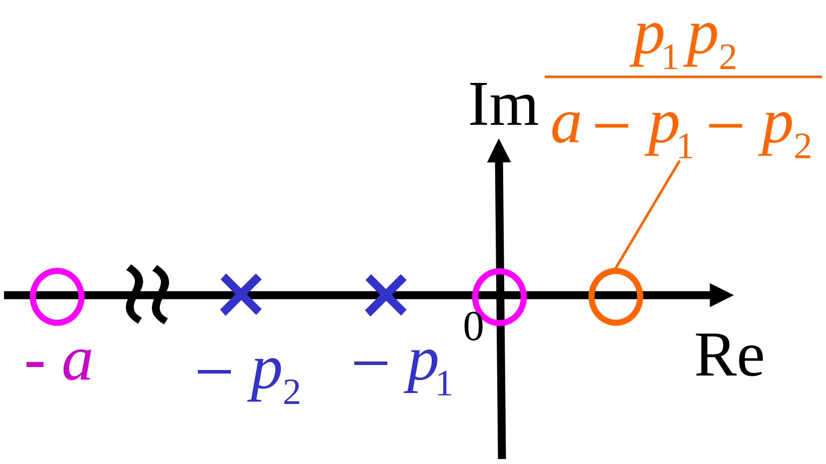
$$k_p = \frac{p_1 + p_2 - a}{b} \qquad k_i = \frac{p_1 p_2}{b}$$

Complementary sensitivity

$$T(s) = \frac{(p_1 + p_2 - a)s + p_1 p_2}{(s+p_1)(s+p_2)}$$

Sensitivity

$$S(s) = \frac{s(s+a)}{(s+p_1)(s+p_2)}$$

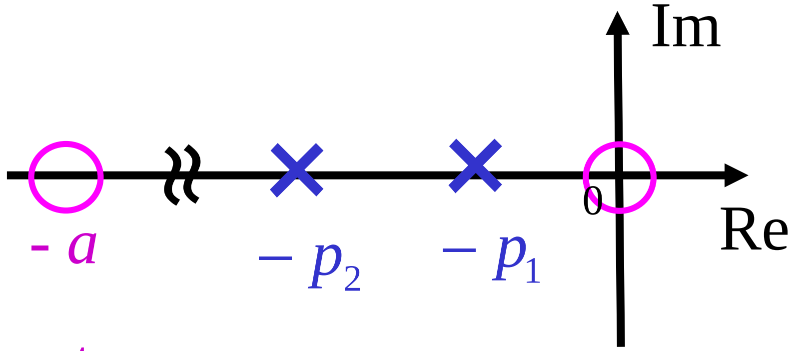


assume $p_1 < p_2 \ll a$

[Ex. 12.9] Fast Stable Process Poles

assume $p_1 < p_2 \ll a$

$$S(s) = \frac{s(s+a)}{(s+p_1)(s+p_2)}$$



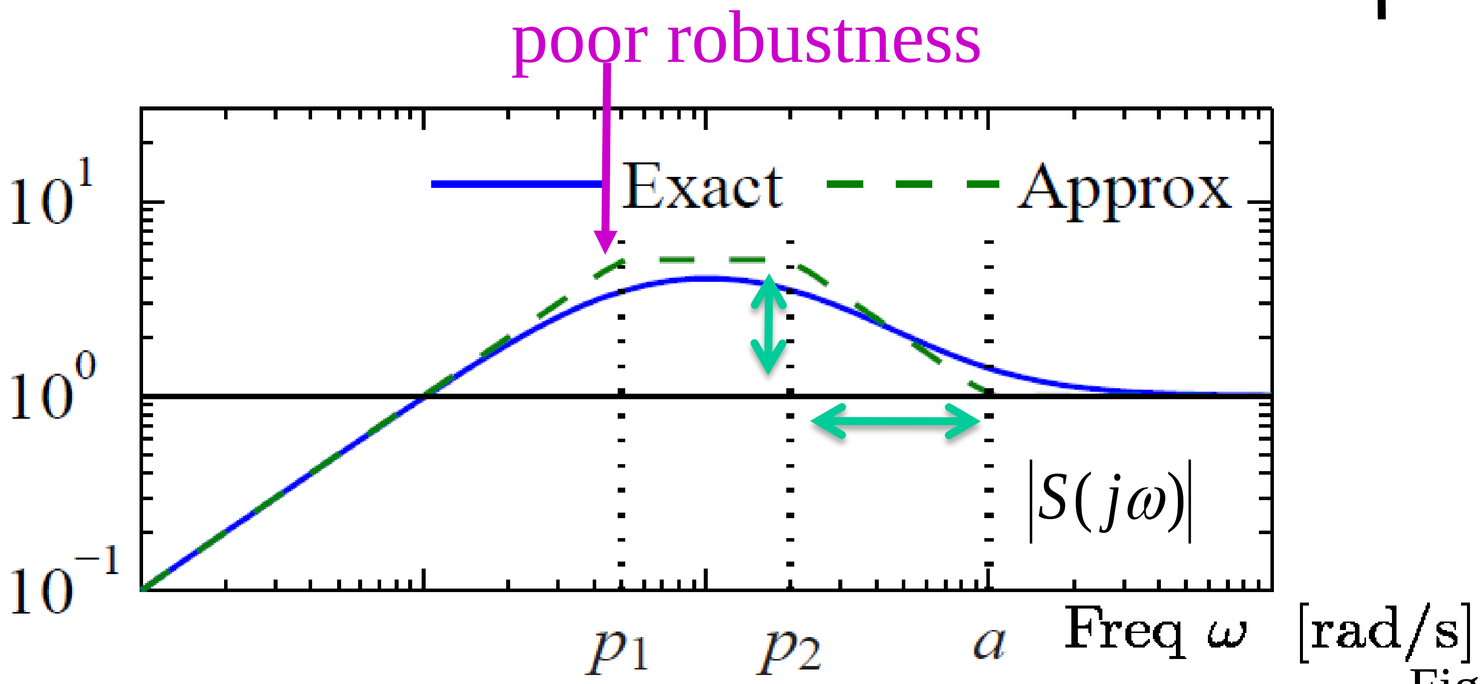


Fig. 12.13

Choose one closed loop pole equal to the process pole

$$p_2 = a$$

[Ex. 12.9] Fast Stable Process Poles

Process

$$P(s) = \frac{b}{s+a}$$

Pole : $p = -a$

PI controller $C(s) = \dfrac{k_p s + k_i}{s}$

$$k_p = \frac{p_1 + p_2 - a}{b}, \quad k_i = \frac{p_1 p_2}{b}$$

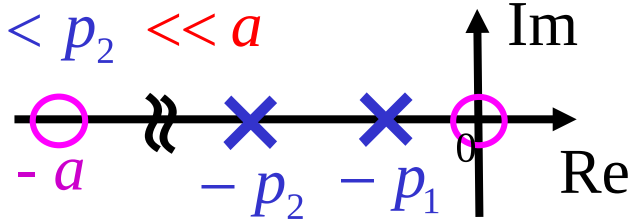
Pole : $p = 0$

Zero : $z = -k_i / k_p$

Sensitivity

$$S(s) = \frac{s(s+a)}{(s+p_1)(s+p_2)}$$

$p_1 < p_2 << a$



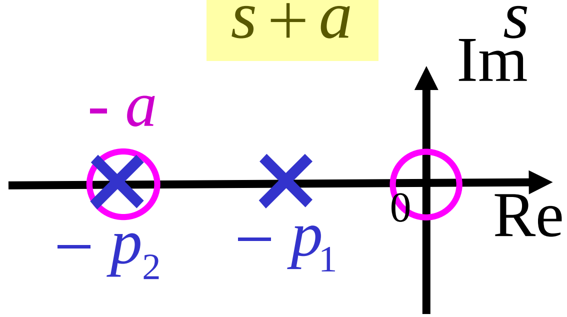
Choose one closed loop pole equal to the process pole.

$p_2 = a$ $\quad k_p = \dfrac{p_1}{b} \quad k_i = \dfrac{ap_1}{b} \quad C(s) = \dfrac{k_p(s+a)}{s}$

Pole : $p = 0$

Zero : $z = -a$

$$L(s) = \frac{b}{s+a} \cdot \frac{k_p(s+a)}{s} = \frac{bk_p}{s} \qquad S(s) = \frac{s}{s+bk_p} \qquad T(s) = \frac{bk_p}{s+bk_p}$$



The fast process pole is canceled by a controller zero

[Ex. 12.9] Fast Stable Process Poles

Process

$$P(s) = \frac{b}{s+a}$$

Pole : $p = -a$

PI controller $C(s) = \dfrac{k_p(s+a)}{s}$

$$k_p = \frac{p_1}{b} \qquad k_i = \frac{ap_1}{b}$$

Pole : $p = 0$

Zero : $z = -a$

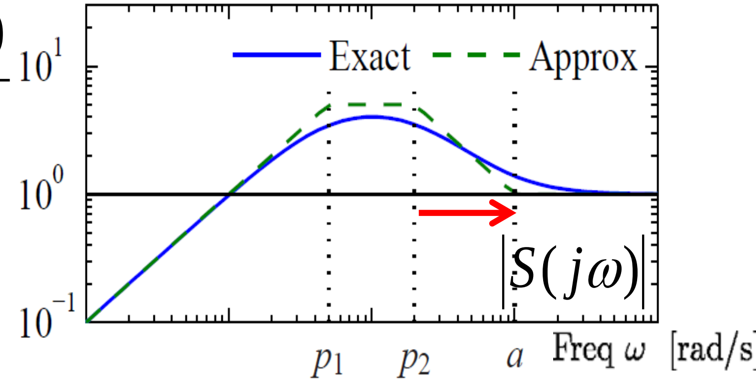
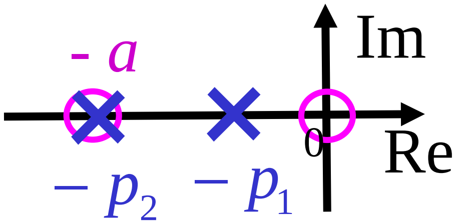
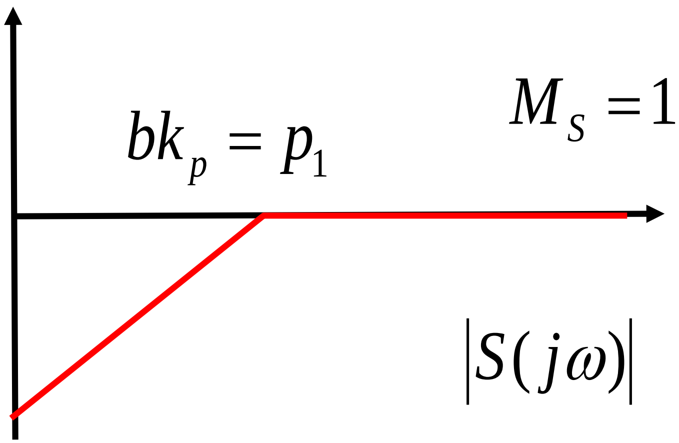


Fig. 12.13

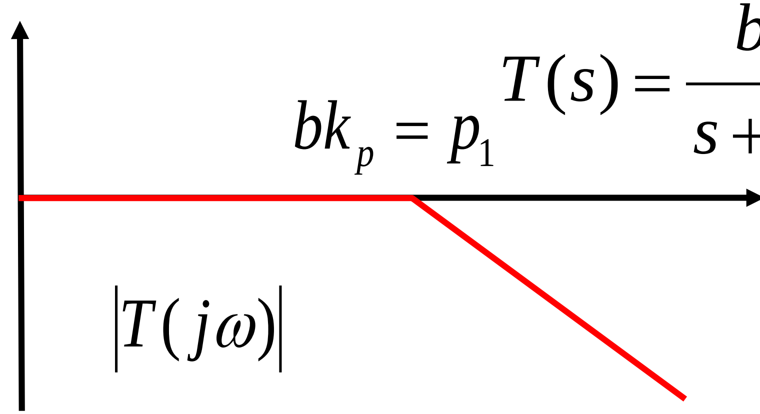
Sensitivity

$$S(s) = \frac{s(s+a)}{(s+p_1)(s+p_2)} \quad \xrightarrow{p_2 = a} \quad S(s) = \frac{s}{s+bk_p}$$





$$T(s) = \frac{bk_p}{s+bk_p}$$



Choose one closed loop pole equal to the process pole

5th Lecture

12 Robust Performance

12.4 Robust Pole Placement: Examples

Example 12.8 (Slow Stable Process Zeros)

(pp. 362--364)

Example 12.9 (Fast Stable Process Poles)

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