Analysis and Design of Linear Control System –Part2-

Instructor: Prof. Masayuki Fujita

6th Lecture

10 PID Control (pp. 293--313)

Keyword : PID Control, Ziegler–Nichols' Tuning

11 Frequency Domain Design

11.4 Feedback Design via Loop Shaping (pp. 326--331)

Keyword : Lead and Lag Compensation

Basic Feedback Loop

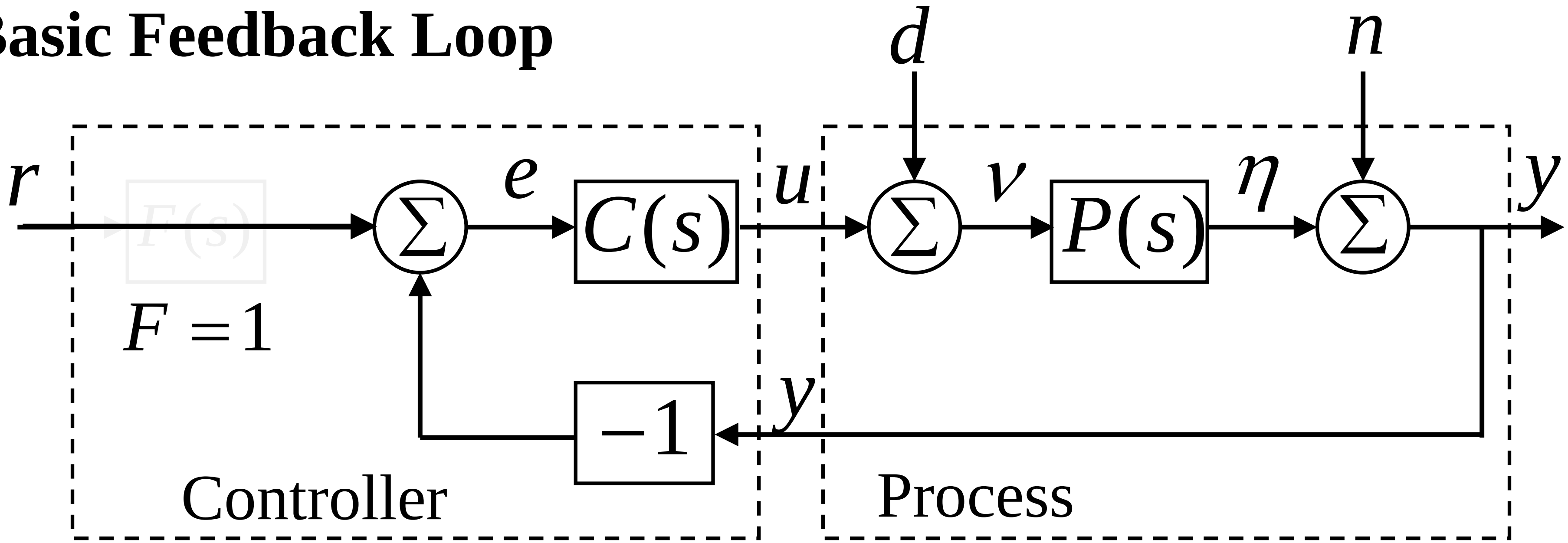


Fig. 11.1

$P(s)$: Process $C(s)$: Feedback block

($F(s)$: Feedforward block)

r : Reference signal

d : Load disturbance

n : Measurement noise

e : Output error

u : Control variable

η : Process output

y : Measured signal

PID Control

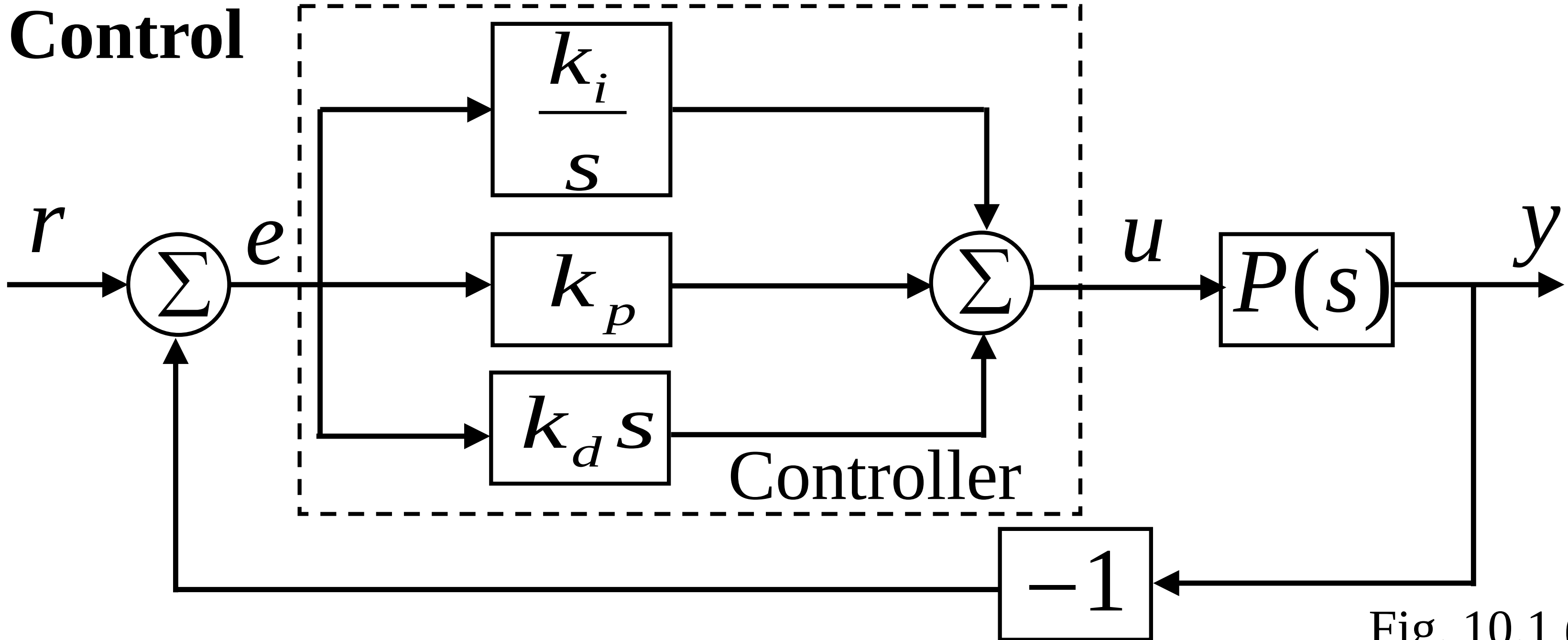


Fig. 10.1 (a)

$$e = r - y$$

$$u = k_p e + k_i \int_0^t e(\tau) d\tau + k_d \frac{de}{dt} = k_p \left(e + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de}{dt} \right) \quad (10.1)$$

k_p : proportional gain

k_i : integral gain

k_d : derivative gain

T_i : integral time

T_d : derivative time

$$C(s) = k_p + \frac{k_i}{s} + k_d s \quad (10.4)$$

PID Control

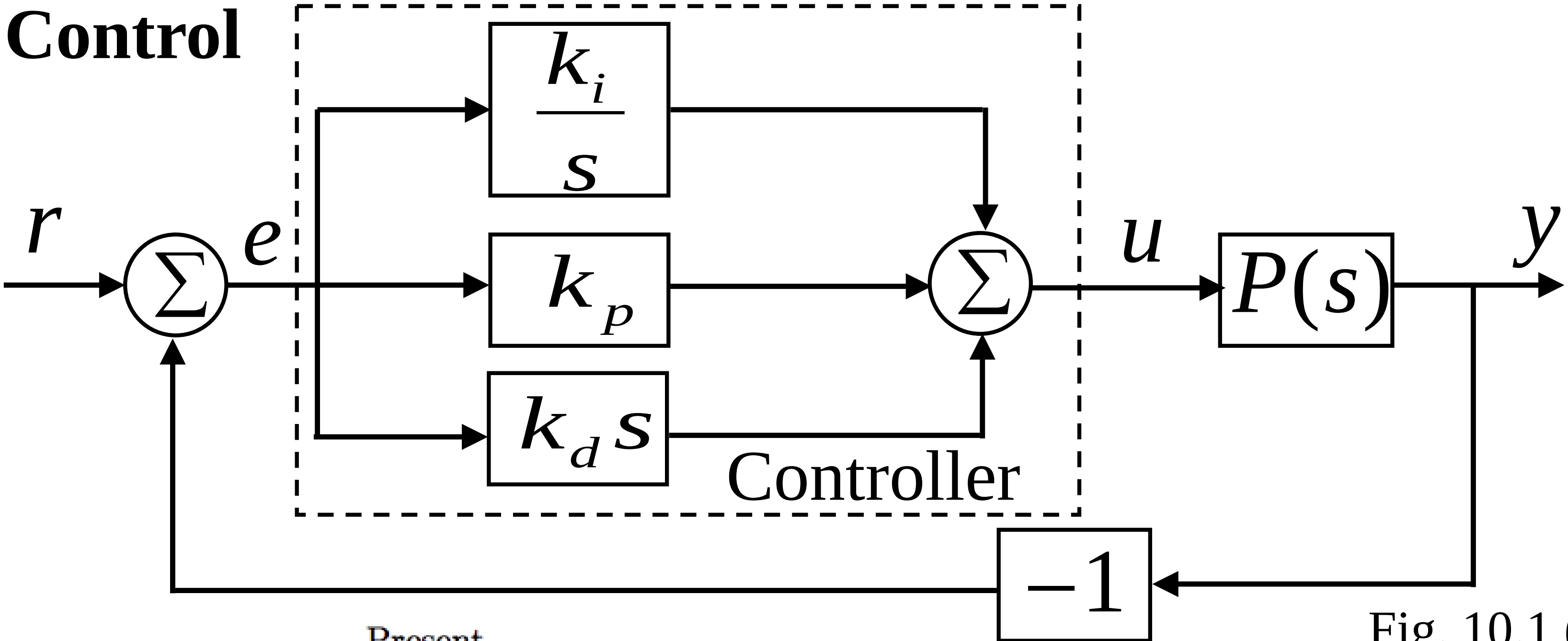


Fig. 10.1 (a)

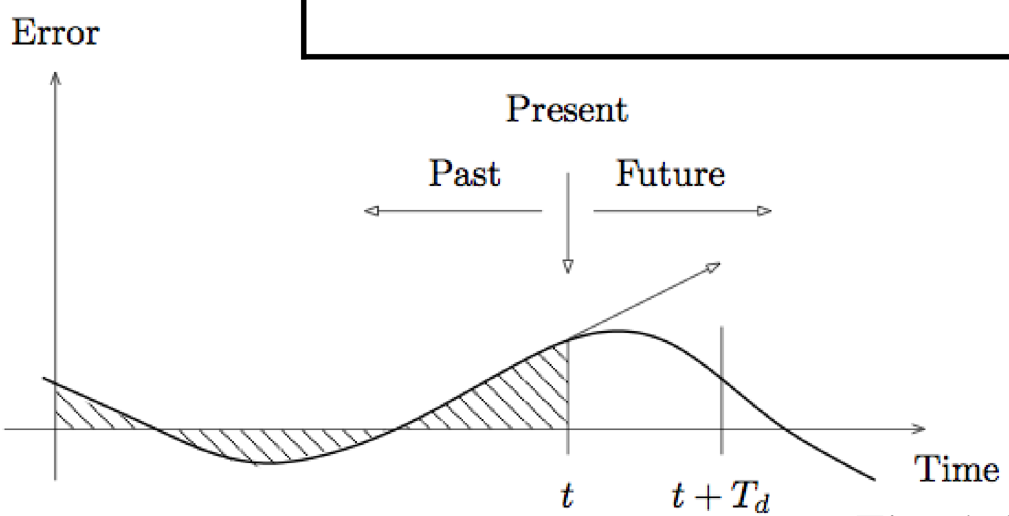


Fig. 1.17

Past	→	integral
Present	→	proportional
Future	→	derivative

Based on a survey of over eleven thousand controllers in the refining, chemicals and pulp and paper industries, 97% of regulatory controllers utilize PID feedback. [L. Desborough and R. Miller, 2002]

Response of the process output to a unit step

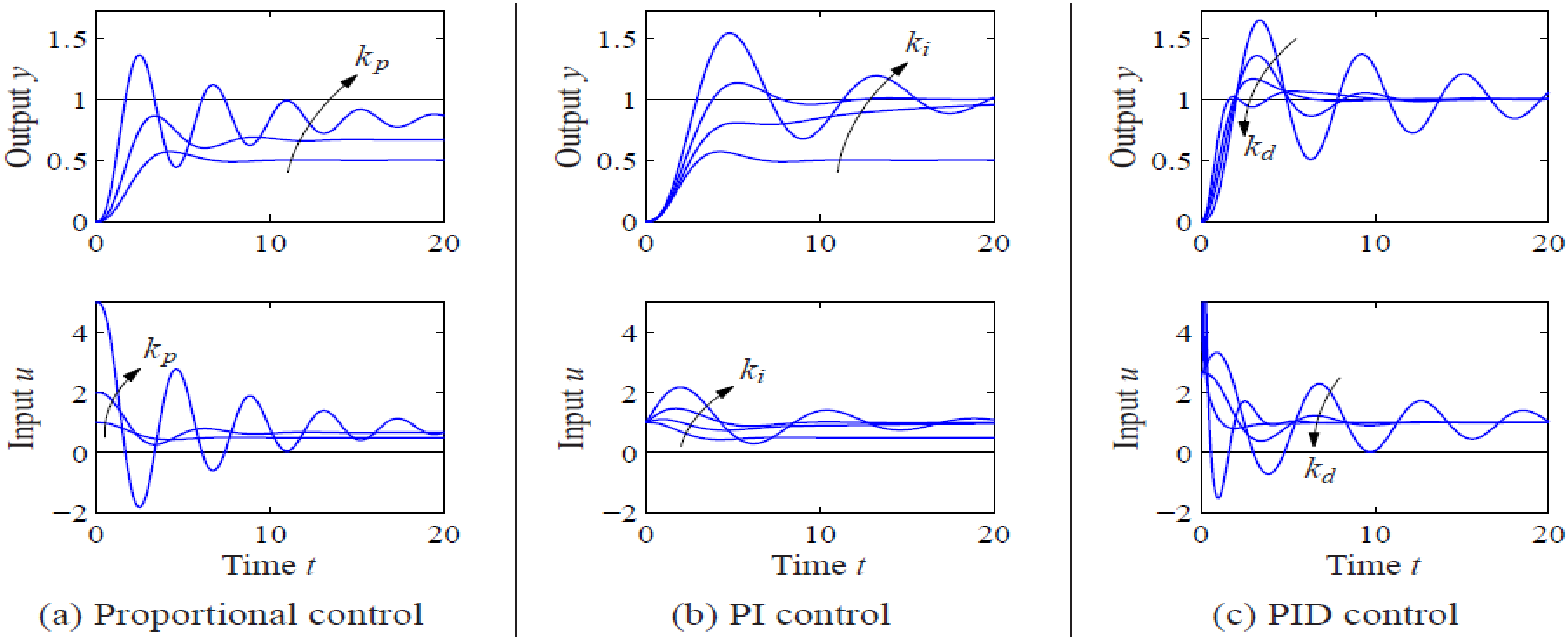


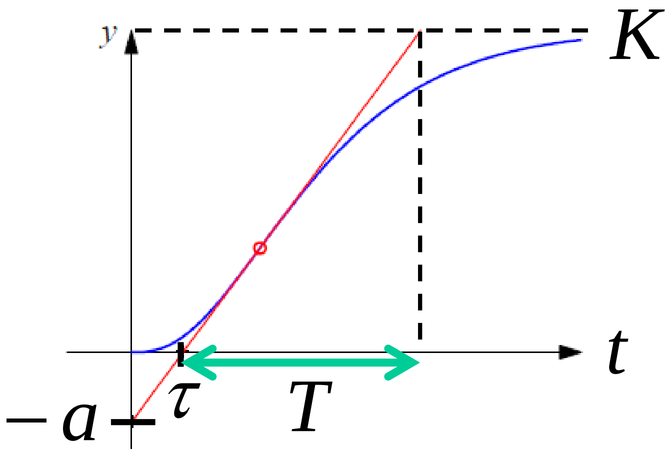
Fig. 10.2

proportional gain k_p increases
error decreases
but
more oscillatory

integral gain k_i increases
steady-state error
is eliminated
but
more oscillatory

derivative gain k_d increases
more damped

Ziegler – Nichols' Tuning (1942) (Step response method)



(a) Step response method

Fig. 10.7 (a)

τ : time delay

a/τ : steepest slope

$$u = k_p\left(e + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de}{dt}\right) \quad (10.1)$$

Type	k_p	T_i	T_d
P	$1/a$		
PI	$0.9/a$	3τ	
PID	$1.2/a$	2τ	0.5τ

(a) Step response method

Tab. 10.1 (a)

From a unit step response

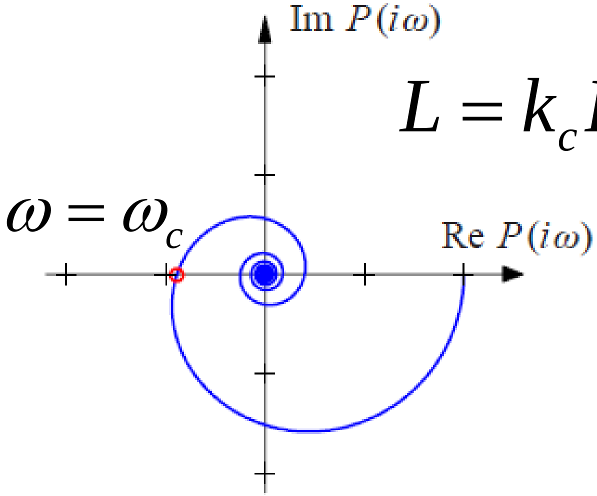
$$P(s) = \frac{K}{1+sT}e^{-\tau s} \quad (10.10)$$

K : steady state value

K/T : slope

$$a/\tau = K/T \rightarrow a = K\tau/T$$

Ziegler – Nichols' Tuning (Frequency response method)



(b) Frequency response method

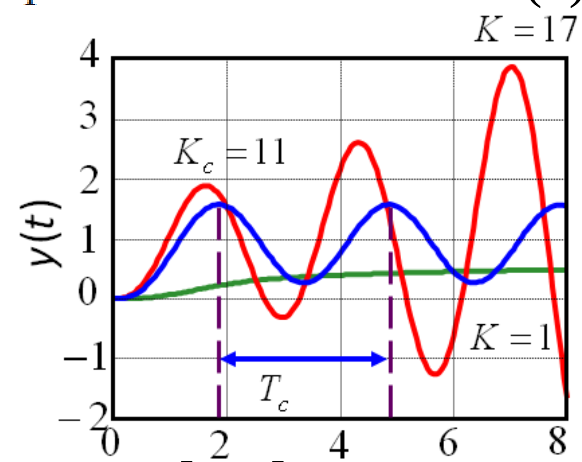
Fig. 10.7 (b)

$$L = k_c P(s)$$

$$u = k_p \left(e + \frac{1}{T_i} \int_0^t e(\tau) d\tau + T_d \frac{de}{dt} \right) \tag{10.1}$$

Type	k_p	T_i	T_d
P	$0.5k_c$		
PI	$0.4k_c$	$0.8T_c$	
PID	$0.6k_c$	$0.5T_c$	$0.125T_c$

(b) Frequency response method   Tab. 10.1 (b)



Step1 Integral and derivative gains are set to zero

Step2 Increase proportional gain k_c until the system starts to oscillate

Step3 From fig. 10.7(b) ω_c is the frequency which $L = k_c P(s)$ intersects the critical point → $T_c = 2\pi / \omega_c$

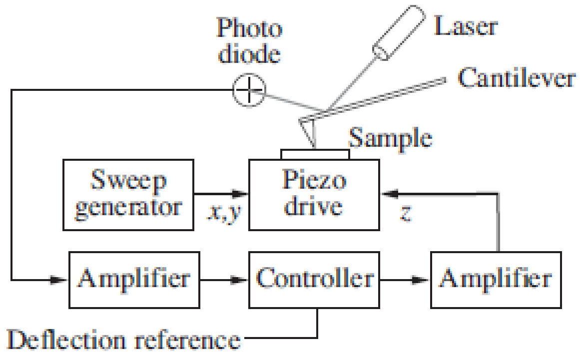
Relay Feedback

[Ex. 10.4] Atomic force microscope in tapping mode

$$P(s) = \frac{1 - e^{-sT_n}}{sT_n(s+1)}$$

$$T_n = 2n\pi\zeta \simeq 0.251$$

$$(\zeta = 0.002, n = 20)$$



(a) Schematic diagram

Fig. 3.14 (a)

Ziegler – Nichols' Tuning

$$k_c = 21.7 \quad T_c = 0.48$$

PI Controller

$$k_p = 0.4k_c = 8.68$$

$$T_i = 0.8T_c = 0.384$$

$$\Rightarrow k_i = \frac{k_p}{T_i} = 23.1$$

Many versions

$$u = k_p\left(e + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de}{dt}\right) \quad (10.1)$$

Type	k_p	T_i	T_d
P	$0.5k_c$		
PI	$0.4k_c$	$0.8T_c$	
PID	$0.6k_c$	$0.5T_c$	$0.125T_c$

(b) Frequency response method Tab. 10.1 (b)

- Nathaniel B. Nichols (1914-1997) -

- Born in Michigan
- B.S degree from Central Michigan University in1936
- M.S degree from the University of Michigan in1937
- An Honorary Doctor of Science degree from Central Michigan University in1964
- An Honorary Doctor of Science degree from Case Western Reserve University in1968

Professional

Automatic control, automatic radar tracking

Fire control computers, power-driven servomechanisms

Industrial process controllers ,spacecraft attitude controls

Well known for

Ziegler 数学が大の苦手で，正弦波は私の数学能力をはるかに超えている

- Ziegler and Nichols PID tuning
- Nichols chart (created to facilitate the calculation of Closed-loop frequency response in Aerospace Corporation)

Windup

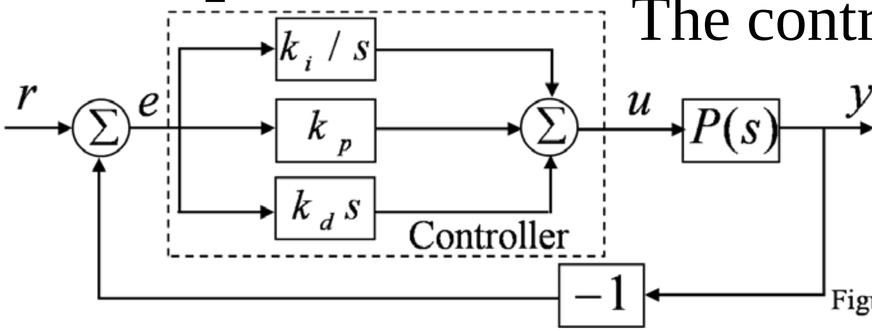


Figure 10.1 (a)

The control variable reaches the actuator limits

→ Output is saturated

→ The integral term and the controller output become very large (windup)

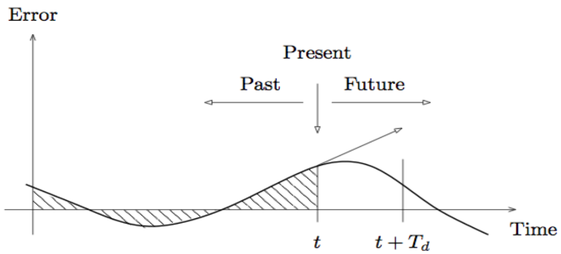
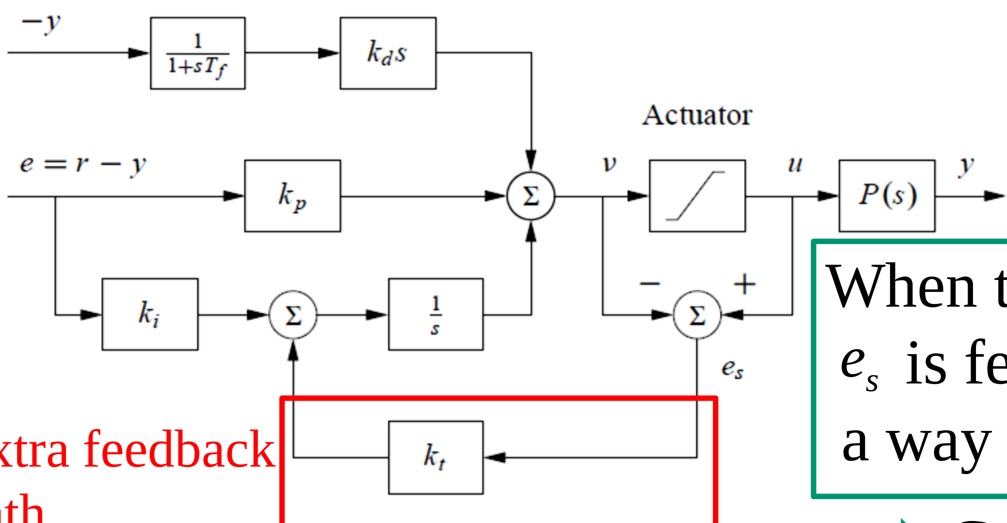


Fig. 1.17

Anti-Windup



Extra feedback path

Fig. 10.11

Add extra feedback path

e_s : error signal

$e_s = 0$ → No saturation

When the actuator saturates, the signal e_s is fed back to the integrator in such a way that e_s goes toward zero

→ Controller output is kept close to the saturation limit

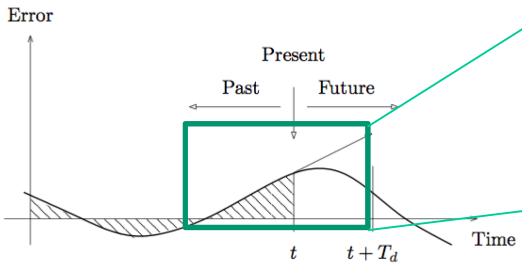
Implementation

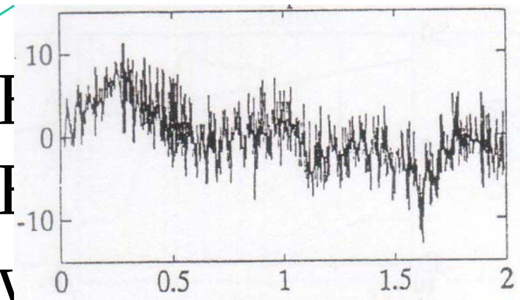
Filtering the Derivative

Ideal derivative → I… ı-frequency sig.

→ I… ıeasurement noise

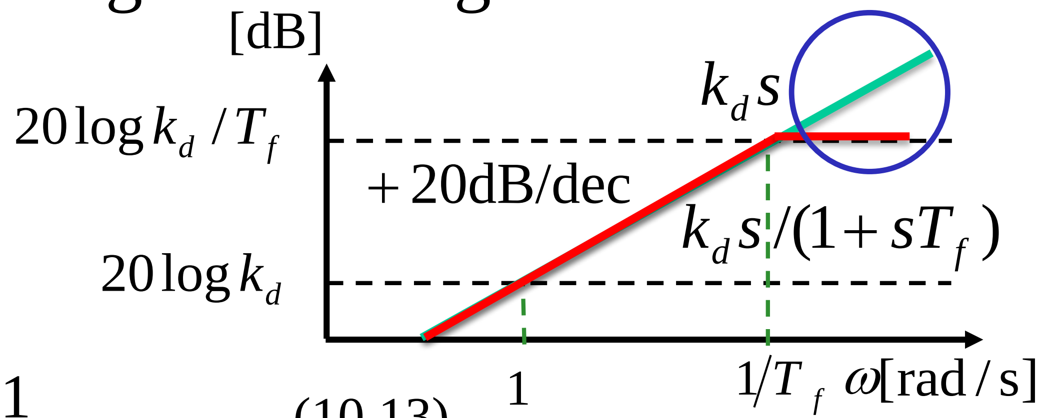
… e variations





$k_d s \rightarrow k_d s/(1+sT_f)$

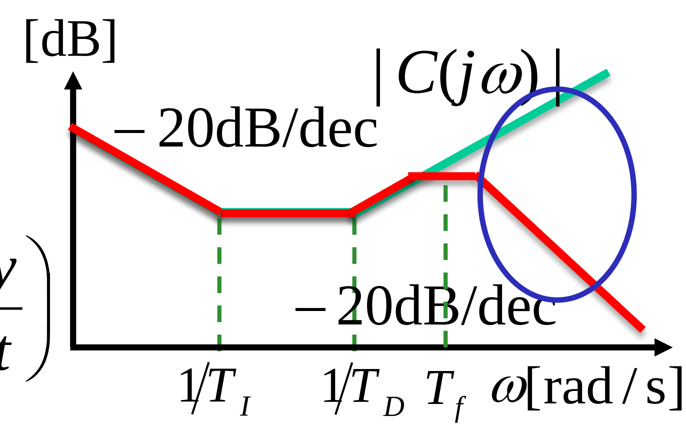
Low-pass filter



Filtering ideal controller

$$C(s) = k_p\left(1+\frac{1}{sT_i}+T_d s\right)\frac{1}{1+sT_f+(sT_f)^2/2} \qquad (10.13)$$

Low-pass filter



Weighting

peak can be avoided

$$u = k_p(\beta r - y) + k_i\int_0^t (r(\tau)-y(\tau))d\tau + k_d\left(\gamma\frac{dr}{dt}-\frac{dy}{dt}\right)$$

β : reference weight $\quad \gamma$: setpoint weight

Computer Implementation

$$u = k_p(\beta r - y) + k_i \int_0^t (r(\tau) - y(\tau))d\tau + k_d\left(-\frac{dy}{dt}\right)\frac{1}{(1+sT_f)}$$

$$k_p(\beta r - y)$$

$$\Rightarrow \quad P(t_k) = k_p(\beta r(t_k) - y(t_k))$$

$$k_i \int_0^t (r(\tau) - y(\tau))d\tau$$

$$\Rightarrow \quad I(t_{k+1}) = I(t_k) + k_i h e(t_k) + \frac{h}{T_t}(sat(v) - v)$$

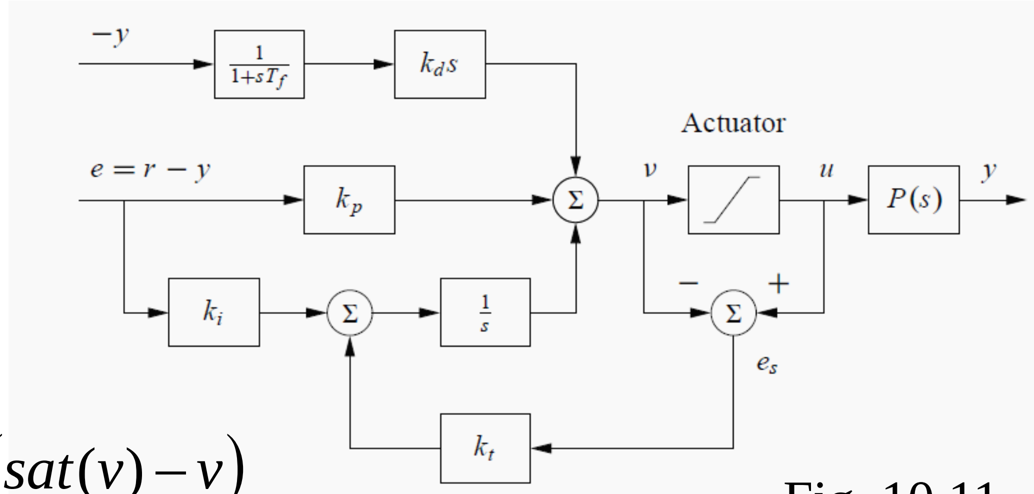


Fig. 10.11

h : sampling time $\quad T_t = h/k_t$: anti windup term

$$k_d\left(-\frac{dy}{dt}\right)\frac{1}{(1+sT_f)}$$

$$\Rightarrow \quad D(t_k) = \frac{T_f}{T_f + h}D(t_{k-1}) - \frac{k_d}{T_f + h}(y(t_k) - y(t_{k-1}))$$

(from backward difference)

Time – Delay, Smith method

Feedback Design via Loop Shaping

Basic Feedback Loop

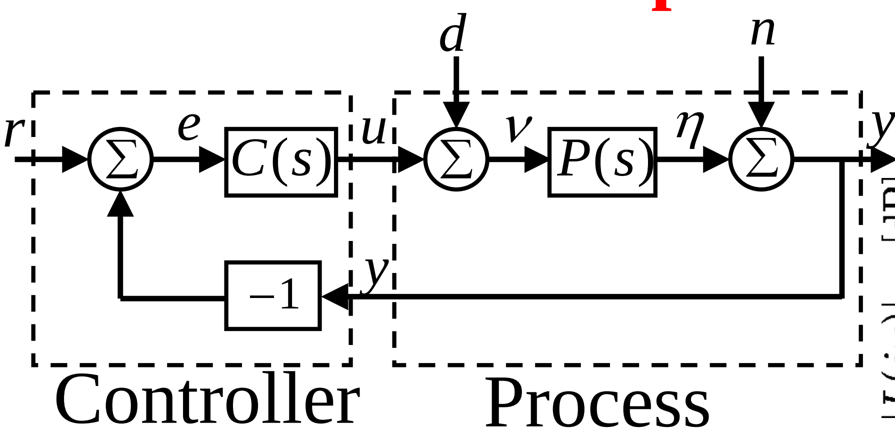


Fig. 11.1

PID Controller

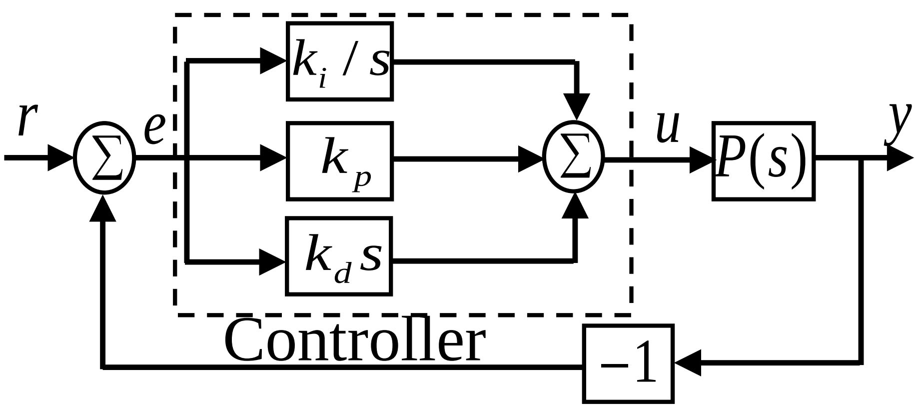
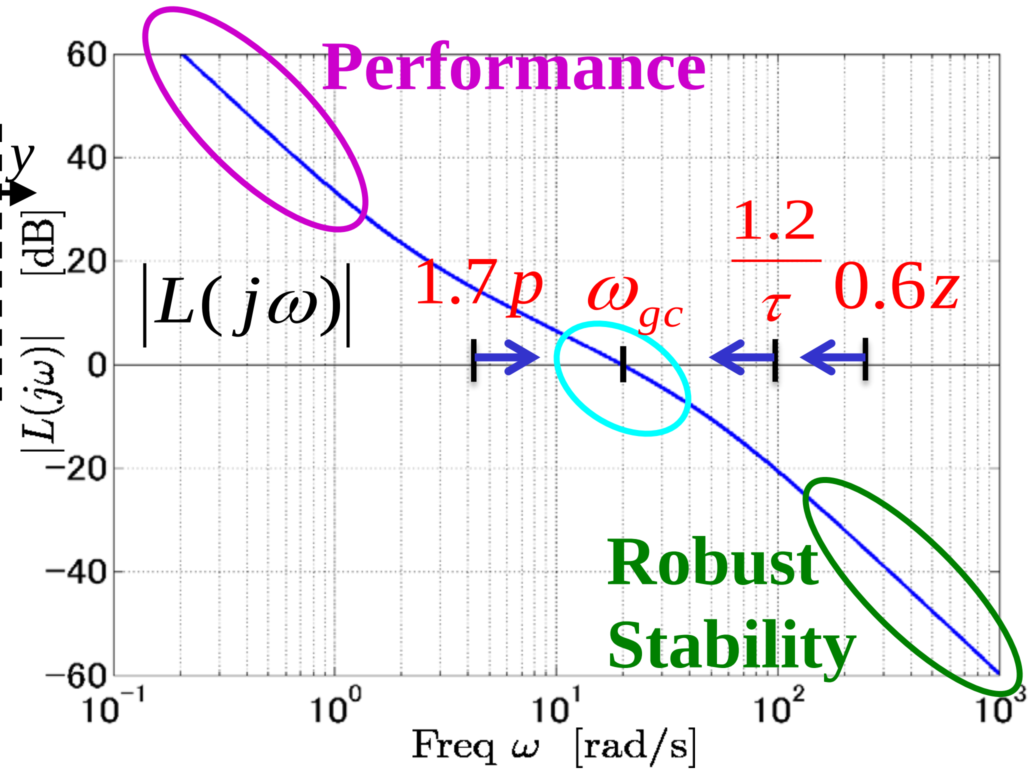


Fig.10.1 (a)



(a) Frequency response ($L(s)$)

Fig. 11.8

$$g_m = 2 - 5$$

$$\varphi_m = 30^\circ - 60^\circ$$

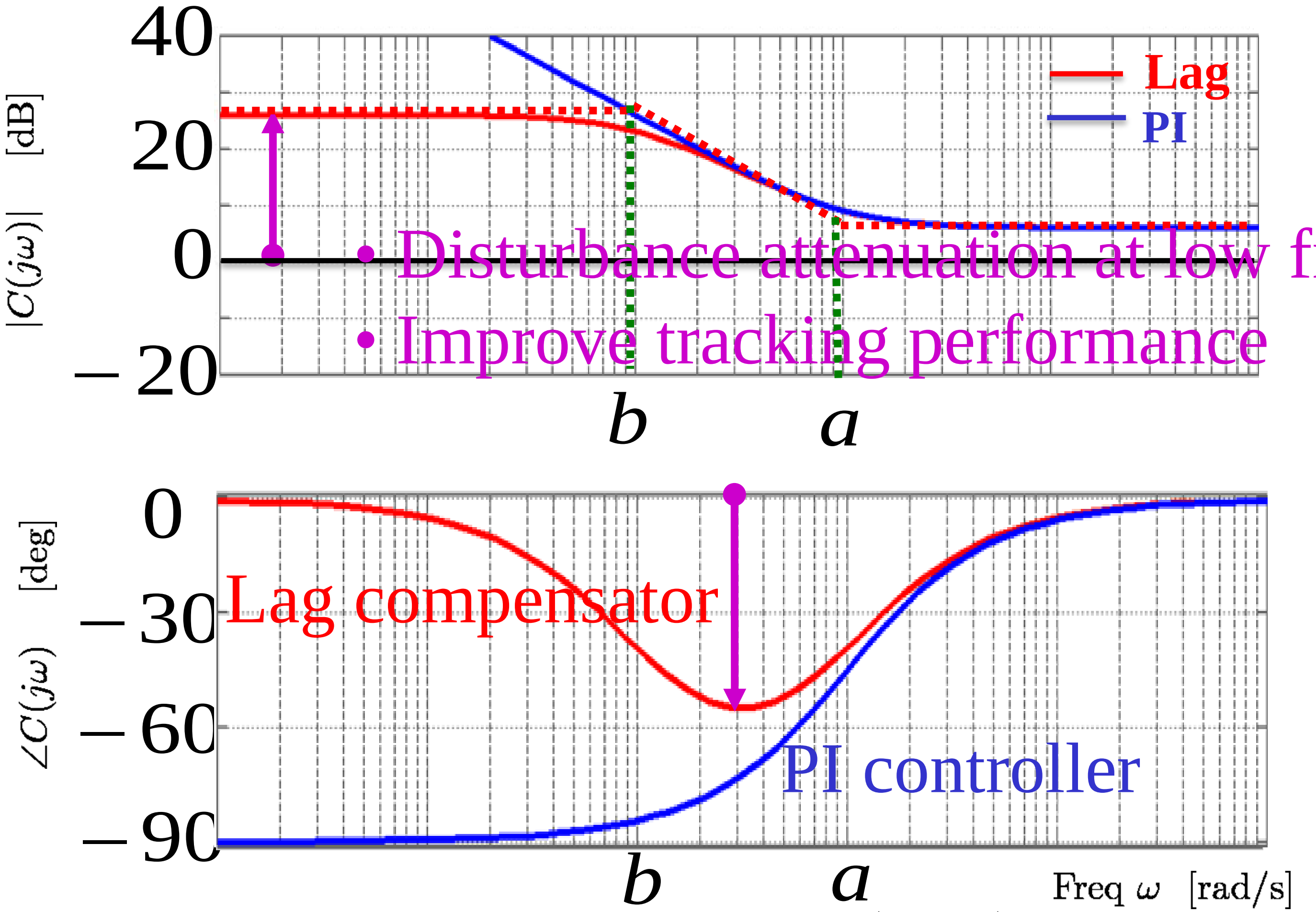
$$s_m = 0.5 - 0.8$$

Lag Compensator and PI Controller

$$C(s) = k\frac{s+a}{s+b} \qquad (11.12)$$

pole: $-b$, zero: $-a$
corner freq.: a, b
(break point)

$b < a$: **Lag compensator** $\qquad b = 0 \rightarrow$ **PI controller**



Frequency response ($C(s)$) Fig. 11.9 (b)

Lead Compensator and PD Controller

$$C(s) = k\frac{s+a}{s+b} \qquad (11.12)$$

pole: $-b$, zero: $-a$
corner freq.: a, b
(break point)

$a < b$: **Lead compensator** $\qquad a = 0 \rightarrow$ **PD controller**

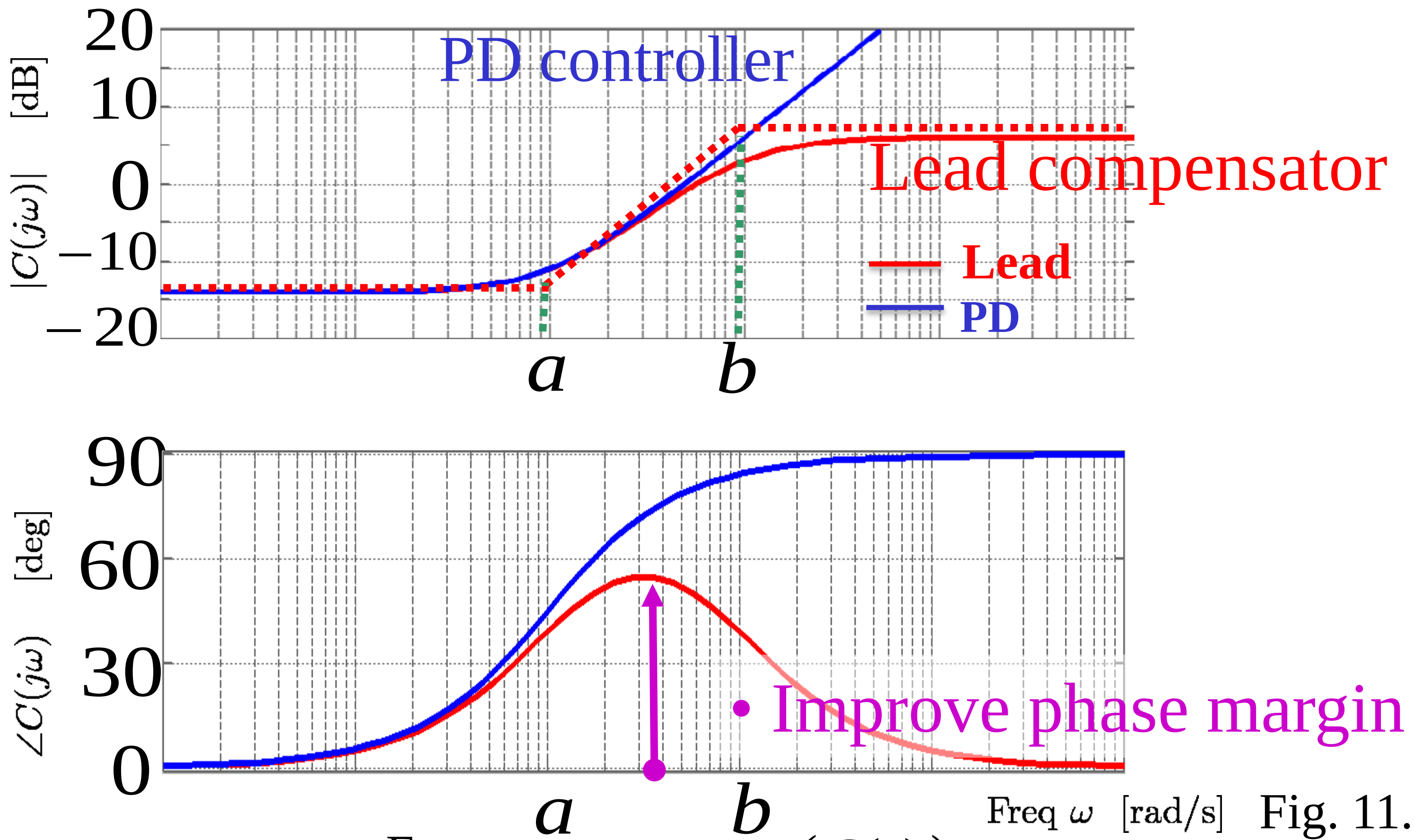


Fig. 11.9 (a)

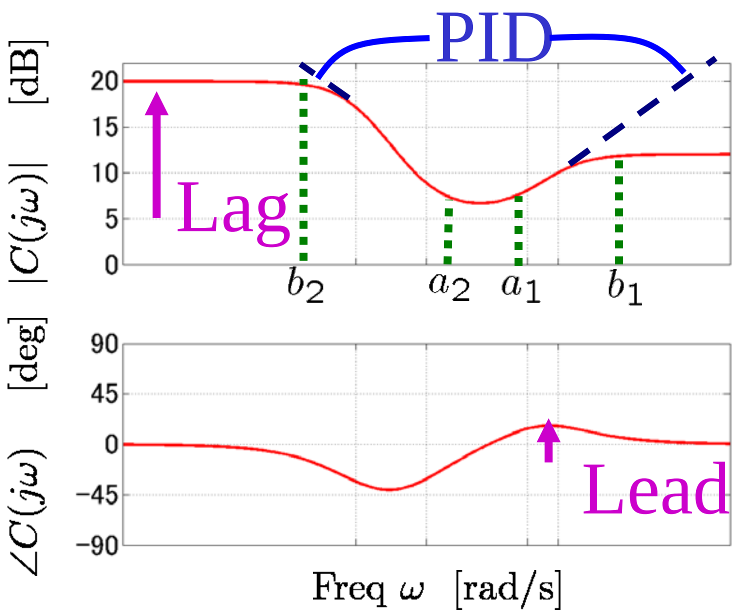
Frequency response ($C(s)$)

Lead and Lag Compensator

$$C(s) = k\left(\frac{s+a_1}{s+b_1}\right)\left(\frac{s+a_2}{s+b_2}\right)$$

$$a_1 < b_1$$
$$a_2 > b_2$$

Lead compensator + Lag compensator



Frequency response ($C(s)$)

- Conditional Stability
- Integral Windup
- Derivative Factor, etc.

Derivative has a high gain for high freq. signals → Filtering

$$k_d s \rightarrow \frac{k_d s}{1 + sT_f}$$

$$T_f = \frac{k_d / k}{N}, \quad N \approx 2 - 20$$

Alternative: (Ideal Controller) + (Noise Filter)

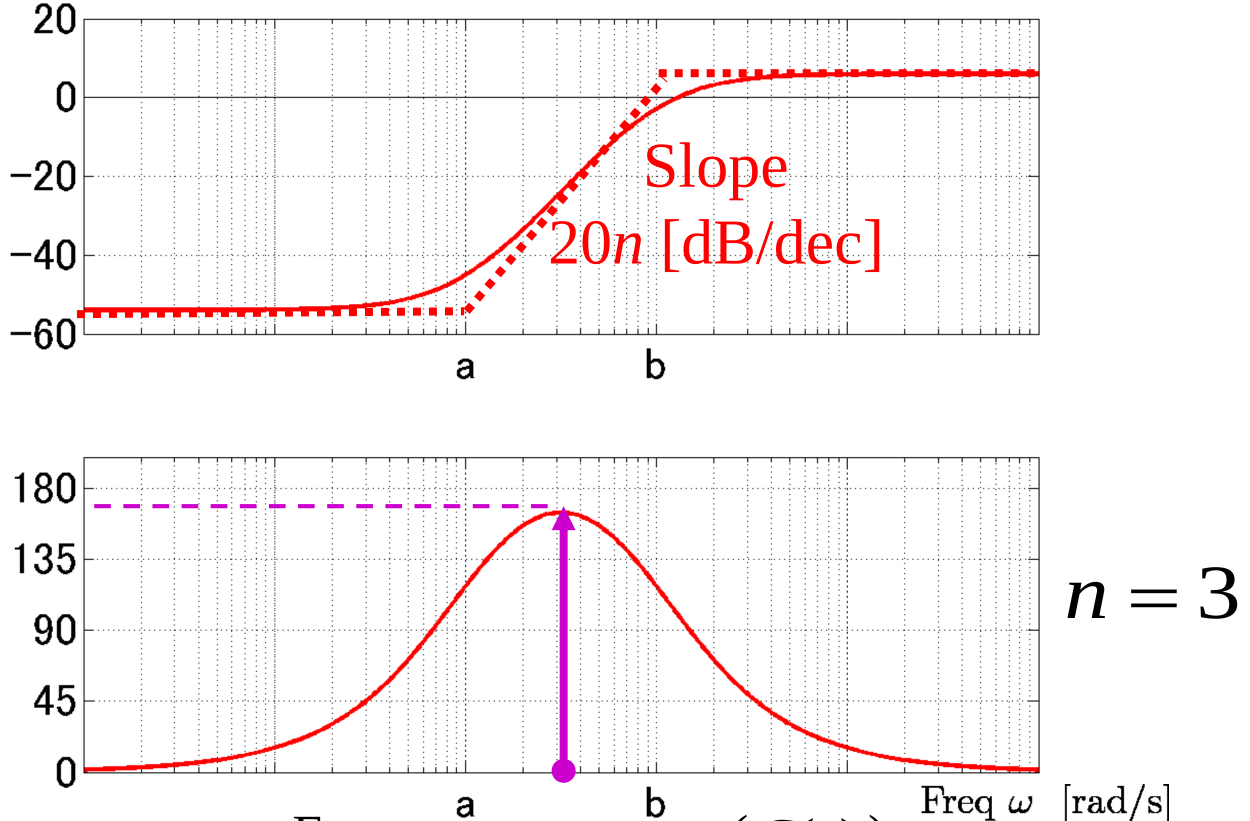
e.g. $$C(s) = \left(k_p + \frac{k_i}{s} + k_d s\right)\frac{1}{T_f^2 / 2s^2 + T_f s + 1}$$

Lead compensation

Higher-order lead compensator

$$C(s) = k\frac{(s+a)^n}{(s+b)^n} \qquad a < b$$

higher-order lead compensator can provide larger phase lead than 90°



Frequency response ($C(s)$)

[Ex. 11.5] Atomic force microscope in tapping mode

(§ 3.5, Exe. 9.2)

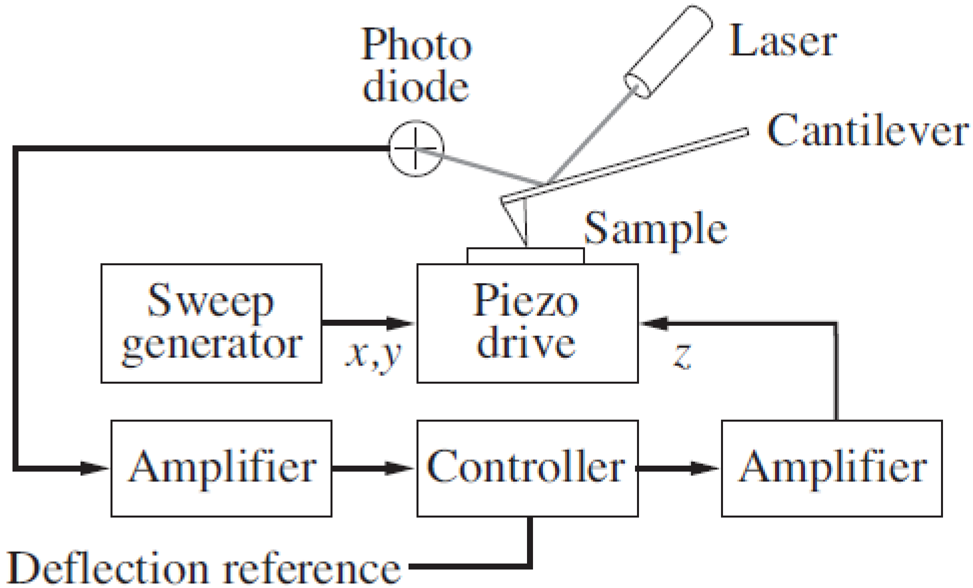


Fig. 3.14 (a) Schematic diagram (b) AFM image of DNA

system dynamics (dynamics of the vertical motion)

$$P(s) = \frac{a\left(1 - e^{-s\tau}\right)}{s\tau(s+a)}$$

$$a = \zeta\omega_0 = 1 \qquad \tau = 2n\pi / \omega_0 = 0.25$$

ζ : damping ratio ω_0 : undamped natural frequency

[Ex. 11.5] Atomic force microscope in tapping mode

System dynamics

$$P(s) = \frac{a\left(1 - e^{-s\tau}\right)}{s\tau(s+a)}$$

Integral controller

$$C_i(s) = \frac{k_i}{s} \quad k_i = 8.3$$

Phase margin: $\varphi_m = 0^\circ$

PI controller

$$C_{pi}(s) = \frac{k_p s + k_i}{s} \quad k_p = 3.5$$

- disturbance attenuation (by k_i / s^2)
- improve phase margin (by k_p)

$$\varphi_m = 45^\circ$$

- bound on the crossover frequency

$$\omega_{gc} = 4[\text{rad/s}] < \frac{1.2}{\tau} = \frac{1.2}{0.25} = 4.8[\text{rad/s}]$$

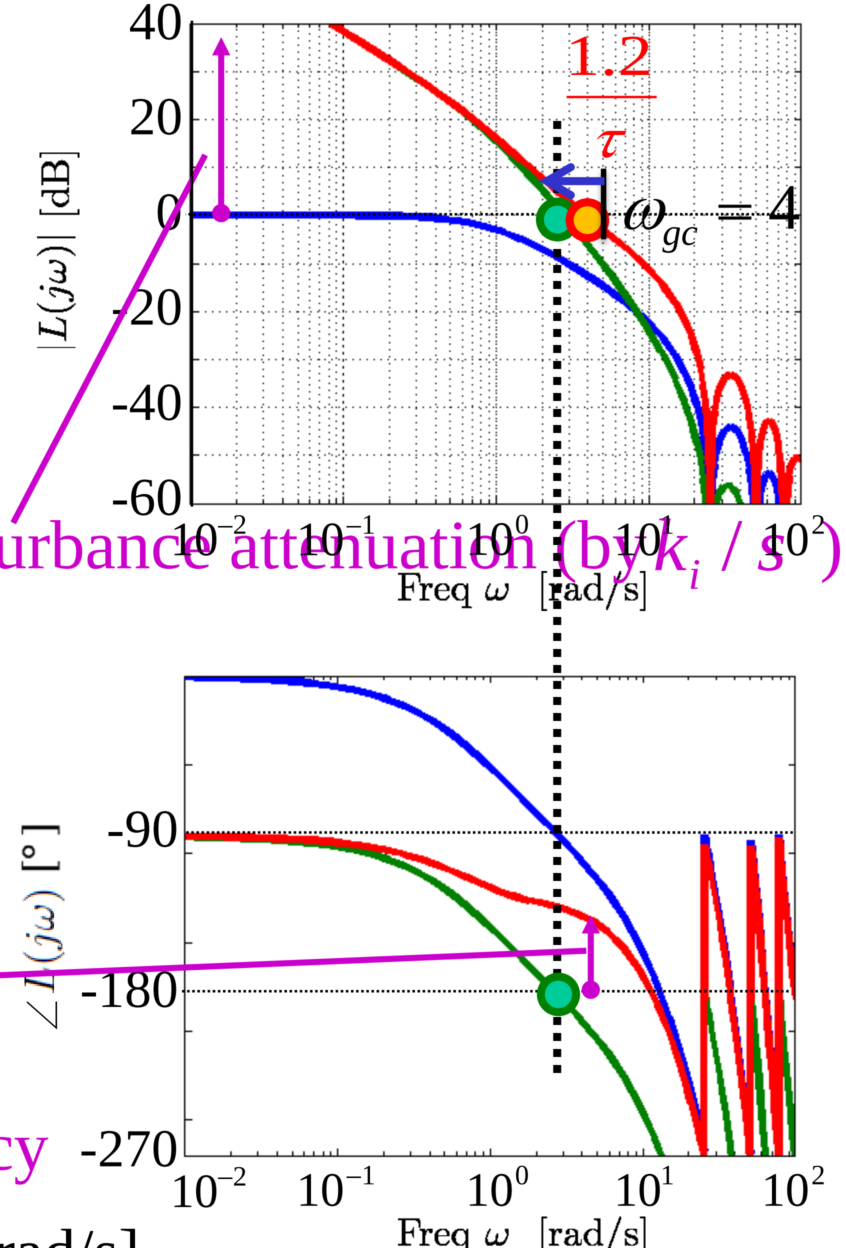


Fig. 11.10 (a) Loop shaping

[Ex. 11.5] Atomic force microscope in tapping mode

System dynamics

$$P(s) = \frac{a\left(1 - e^{-s\tau}\right)}{s\tau(s+a)}$$

PI controller

$$C_{pi}(s) = \frac{k_p s + k_i}{s}$$

$$k_i = 8.3 \qquad k_p = 3.5$$

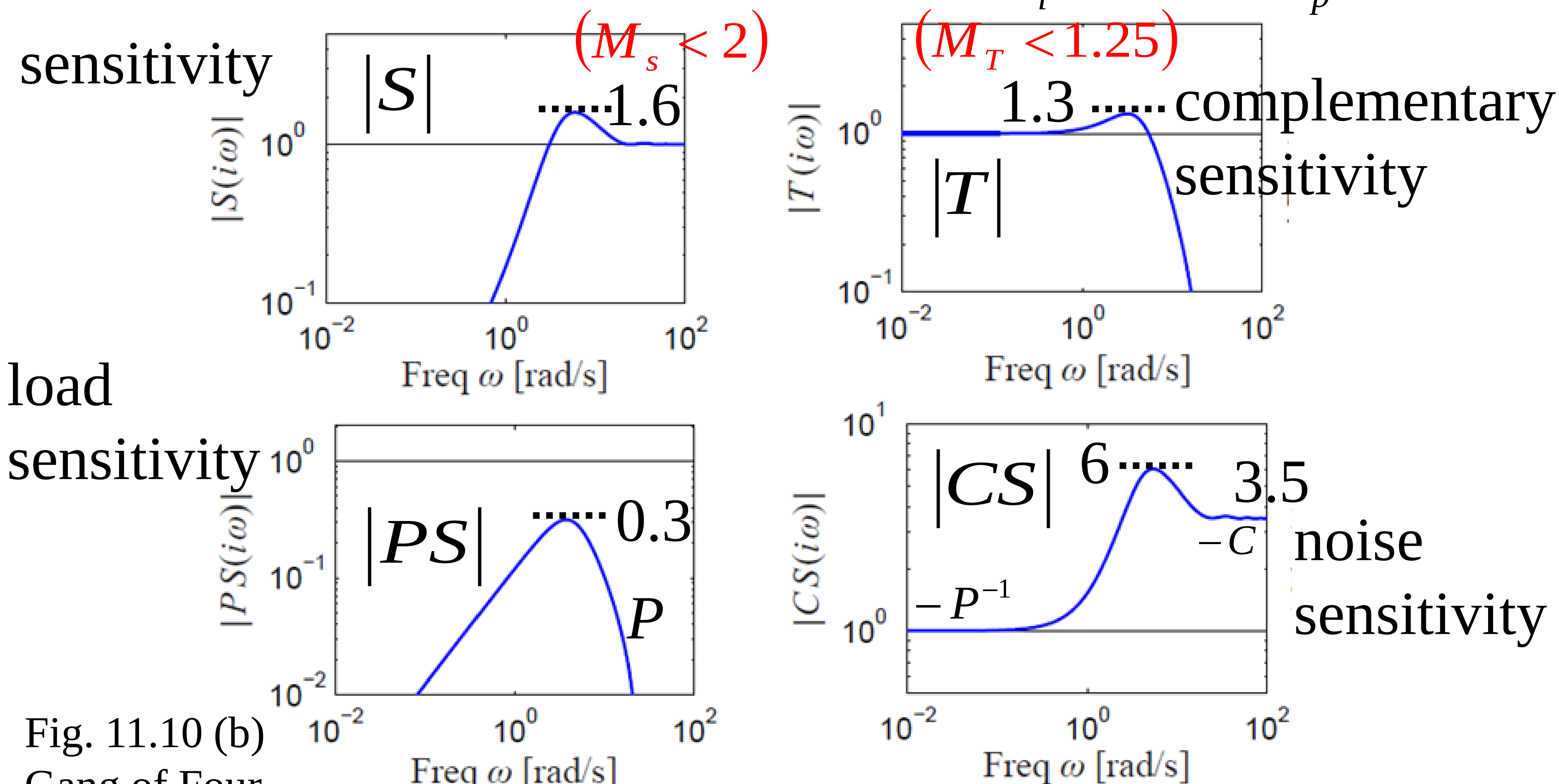


Fig. 11.10 (b)
Gang of Four

[Ex. 11.5] Atomic force microscope in tapping mode

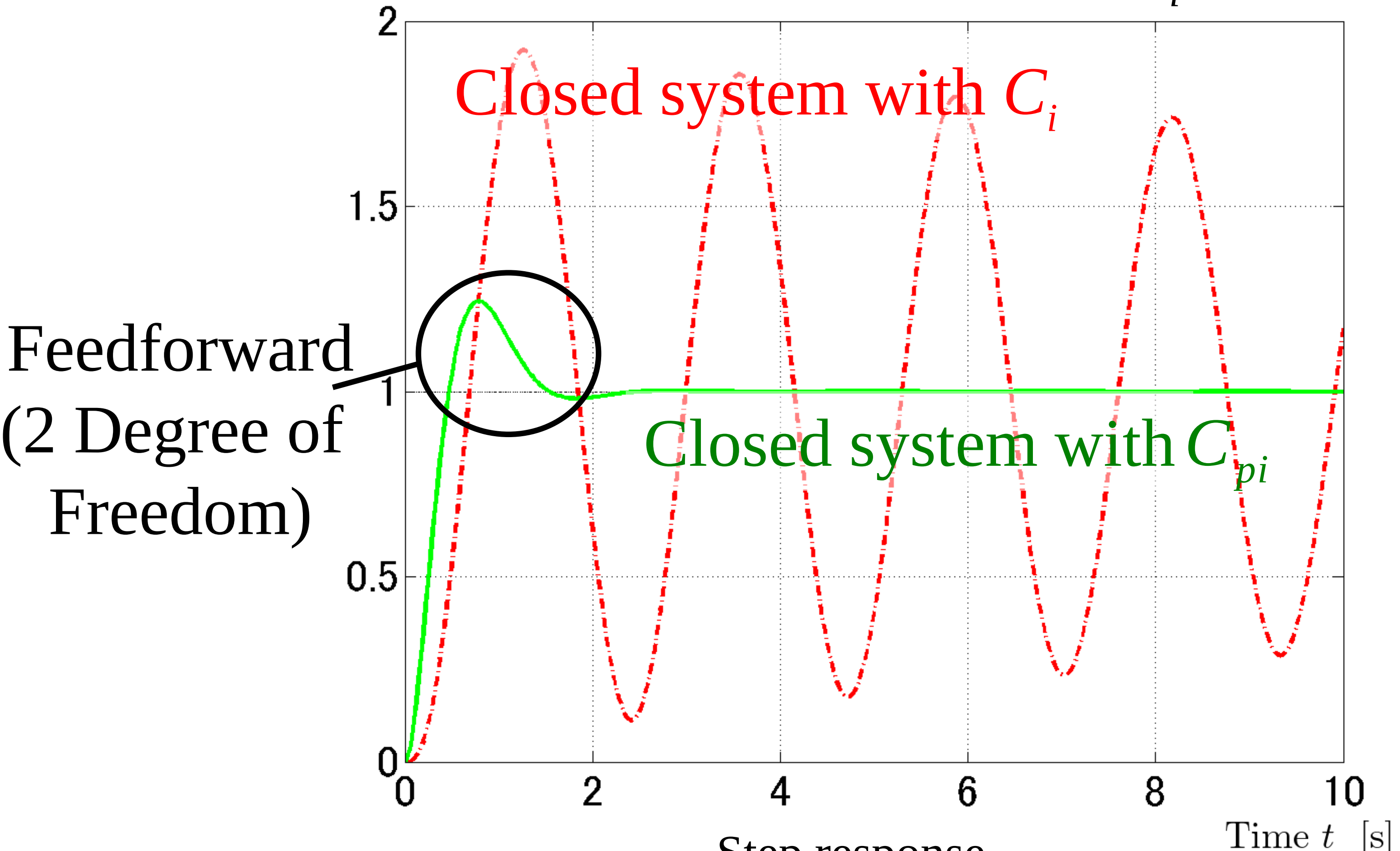
Integral controller

$$C_i(s) = \frac{k_i}{s}$$

PI controller

$$C_{pi}(s) = \frac{k_p s + k_i}{s}$$

$$k_i = 8.3 \quad k_p = 3.5$$



Step response

6th Lecture

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